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A practical approach to open-pit mine planning under price uncertainty using information gap decision theory

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Abstract
In the context of open-pit mine planning, uncertainties including commodity price would significantly affect the technical and financial aspects of mining projects. A mine planning that takes place regardless of the uncertainty in price just develops an optimized plan at the starting time of the mining operation. Given the price change over the life of mine, which is quite certain, optimality of the proposed plan will be eliminated. This paper presents a risk-averse decision-making tool to help mine planners in mining activities under price uncertainty. The objective is to propose mine planning in a way that a target Net Present Value (NPV) is guaranteed. In order to reach this goal, Information Gap Decision Theory (IGDT) is developed to hedge the mining project against the risk imposed by the information gap between the forecasted and actual price. The proposed approach is of low sensitivity to the price change over the life of mine, and can use the estimated prices with uncertainty. A case study at an existing iron mine demonstrates the performance of the proposed approach. The results obtained showed that the proposed method could provide a robust solution to mine planning under price uncertainty. Moreover, it was concluded that the method could present more reliable mine plans under condition of price uncertainty.

Keywords: *IGDT, Open-Pit, Mine Planning, Uncertainty Modelling, Price Uncertainty.*

1. Introduction

An open pit mine design aims to generate the mine optimal pit limits and extraction sequences named pushbacks or phases, which are generated using the optimization methods [1]. Mine production planning accomplished by a production schedule recognizing blocks to be mined over a number of periods is subjected to the mining and operating constraints. This is carried out to make decisions on how the extraction process should be performed to obtain the best outcomes defined by the management intentions [2]. These usually include maximizing the financial value of the mining project as well as meeting the defined expectations in terms of ore tonnage and ore quality characteristics to be delivered [3]. Production planning for open-pit mines is a critical issue in mine planning, and can have a huge impact on the economic value of a mining operation. Even each percent increase in the efficiency of the exploitation scheme can meaningfully change the profitability of an operation [4, 5].

Due to the uncertainty of ore price, and consequently, uncertainty in block economic value, there is a probability of a block being incorrectly identified as an ore block when it may in fact be a waste block, and vice versa. The assumption of certain block values may result in unrealistic mine production planning, significant economic losses, and a probable failure of a project [6-8]. Sensitivity analyses correspondingly illustrate that the ore price is the most sensitive factor that affects the ore block economic value [9, 10]. Over the recent years, considerable efforts have been made to integrate geological uncertainty into mine planning [11, 12]. A standard tool used to model ore grade uncertainty is the use of conditional simulations of orebodies

[13, 3, 14]. A majority of the discussed methods consider the geological (ore grade of mining blocks) uncertainty but consider the price to be constant over the life of the mining operation. Some research works have considered price uncertainty [15, 8] using a mean reverting price or value model [16] for generating price realizations, which considers the present price as a basis for calculation of the future price. The mining industry commodity prices, especially those commodities whose price is listed on open markets, are normally modeled as the average price for the last three years [10]. It is clear that it may prevent the use of optimistic prices, and this can be misleading in mine planning. Dehghani et al. [17] have developed an evaluation method (pyramid technique) based on the multi-dimensional binomial tree method to evaluate mining projects under economic uncertainties. They concluded that applying the economic uncertainties caused the net present value to be calculated more realistically than the certainty conditions. Mokhtarian and Sattarvand [18] have introduced an approach for integration of the commodity price uncertainty into long-term production planning of open-pit mines. The procedure involves solving the problem by the integer programming method based on a series of economic block models that are realized based on the sampled prices from commodity price distribution function using the median Latin hypercube sampling method. Bakhtavar et al. [19] have presented a stochastic model to create an optimal strategy for producing bimetallic deposit open-pit mines under certain and uncertain conditions. The uncertainties in grade, product price, and capacities of the various stages in the process of production of the final product were considered. They showed that the stochastic model had a greater compatibility and performance thn the other ones.

Considering the importance of price uncertainty impacts in mine planning, this paper proposes an IGDT-based approach for open-pit mine planning to deal with commodity price uncertainty. Uncertainty models using IGDT have been used in studies such as engineering control theory, environmental studies, electrical engineering, mechanical reliability, and water resource management [20-25].

An application at an existing open-pit mine shows the practical aspects of the approach over conventional methods as well as the helpfulness of the proposed approach in solving mine planning and design problems under uncertainties.

In the following sections, the paper presents: (i) an introduction to IGDT; (ii) a deterministic mathematical formulation of the model including the objective function and constraints; (iii) robust counterpart of the deterministic formulation using the IGDT-based method and the solution procedure; (iv) application of this procedure to an iron mining operation; and then (v) discussion and conclusions follow.

2. Information gap decision theory (IGDT)

IGDT seeks to assist decision-making under uncertainty. This theory seeks to provide a framework for rational decision-making in situations of severe uncertainty, and proposes non-probabilistic models of uncertainty and requires relatively small information inputs when compared to the alternative theories of uncertainty. The IGDT method maximizes the uncertainty horizons (as shown in Figure 1) and finds a solution that guarantees a certain expectation for the objective [26]. The IGDT method essentially relies on the gap between the actual and forecasted values of uncertain variables. The uncertainty model in this method does not hold any assumption on the probability distributions, which make it suitable in the cases with a high level of uncertainty or a lack of sufficient historical data [27].

The IGDT model can be described using three elements, namely a system model, an uncertainty model, and a performance requirement [26]. The info-gap decision theory is based upon the following elements [28]:

1- A decision space Q that includes a number of alternative decisions, actions or choices ($q \in Q$) available to a manager;

2- An uncertainty space S that includes all the uncertain elements of a problem;

3- A reward function R as the system model that measures how successful the decision is. The reward function is a mapping from the domains of the decision space and the uncertainty space to the real numbers $\mathfrak{R}$. In a mine planning application, the reward function is typically NPV of the mining project, and the uncertain element in this work is commodity price. Associated with the reward function is a critical value r_c that the manager specifies and aims to meet or exceed;

4- A non-probabilistic model $U\left(\alpha,\hat{u}\right)$ for the uncertain quantities u in the reward function, parameterized by the non-negative parameter α that measures uncertainty in terms of the disparity between an initial estimate of the uncertain

quantities $\hat{u}$ and the other possible values.
IGDT uses these constructs to identify decisions based on their robustness or opportunity. This work focuses on the robustness of decisions. Robustness function is considered in this work due to its capability for modeling worst-case scenarios. The robustness function models the immunity of the decision against the unfavorable deviations of the uncertain parameter from the forecasted value [29].

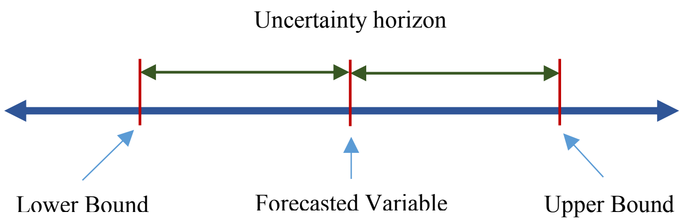

Figure 1. Illustration of uncertainty horizons for uncertain parameters [27].

2.1. System model and robustness function

The input/output structure of the studied system is described in the system model, i.e. $R(q,u)$, considering a decision variable q and the uncertain parameter u. The robustness function, denoted as $\hat{\alpha}(q,r_c)$, defines the robustness of a decision q to be the maximum amount of uncertainty α such that the minimum reward (influenced by uncertain quantities u) associated with the decision, $\min R(q,u)$, is greater than the critical reward r_c. Equation (1) shows the general form of the robustness function in IGDT [26]:

$$\hat{\alpha}(q,r_c) = \max\left\{\alpha : r_c \le \min_{u \in U(\alpha,\tilde{u})} R(q,u)\right\} \qquad (1)$$

This immunity function ensures that the reward (the NPV in this paper) will not be less than a critical value r_c, provided that the uncertain parameters fall within the region of uncertainty. The IGDT robustness function represents the maximum info-gap uncertainty that decision variable q can tolerate with the performance not being worse than r_c [29].

2.2. Uncertainty model

The objective of IGDT is to help the decision-makers in selecting the best plan where best is defined by the criteria, i.e. the decision with the greatest robustness for a given uncertainty model, reward function, and initial estimates of the uncertain elements in this function. The info-gap decision theory prescribes a variety of non-probabilistic models of uncertainty. There are different methods for representing the uncertainty model using IGDT [26]. A more common uncertainty model in IGDT is the fractional uncertainty model. This uncertainty model implies that the length of horizon of uncertainty is proportional to the forecasted value of the uncertain parameter [29]. The fractional uncertainty model can be represented as follows [28]:

$$U(\alpha,\hat{u}_i) = \left\{u_i ; \frac{|u_i - \hat{u}_i|}{\hat{u}_i} \le w_i \alpha\right\}, \quad \alpha \ge 0 \qquad (2)$$

where $\hat{u}$ shows the forecasted values for the uncertain parameters and α is the horizon of uncertainty. The fractional error model creates an expanding interval around the initial estimates of uncertain parameters. Weight parameter w_i such that $0 \le w_i \le 1$ allows the analyst to moderate the influence of individual parameters on the horizon of uncertainty [28]. The forecasted values of the uncertain parameters are input while the horizon of uncertainty is a variable that is determined in the decision-making process [29].

3. Problem formulation

The open-pit mine planning problem is described in this section in terms of the deterministic and uncertainty conditions.

3.1. Mine planning without considering uncertainty

In this section, the deterministic mathematical formulation of the model including the objective function and constraints is comprehensively described. The basic input to mine production planning is a typical of the 3D orebody model consisting of a large number of units with their individual values. The mining unit is referred to as a mining block whose value is derived from an estimated grade and the economic factors such as the commodity price and exploitation costs [30]. The fundamental assumption in conventional approaches is that the inputs derived from economic parameters such as market price are constant; consequently, the economic values of mining blocks calculated are also treated as constant. The economic value for block *i* is calculated by [31]:

$$\hat{V}_i = \left(\hat{P} - r\right) Q_{oi}\, g_i\, y - mQ_{bi} - pQ_{oi} \tag{3}$$

where $\hat{V}_i$ corresponds to the forecasted economic value of block *i*, which is adjusted such that:

$$\hat{V}_i = \begin{cases} \hat{V}_i & if\ \hat{V}_i > 0 \\ -mQ_{bi} & if\ \hat{V}_i \le 0 \end{cases} \tag{4}$$

The other parameters are defined as:
i = block indicator, where i = 1, 2, ..., N;
Q_{oi} = tonnage of ore in block *i*;
Q_{bi} = tonnage of block *i*;
$\hat{P}$ = forecasted (expected) price of metal (USD/ton of metal);
g_i = ore grade of block *i;*
y = metallurgical recovery (%);
m = mining cost (USD/ton of material);
p = processing cost (USD/ton of ore);
r = refining cost (USD/ton of metal);
NPV of an open-pit mine for a T time period and N blocks is calculated by the following formula:

$$NPV = \sum_{i=1}^{N}\sum_{t=1}^{T}\frac{\hat{V}_i}{\left(1+d\right)^t}x_{it} \tag{5}$$

where x_{it} is a binary variable that is equal to 1 if the block is mined in a period and 0, otherwise, t is the period indicator, where t = 1, ...,T, and d is the discount rate at period t. Cutoff grade is the criterion for recognizing a mining block as an ore block or a waste block. It is the established practice to assume a fixed cutoff grade at least for determining the ultimate pit limit [32, 31, 8], and the work herein also assumes a fixed cutoff grade.

3.1.1. Objective function

The objective function of the open-pit mine planning problem is to maximize the NPV of the operation based on the Johnson's [33] linear programming formulation that can be described as follow:

$$NPV_0 = max\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{\hat{V}_i}{\left(1+d\right)^t}x_{it} \tag{6}$$

where NPV_0 is expected as NPV of mining operation.

3.1.2. Constraints

The objective function in the previous section is subjected to the following constraint. The reserve constraint determines a block that can be mined only at once.

$$\sum_{t=1}^{T}x_{it} \le 1,\ \forall i = 1,2,\ldots,N \tag{7}$$

A block can be mined if its predecessors, j, are already mined in or before period t as the following slope constraint:

$$x_{it} - \sum_{t=1}^{T}x_{jt} \le 0,\ \forall i = 1,2,\ldots,N;t = 1,2,\ldots,T \tag{8}$$

The mining and processing capacity constraints determine that the total material mined and the total ore processed during each period should be within the pre-defined upper and lower limits as (9)-(12).

$$\sum_{i=1}^{I}Q_i x_{it} \ge \underline{M}_t,\ \forall t = 1,2,\ldots,T \tag{9}$$

$$\sum_{i=1}^{I}Q_i x_{it} \le \bar{M}_t,\ \forall t = 1,2,\ldots,T \tag{10}$$

$$\sum_{i=1}^{I}o_i Q_i \times x_{it} \ge \underline{O}_t,\ \forall t = 1,2,\ldots,T \tag{11}$$

$$\sum_{i=1}^{I}o_i Q_i \times x_{it} \le \bar{O}_t,\ \forall t = 1,2,\ldots,T \tag{12}$$

where $\underline{M}_t$ and $\bar{M}_t$ represent the lower and upper limits of the available mining capacity, and $\underline{O}_t$ and $\bar{O}_t$ represent the lower and upper limits of the available processing capacity, respectively. Value of the o_i indicator is equal to 1 if block i is considered as ore, and 0 otherwise.

3.2. Proposed IGDT-based mine planning under price uncertainty

In the deterministic mathematical formulation, it has been assumed that the commodity price is precisely forecasted. However, due to the lack of accurate forecasting methods, it should appropriately take into account inherent uncertainties associated with the forecasting variables in the planning. In this section, the deterministic formulation of open -pit mine planning is recast into its robust counterpart using the IGDT-based method to cope with price uncertainty. The IGDT-based decisions guarantee a specified target, provided that the prices fall into their maximized uncertainty horizons centered on the given estimates. Therefore, the IGDT method helps a risk-averse planning to maximize the robustness of decisions against the uncertain variables. The robustness is defined here as the immunity of target NPV satisfaction at variations of the uncertain forecasted price. In this regard, greater uncertainty horizons can provide more robustness for plans. In this section, the fractional uncertainty model is utilized to model uncertain market price, as given in Equation (13).

$$U\left(\alpha,\hat{P}\right)=\left\{P;\frac{\left|P-\hat{P}\right|}{\hat{P}}\leq\alpha\right\},\ \alpha\geq 0 \quad (13)$$

where $\hat{P}$ shows the forecasted value of commodity price and α is the horizon of price uncertainty. Uncertainty set can be defined for all planning horizon intervals. In the IGDT approach, it is assumed that the forecasted value of uncertain parameter P is available. The objective of IGDT in this work is to maximize the robustness, i.e. uncertainty horizon of the uncertain variable while a critical NPV as Equation (14) is guaranteed. The critical NPV is a percentage of the expected NPV when scheduling is done based on the forecasted price values without considering uncertainty, i.e. NPV_0.

$$NPV_c=\left(1-\delta\right)NPV_0 \quad (14)$$

where NPV_c and δ are the critical NPV and the deviation factor, respectively. The deviation factor δ indicates the level of risk-averseness of the planning. Higher values of the deviation factor leads to a more robust plan, and the critical NPV obtained will be valid for a wider range of realizations of the uncertain variable P . Deviation factor δ is the only parameter required in addition to the input parameters for deterministic planning. The robustness degree of the problem can be controlled thorough adjusting the NPV deviation factor δ , which is specified by the mine planners based on its management policy. The robustness function for an open-pit mine planning problem can be presented as follows:

$$\hat{\alpha}\left(NPV_c\right)=\max\left\{\alpha : minNPV \geq NPV_c\right\} \quad (15)$$

The function $\hat{\alpha}\left(NPV_c\right)$ is related to the NPV lower than the minimum NPV, and works as a risk-averse mechanism. In other words, $\hat{\alpha}\left(NPV_c\right)$ measures the protection level of the mine planning decision against experiencing low NPV. Thus a high value of this function, which corresponds to the low target, NPV_c , means that the associated decision is highly robust against low market prices. It is expected that $\hat{\alpha}\left(NPV_c\right)$ increases with decrease in NPV_c . Note that NPV_c represents the mine planning NPV target that the mine manager is willing to face. Then the robustness function and, consequently, a robust mine plan can be derived by providing the largest possible α for a given target NPV_c . Using (13), the low commodity price can be expressed as (16).

$$P=\hat{P}-\alpha\hat{P} \quad (16)$$

Inserting (16) in (5), the minimum value for the optimum NPV can be calculated. The uncertainty-based problem can be modeled using (17)-(25).

$$\hat{\alpha}\left(NPV_c\right)=\max\alpha \quad (17)$$

s.t.:

$$\min_{\alpha} NPV \geq NPV_c=\left(1-\delta\right)NPV_0 \quad (18)$$

$$\min_{\alpha} NPV = \left\{ max\sum_{i=1}^{N}\sum_{t=1}^{T}\frac{\left(\hat{P}-\alpha\hat{P}-r\right)Q_{oi}\,g_i\,y-mQ_{bi}-pQ_{oi}}{\left(1+d\right)^t}.x_{it}\right. \quad (19)$$

s.t.:

$$\sum_{t=1}^{T}x_{it}\leq 1,\ \forall i=1,2,\ldots,N \quad (20)$$

$$x_{it}-\sum_{t=1}^{T}x_{jt}\leq 0,\ \forall i=1,2,\ldots,N;t=1,2,\ldots,T \quad (21)$$

$$\left.\begin{array}{l}\sum_{i=1}^{I}o_iQ_i\times x_{it}\leq\bar{O}_t,\ \forall t=1,2,\ldots,T \quad (22)\\ \sum_{i=1}^{I}o_iQ_i\times x_{it}\geq\underline{O}_t,\ \forall t=1,2,\ldots,T \quad (23)\\ \sum_{i=1}^{I}Q_ix_{it}\geq\underline{M}_t,\ \forall t=1,2,\ldots,T \quad (24)\\ \sum_{i=1}^{I}Q_ix_{it}\leq\bar{M}_t,\ \forall t=1,2,\ldots,T \quad (25)\end{array}\right\}$$

where $\min_{\alpha} NPV$ is the lower limit of NPV. It should be noted that obtaining the minimum requirement is not straightforward in mine planning. Hence, it is required to solve an optimization problem to obtain the minimum requirement. This minimum requirement is dependent on the uncertainty horizon, which is the solution to another optimization problem. Due to this cross-relation between the solutions of two optimization problems, a bilevel optimization model should be implemented. The upper level program is formulated in (17)-(18) and is used to determine the maximum possible horizon of uncertainty that guarantees a pre-determined NPV. The lower level program is presented in (19)-(25), which is used to determine the lowest NPV for a certain planning and uncertainty horizon. In the lower level problem, the value of price in uncertainty horizon that results in a minimum NPV is determined. In this problem, the objective is to maximize the horizon of price uncertainty while the NPV is greater than a pre-determined value of NPV_c.

Deterministic methods for finding a sequence of pushbacks are often based on the nested pit implementation of the Lerchs–Grossman [32] algorithm for finding ultimate pit limits. Typically, a variable in the evaluation of the economic value of a block is scaled, and an ultimate pit algorithm is used to produce a pit smaller than the ultimate pit. The process is repeated using multiple scaling factors, which result in a series of nested pits; these pits are subsequently grouped into possible choices for pushbacks, whereby the nested pit that closely satisfies a set of given constraints is selected as the pushback [1]. Here, this method can be used to solve the lower level of mentioned problem. The steps of the proposed mine planning method based on IGDT are shown in Figure 2. In the first step, the forecasted values and other input parameters are used to determine the optimal deterministic solution. The only output of this step that will be used in the next step is the maximum expected NPV based on the forecasted values, i.e. NPV_0. The solution provides the mine planning considering the uncertainty horizon. The only input parameter that is required in addition to the forecasted values of uncertain variables is the deviation factor δ.

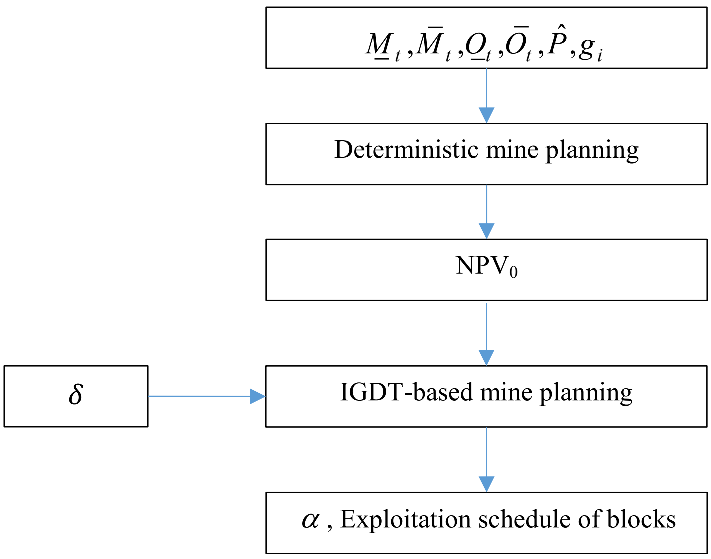

Figure 2. Steps of proposed IGDT-based mine planning.

4. Application in an open-pit mine

In this section, a case study in the Rezvan iron ore mine located in the south of Iran is used to clarify the practical aspects of the proposed approach. The procedure begins with the development of the orebody model. The geological database of the deposit was obtained from 159 vertical exploration boreholes. The holes were drilled in a lattice pattern in a 600 m (E–W) by 1200 m (N–S) area with a 50 m spacing between the adjacent drill holes. The information includes 4170 powder samples. Spatial modelling of the geological information generates grade model. The orebody model in this case study consists of 189,443 blocks. In order to determine the ultimate pit limit and mining phases, the required slope angles were assumed to be 45° in all directions. The upper and lower limits for processing and mining capacity were set to be equal to the average available quantity of ore and rock within the ultimate pit limit for each period of the scheduling horizon, respectively. Using the initial data presented in

Table 1, Datamine studio and NPV Scheduler softwares were utilized to determine the ultimate pit limit, the sequence of phases, and the expected value of NPV of mining operation, i.e. NPV_0. Figure 3 presents the section maps of the ultimate pit limit obtained and the sequence of phases for the forecasted price.

Implementing the Lerchs–Grossman algorithm based on the forecasted price P , 18 phases to the ultimate pit limit were developed. The expected NPV of mining operation is 519,407,190 USD (NPV_0 = 519,407,190). Figure 4 shows the critical NPV versus the deviation factor δ. The horizontal axis denotes the deviation factor to determine the critical NPV. The greater the deviation factor tolerated in the system, the lower the critical NPV becomes. In the case study, four management alternative δ (10, 20, 30, and 40 percent) are suggested. In this way, four critical NPVs 467,466,471 USD, 415,525,752 USD, 363,585,033 USD, and 311,644,314 USD were determined.

Figure 5 shows horizon of uncertainty (α) versus determined critical NPV. The vertical axis denotes the critical NPV to determine horizon of uncertainty. Based upon the four critical NPVs in the previous section, four horizons of uncertainty were determined. Figure 5 endorses that robustness increases with decrease in NPV_c , as expected. If the mine planning desires higher robustness, less NPV should be obtained, and vice versa; if the mine plan obtains less NPV, its decision will be more robust. Note that the values of the robustness function related to the maximum NPV is equal to zero, i.e. $\hat{\alpha}\left(519{,}407{,}190\$\right)=0$.

Figure 6 displays horizon of uncertainty versus total ore in the ultimate pit. The graph illustrates that in the case of total ore, there are no significant fluctuations by changing horizon of uncertainty in the range of 0-40. Table 2 presents the deviation factor, horizon of uncertainty, offered prices, and ultimate pit characteristics for four proposed management options. These values confirm the results of the graph in Figure 6. Consequently, due to no significant difference in total ore in the mentioned options, maximum proposed deviation factor $(\delta = 40\%)$ is considered for planning the mine.

Table 1. Economic and parameter values for case study.

Parameter	Quantity	Unit
Ore production	3	Million ton/year
Overall pit slope	50	Degree
Density	0.038*Fe+1.86	ton/m^3
Cutoff grade	15	%
Recovery	90	%
Dilution	5	%
Ore mining cost	6	USD/m^3
Waste mining cost	4	USD/m^3
Crushing cost	1	USD/ton
Discount rate	15	%
Iron ore price	58	USD/ton
Processing capacity	3	Million ton/year

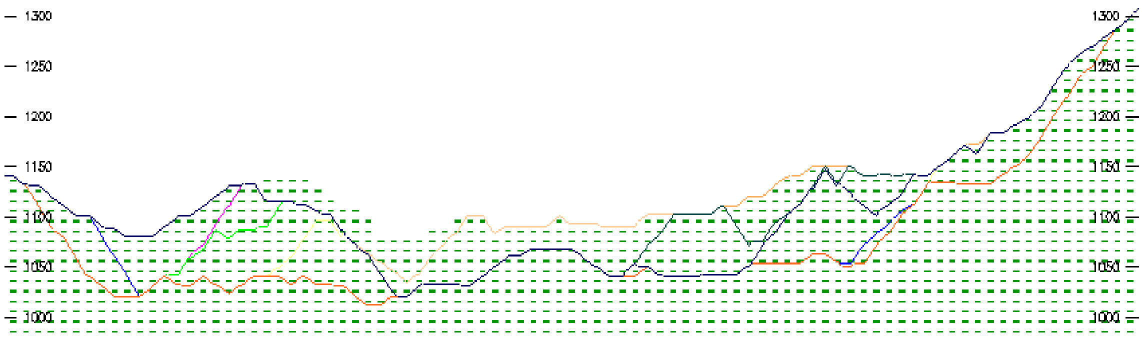

Figure 3. A representative section showing phases and ultimate pit of the mine.

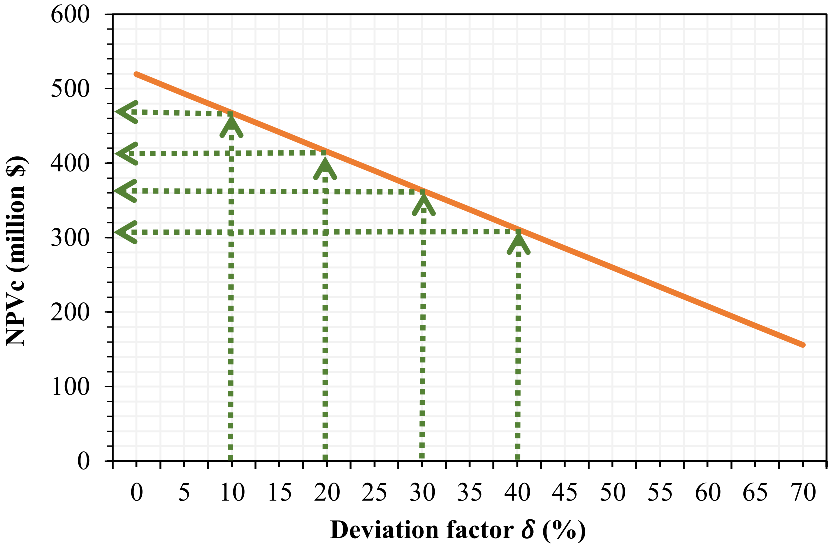

Figure 4. Critical NPV *vs.* deviation factor (δ).

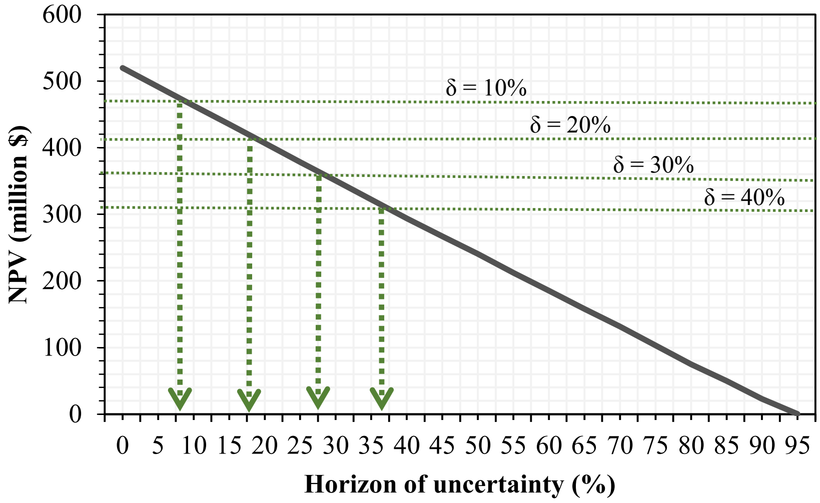

Figure 5. Horizon of uncertainty (α) *vs.* expected NPV.

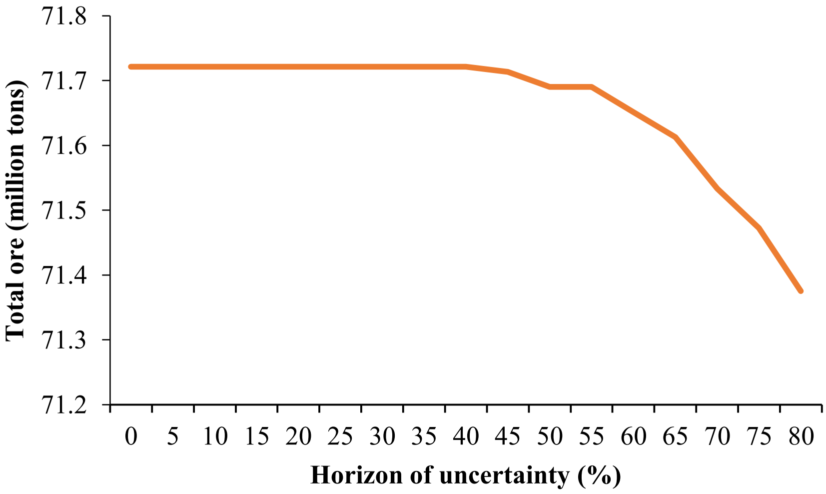

Figure 6. Horizon of uncertainty (α) *vs.* total ore in the ultimate pit.

Table 2. Four management options and offered prices for mine planning.

Management option No.	δ (%)	α (%)	P ($/ton)	NPV (million $)	Blocks in ultimate pit	Total ore (million tons)
1	10	8	53.36	474.338	10,804	71.721
2	20	18	47.56	418.002	10,804	71.721
3	30	28	41.76	361.665	10,804	71.721
4	40	37	36.54	310.962	10,804	71.721

Table 3. Phase design and ultimate pit limit material distribution for 4th proposed management option (δ = 40%).

Phase number	Total rock (ton)	Total ore (ton)
1	3,298,772	3,158,147
2	4,056,086	4,005,461
3	4,889,630	4,827,755
4	3,036,698	3,014,198
5	4,513,092	4,276,842
6	11,142,092	10,877,717
7	3,980,829	3,873,954
8	3,207,590	3,173,840
9	3,081,290	3,013,790
10	3,554,830	3,009,205
11	3,330,105	3,105,105
12	4,286,621	3,988,496
13	4,262,399	3,351,149
14	3,970,437	3,520,437
15	3,104,134	3,019,759
16	4,613,143	3,600,643
17	11,297,861	5,751,611
18	3,210,759	2,153,259
Ultimate pit	**82,836,377**	**71,721,377**

5. Conclusions

This paper proposes a practical approach risk-averse decision-making tool to help mine planners in mining activities under price uncertainty. The main innovative contributions of this paper is to utilize IGDT to define a new non-deterministic and non-probabilistic manner for open-pit mine planning under uncertainty. The IGDT-based approach finds a solution that guarantees a certain expectation of NPV in mine planning. It essentially relies on the gap between the actual and forecasted values of uncertain variables (commodity price in this work). The proposed approach is straightforward to implement in practice. It requires less information compared to the other non-deterministic mine plannings such as stochastic programming, and fuzzy and scenario planning methods. In other words, no probability distribution functions, fuzzy membership functions or confidence intervals of the uncertain variables are required for the suggested methodology, which is an important advantage of the proposed approach. After implementing the proposed IGDT-based approach, the deviation factor acts as a setting for it. By changing the deviation factor, planners can obtain the best trade-off between the acceptable uncertainty ranges and critical NPV based on its preferences. In other words, a planner can adjust the robustness level of its strategy based on the minimum tolerable NPV, which is another advantage of the proposed approach. There is thus a confident mine planning that can reduce risk and establish an acceptable mine production plan that enables the mine managers to meet production targets and make the arbitrary return on investment. It is worth mentioning that the method is not only useful in conditions of reducing the price; given the direct impact of the price on the NPV, if the price increases from the expected values, the NPV value will certainly be greater than the critical value. Therefore, in the case of increasing prices, the proposed method will be robust and risk-averse to the changes.

The case study presented an example in which the suggested approach anticipated develops a reliable mine plan, leading to a resistant economic value and ore production compared to the deterministic methods. The results obtained show that the proposed method can provide a robust solution for mine planning under price uncertainty. Future research works aim to the additional test method presented above as well as include uncertainty in other parameters for instance mining costs and discount rate.

Acknowledgments
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یک روش کاربردی برنامه‌ریزی معادن روباز در شرایط عدم قطعیت قیمت با استفاده از تئوری تصمیم‌گیری مبتنی بر شکاف اطلاعاتی

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چکیده:

وجود عدم قطعیت در برنامه‌ریزی استخراج معادن روباز، از جمله عدم قطعیت قیمت، تأثیر بسزایی بر مسائل فنی و اقتصادی پروژه‌های معدنی دارد. برنامه‌ریزی استخراج معدن که بدون توجه به عدم قطعیت قیمت انجام شود، فقط در شروع عملیات استخراج معدن می‌تواند بهینگی خود را حفظ کند و با گذشت زمان کوتاه، به دلیل تغییرات قیمت از حالت بهینه خارج می‌شود. این پژوهش یک روش تصمیم‌گیری ریسک گریز برای کمک به برنامه ریزان معادن در شرایط عدم قطعیت قیمت ارائه می‌دهد. هدف این است که برنامه‌ریزی استخراج معدن به صورتی انجام شود که ارزش خالص فعلی (NPV) مورد نظر تضمین شود. برای رسیدن به این هدف، تئوری تصمیم‌گیری مبتنی بر شکاف اطلاعاتی (IGDT) برای محافظت از پروژه معدنی در برابر ریسک‌هایی که به دلیل شکاف بین مقادیر پیش‌بینی شده و مقادیر واقعی قیمت به کار گرفته شد. روش پیشنهادی در برابر تغییرات قیمت حساسیت کمی دارد و می‌تواند قیمت‌های تخمینی که همراه با عدم قطعیت هستند را برای ارائه برنامه‌ریزی مورد استفاده قرار دهد. با انجام مطالعه موردی در یکی از معادن سنگ آهن عملکرد روش پیشنهادی نشان داده شد. نتایج به دست آمده نشان می‌دهد که روش پیشنهادی می‌تواند یک راهکار مقاوم برای برنامه‌ریزی استخراج معادن در شرایط عدم قطعیت قیمت ارائه دهد. علاوه بر آن روش پیشنهادی می‌تواند برنامه‌های استخراجی قابل اعتمادی را در شرایط عدم قطعیت قیمت ارائه دهد.

کلمات کلیدی: IGDT، معدن روباز، برنامه‌ریزی استخراج معدن، مدل‌سازی عدم قطعیت، عدم قطعیت قیمت.