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A COVERING PROPERTY IN PRINCIPAL BUNDLES

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Abstract. Let $p : X \longrightarrow B$ be a locally trivial principal G-bundle and $\widetilde{p} : \widetilde{X} \longrightarrow B$ be a locally trivial principal $\widetilde{G}$-bundle. In this paper, by using the structure of principal bundles according to transition functions, we show that $\widetilde{G}$ is a covering group of G if and only if $\widetilde{X}$ is a covering space of X. Then, we conclude that a topological space X with non-simply connected universal covering space has no connected locally trivial principal $\pi(X, x_0)$-bundle, for every $x_0 \in X$.

1. Introduction and Motivation

Existence of simply connected covering space of a given space X is not always available unless for locally well behaved spaces, i.e, locally path connected and semi-locally simply connected spaces. For the spaces which do not satisfy in these nice conditions, the classification of covering spaces of X is related to some newly discovered subgroups of the fundamental group of X (see [7, 11, 6, 4]). For these spaces, the first author in [8], introduced some new kinds of categorical universal coverings and also proved that every categorical universal covering must be Spanier covering. If $p : \widetilde{X} \longrightarrow X$ is the categorical universal covering, then $p_*(\pi_1(\widetilde{X}, \tilde{x}_0)) = \pi_1^{sp}(X, x_0)$, where $\pi_1^{sp}(X, x_0)$ is the Spanier group of X [8] and so $p : \widetilde{X} \longrightarrow X$ is a principal $\dfrac{\pi_1(X, x_0)}{\pi_1^{sp}(X, x_0)}$-bundle.

Recently, various generalizations of covering theory have been proposed in order to keep useful properties of coverings [1, 3]. Bundles,

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particularly principal bundles are one of the old generalizations of coverings. For the spaces without simply connected universal covering, the existence of some principal bundles will be affected.

A regular covering $q : Y \longrightarrow X$ is a principal bundle where the structure group is $\frac{\pi_1(X,x_0)}{q_*(\pi_1(Y,y_0))}$, acting on the fibers of q by the monodromy action (fundamental group is equipped with discrete topology). If $p : \widetilde{X} \longrightarrow X$ is the simply connected universal covering of X, then it is also a principal $\pi_1(X, x_0)$-bundle. In this case, $\widetilde{X}$ is a covering space of Y and obviously the natural quotient homomorphism $r : \pi_1(X, x_0) \longrightarrow \frac{\pi_1(X,x_0)}{q_*(\pi_1(Y,y_0))}$ is a covering group homomorphism (an open epimorphism with discrete kernel [2]).

In this paper, we show that the above phenomenon holds for general principal bundles. In fact, we prove that for a given first countable topological group $\widetilde{G}$, locally trivial principal G-bundle (X, p, B) and locally trivial principal $\widetilde{G}$-bundle $(\widetilde{X}, \widetilde{p}, B)$, $\widetilde{G}$ is a covering group of G if $\widetilde{X}$ is a covering space of X. Conversely, we prove that when $\varphi : \widetilde{G} \longrightarrow G$ is a covering homomorphism compatible with the transition functions, then $\widetilde{X}$ is a covering space of X. This makes that when $\widetilde{G}$ is a covering group of G and $\widetilde{X}$ is a regular covering space of X, there exists $f : \widetilde{X} \longrightarrow X$ which is a locally trivial principal $\frac{\pi_1(X,x_0)}{f_*(\pi_1(\widetilde{X},\widetilde{x_0}))}$-bundle. Using this, we prove that spaces with non-simply connected universal covering has no connected locally trivial principal $\pi(X, x_0)$-bundle.

2. Preliminaries

In this section, we recall some preliminary definitions and results from [5] to drive the main results of the paper. It is notable that all the spaces will be assumed to be connected and locally path connected and the fundamental groups have discrete topology.

Let G be a topological group. A space X is called G-space if G acts on X continuously. A bundle (X, p, B) is called G-bundle if it is isomorphic (as a bundle) to the bundle $(X, \pi, X \ mod\ G)$, where $\pi : X \longrightarrow X \ mod\ G$ is the canonical quotient map. Let X^* be the subspace of all $(x, xs) \in X \times X$, where $x \in X$ and $s \in G$. Then, there is a function $\tau : X^* \longrightarrow G$, called translation function, such that $x\tau(x, x') = x'$ for all $(x, x') \in X^*$.

A principal G-bundle is a G-bundle (X, p, B) where the action of G on X is free ($xs = x \Leftrightarrow s = 1$) and its translation function is continuous.

A system of transition functions on the space B relative to an open covering $\{V_i\}_{i\in J}$ of B is a family of maps $g_{i,j} : V_i \cap V_j \to G$ for each $i, j \in J$ such that $g_{i,j}(b)g_{j,k}(b) = g_{i,k}(b)$. Every locally trivial principal G-bundle has a system of transition function.

Let $\{V_i\}_{i\in J}$ be an open cover of B and $\{g_{i,j}\}_{i,j\in J}$ be a system of transition function of a principal G-bundle (X, p, B) associated with the open covering $\{V_i\}$. We can construct X, up to homeomorphism by the following way.

Let Z be the sum space of the family $\{V_i \times G\}_{i\in J}$. Indeed,

$$Z := \sum_{i\in J}(V_i \times G) = \{(b, s, i);\ b \in V_i,\ s \in G\},$$

is equipped by the largest topology such that all the inclusion maps $q_i : V_i \times G \longrightarrow Z$ with $q_i(b, s) = (b, s, i)$ are continuous.

Define the relation R on the space Z by

$$(b, s, i)\ R\ (b', s', j) \Leftrightarrow b = b',\ s' = g_{j,i}(b)s,$$

which is an equivalence relation by the properties of transition functions. Let $Y := Z\ mod\ R$ and denote the class of (b, s, i) in Y by $\langle b, s, i\rangle$. The space Y becomes a G-space by the action $\langle b, s, i\rangle t = \langle b, st, i\rangle$, for $t \in G$ and (Y, q, B) becomes a principal G-bundle, isomorphic to (X, p, B) where $q : Y \longrightarrow B$ is defined by $q(\langle b, s, i\rangle) = b$. Therefore X is homeomorphism to $\frac{\sum(V_i\times G)}{R}$ and so we may assume that $X = \frac{\sum(V_i\times G)}{R}$.

Now consider principal G-bundle (X, p, B) and $\widetilde{G}$-bundle $(\widetilde{X}, \tilde{p}, B)$ with transition functions $\{g_{i,j}\}$ and $\{\widetilde{g_{i,j}}\}$, respectively. We say a homomorphism $\varphi : \widetilde{G} \longrightarrow G$ is compatible with the structure groups of (X, p, B) and $(\widetilde{X}, \tilde{p}, B)$ if $\varphi \circ \widetilde{g_{i,j}} = g_{i,j}$.

A covering map is a continuous map $p : \widetilde{X} \longrightarrow X$ if for every $x \in X$, there exists an open subset U of X with $x \in U$ such that $p^{-1}(U) = \sqcup_{j\in J} V_j$ and $p|_{V_j} : V_j \longrightarrow U$ is a homeomorphism for every $j \in J$. By a categorical universal covering map we mean a covering map $p : \widetilde{X} \longrightarrow X$ with the property that for every covering map $q : \widetilde{Y} \longrightarrow X$ with a path connected space $\widetilde{Y}$ there exists a unique covering $f : \widetilde{X} \longrightarrow \widetilde{Y}$ such that $q \circ f = p$.

If $\mathcal{U}$ is an open cover of X, then the subgroup of $\pi_1(X, x)$ consisting of all homotopy classes of loops that can be represented by a product of the following type

$$\prod_{j=1}^{n} \alpha_j * \beta_j * \alpha_j^{-1},$$

where the α_j's are arbitrary paths starting at the base point x and each β_j is a loop inside one of the neighborhoods $U_i \in \mathcal{U}$, is called the *Spanier group with respect to* $\mathcal{U}$, and denoted by $\pi(\mathcal{U},x)$ [4, 10]. For two open covers $\mathcal{U},\mathcal{V}$ of X, we say that $\mathcal{V}$ refines $\mathcal{U}$ if for every $V\in\mathcal{V}$ there exists $U\in\mathcal{U}$ such that $V\subseteq U$. The Spanier group of a topological space X, denoted by $\pi_1^{sp}(X,x)$ is

$$\pi_1^{sp}(X,x)=\bigcap\{\pi(\mathcal{U},x);\mathcal{U} \text{ is an open cover of } X\}.$$

A space X is called Spanier space if $\pi_1(X,x)=\pi_1^{sp}(X,x)$, for each $x\in X$ and a covering $p:\widetilde{X}\longrightarrow X$ is called Spanier covering if $\widetilde{X}$ is a Spanier space.

3. Covering behaviors

Throughout this article, G is a topological group, (X,p,B) is a locally trivial principal G-bundle with translation map τ and a system of transition functions $g_{i,j}$, for open covering $\{V_i\}_{i\in J}$ consisting of path connected subsets. Also, by a covering group we mean an open epimorphism with discrete kernel.

At first, we prove the following lemma which is useful in the next results.

Lemma 3.1. *The canonical quotient map* $\pi:\sum_{i\in J}(V_i\times G)\longrightarrow \frac{\sum_{i\in J}(V_i\times G)}{R}$ *is open.*

Proof. Let U be an open subset of $\sum_{i\in J}(V_i\times G\times\{i\})$. Then $U\cap(V_i\times G\times\{i\})$ is open in $V_i\times G\times\{i\}$, for each $i\in J$. It suffices to show that $\pi^{-1}(\pi(U))\cap(V_i\times G\times\{i\})$ is open in $(V_i\times G\times\{i\})$, for each $i\in J$. Let $(b,s,i)\in\pi^{-1}(\pi(U))\cap(V_i\times G\times\{i\})$. There exists $x\in U$ such that $\pi(x)=\langle b,s,i\rangle$ and hence we can consider $x=(b,g_{j,i}(b)s,j)$, by the definition of R. Therefore, $(b,g_{j,i}(b)s,j)\in U\cap(V_j\times G\times\{j\})$. Thus there exists an open subset $W_j\times O\times\{j\}$ of $U\cap(V_j\times G\times\{j\})$ in which contains $(b,g_{j,i}(b)s,j)$ and $W_j\subseteq V_i\cap V_j$.

Consider the following continuous map

$$\begin{cases}\theta:W_j\times G\longrightarrow G\\ \qquad (a,t)\mapsto g_{j,i}(a)t.\end{cases}$$

Since O is an open subset containing $g_{j,i}(b)s$, there exist open subsets O' and $W_j'\subset W_j$ containing s and b, respectively, such that $g_{j,i}(W_j')O'\subseteq O$. We show that $W_j'\times O'\times\{i\}\subseteq\pi^{-1}(\pi(U))$. If $(b',t,i)\in W_j'\times O'\times\{i\}$, then

$$\pi((b',t,i))=\langle b',t,i\rangle=\langle b',g_{j,i}(b')t,j\rangle\in\pi(W_j\times O)\subseteq\pi(U).$$

Hence, $(b,s,i)\in W_j'\times O'\times\{i\}\subseteq\pi^{-1}(\pi(U))\cap(V_i\times G\times\{i\})$, as desired. $\square$

It is known that a topological group homomorphism $\varphi : G \longrightarrow H$ is a covering map if and only if it is an open epimorphism with discrete kernel [9, 94.2].

Theorem 3.2. *Let (X, p, B) be a locally trivial principal G-bundle and $(\widetilde{X}, \widetilde{p}, B)$ be a locally trivial principal $\widetilde{G}$-bundle. If $\varphi : \widetilde{G} \longrightarrow G$ is a covering homomorphism, compatible with their systems of transition functions, then $\widetilde{X}$ is a covering space of X.*

Proof. We can assume that $X = \frac{\sum_i (V_i \times G)}{R}$, $\widetilde{X} = \frac{\sum_i (V_i \times \widetilde{G})}{\widetilde{R}}$ and define $f : \widetilde{X} \longrightarrow X$ by $f(\langle b, \tilde{s}, i\rangle) = \langle b, \varphi(\tilde{s}), i\rangle$. If $\langle b, \tilde{s}, i\rangle = \langle b', \tilde{t}, j\rangle$, then $b = b'$ and $\tilde{t} = \widetilde{g_{j,i}}(b)\tilde{s}$ which implies $\varphi(\tilde{t}) = \varphi(\widetilde{g_{j,i}}(b))\varphi(\tilde{s}) = g_{j,i}(b)\varphi(\tilde{s})$ by compatibility of φ. Then $\langle b, \varphi(\tilde{s}), i\rangle = \langle b', \varphi(\tilde{t}), j\rangle$ and so f is well defined.

For an arbitrary point $x = \langle b, s, i\rangle \in X$ we have $p(x) = b$ and since the bundles are locally trivial, there exist local trivializations (V, h) and (V, k) such that $h : p^{-1}(V) \longrightarrow V \times G$ and $k : \tilde{p}^{-1}(V) \longrightarrow V \times \tilde{G}$ are fiber preserving homeomorphisms and also $b \in V$. Then, $V \times \tilde{G}$ is a covering of $V \times G$ by $1 \times \varphi$.

Consider the following diagram

$$\begin{array}{ccc} V \times \widetilde{G} & \xrightarrow{1\times\varphi} & V \times G \\ {\scriptstyle k^{-1}}\downarrow & & \downarrow{\scriptstyle h^{-1}} \\ \tilde{p}^{-1}(V) & \xrightarrow{f} & p^{-1}(V), \end{array}$$

which is commutative since f, h and k are fiber preserving.

Now, $g := f|_{\tilde{p}^{-1}(V)} : \tilde{p}^{-1}(V) \longrightarrow p^{-1}(V)$ is a covering map and $\langle b, s, i\rangle \in p^{-1}(V)$. Hence there exists an open neighborhood $O \subseteq p^{-1}(V)$ containing $\langle b, s, i\rangle$ such that $g^{-1}(O) = \sqcup_\alpha O_\alpha$ and $g|_{O_\alpha}$ is a homeomorphism for every α. Since $p \circ f = \tilde{p}$, we have $f^{-1}(O) \subseteq f^{-1} \circ p^{-1}(V) = \tilde{p}^{-1}(V)$ which implies that $f^{-1}(O) = \sqcup_\alpha O_\alpha$ and $f|_{O_\alpha} = g|_{O_\alpha}$ is a homeomorphism for every α. $\square$

Every regular covering $q : (Y, y_0) \longrightarrow (X, x_0)$ is a principal bundle with the structure group $\frac{\pi_1(X,x_0)}{q_*(\pi_1(Y,y_0))}$. Therefore, we have the following result.

Corollary 3.3. *Let (X, p, B) be a locally trivial principal G-bundle, $(\widetilde{X}, \widetilde{p}, B)$ be a locally trivial principal $\widetilde{G}$-bundle, $\varphi : \widetilde{G} \longrightarrow G$ be a compatible covering homomorphism and $f : \widetilde{X} \longrightarrow X$ be defined by $f(\langle b, \tilde{s}, i\rangle) = \langle b, \varphi(\tilde{s}), i\rangle$. If $f_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$ is a normal subgroup of $\pi_1(X, x_0)$, then $(\widetilde{X}, f, X)$ is a locally trivial principal $\frac{\pi_1(X,x_0)}{f_*(\pi_1(\widetilde{X},\widetilde{x}_0))}$-bundle.*

Now, we state the second main theorem of the paper.

Theorem 3.4. *Let (X, p, B) be a locally trivial principal G-bundle, $(\widetilde{X}, \widetilde{p}, B)$ be a locally trivial principal $\widetilde{G}$-bundle and $\varphi : \widetilde{G} \longrightarrow G$ be an epimorphism. If $f : \widetilde{X} \longrightarrow X$ by $f(\langle b, \tilde{s}, i\rangle) = \langle b, \varphi(\tilde{s}), i\rangle$ is a covering map and $\widetilde{G}$ is first countable, then $\varphi : \widetilde{G} \longrightarrow G$ is a covering homomorphism.*

Proof. At first, we show that $ker\varphi$ is discrete. Let $\tilde{g} \in ker\varphi$ and every open neighborhood of $\tilde{g}$ intersects $ker\varphi - \{\tilde{g}\}$. Since $\widetilde{G}$ is first countable, there exists a sequence $\{\tilde{g}_n\}_{n\in\mathbb{N}} \subseteq ker\varphi$ converging to $\tilde{g}$. The sequence $\{(b, \tilde{g}_n, i)\}$ converges to $(b, \tilde{g}, i)$, for every $b \in V_i$ and $i \in J$. Let W be an open neighborhood of $\langle b, \tilde{g}, i\rangle$. Then $\widetilde{\pi}^{-1}(W)$ is an open neighborhood of $(b, \tilde{g}, i)$ and so there exists $N_0 \in \mathbb{N}$ such that $(b, \tilde{g}_n, i) \in \widetilde{\pi}^{-1}(W)$ for every $n \geq N_0$. Therefore $\langle b, \tilde{g}_n, i\rangle \in \widetilde{\pi}(\widetilde{\pi}^{-1}(W)) = W$ which implies that $\langle b, \tilde{g}_n, i\rangle$ converges to $\langle b, \tilde{g}, i\rangle$. Since $f(\langle b, \tilde{g}_n, i\rangle) = f(\langle b, \tilde{g}, i\rangle)$ and f is a local homeomorphism, we have a contradiction and hence $ker\varphi$ is discrete.

Secondly, we show that φ is open. Clearly, f is a bundle map since

$$p \circ f(\langle b, \tilde{g}, i\rangle) = b = \widetilde{p}(\langle b, \tilde{g}, i\rangle).$$

For every $b \in B$, $f|_{\widetilde{p}^{-1}(b)}$ is a bundle map and hence is open. Fix an arbitrary point $\tilde{x} \in \widetilde{p}^{-1}(b)$ and define

$$\begin{cases} \eta : \widetilde{p}^{-1}(b) \longrightarrow \widetilde{G} \\ \qquad \tilde{y} \mapsto \widetilde{\tau}(\tilde{x}, \tilde{y}), \end{cases}$$

where $\widetilde{\tau} : \widetilde{X}^* \longrightarrow \widetilde{G}$ is the translation function of $(\widetilde{X}, \widetilde{p}, B)$ which implies that η is continuous. The inverse of η is given by

$$\begin{cases} \eta^{-1} : \widetilde{G} \longrightarrow \widetilde{p}^{-1}(b) \\ \qquad \tilde{g} \mapsto \tilde{x}\tilde{g}, \end{cases}$$

which is continuous and hence η is a homeomorphism. Similarly,

$$\begin{cases} \zeta : p^{-1}(b) \longrightarrow G \\ \qquad y \mapsto \tau(f(\tilde{x}), y), \end{cases}$$

is homeomorphism.

Since η, ζ and $f|_{\widetilde{p}^{-1}(b)}$ are open, to prove the openness of φ, it suffices to show that the following diagram is commutative.

$$\begin{tikzcd} \widetilde{p}^{-1}(b) \arrow[r, "f|"] \arrow[d, "\eta"'] & p^{-1}(b) \arrow[d, "\zeta"] \\ \widetilde{G} \arrow[r, "\varphi"] & G. \end{tikzcd}$$

For every $\tilde{y} \in \widetilde{p}^{-1}(b)$ we have

$$\zeta f(\tilde{y}) = \varphi\eta(\tilde{y}) \Leftrightarrow \tau(f(\tilde{x}), f(\tilde{y})) = \varphi(\widetilde{\tau}(\tilde{x}, \tilde{y})).$$

We claim that $f(\tilde{x}\tilde{g}) = f(\tilde{x})\varphi(\tilde{g})$, for every $\tilde{g} \in \widetilde{G}$. Set $\tilde{x} = \langle b, \tilde{s}, i\rangle$. Then $\tilde{x}\tilde{g} = \langle b, \tilde{s}\tilde{g}, i\rangle$ which implies

$$f(\tilde{x}\tilde{g}) = \langle b, \varphi(\tilde{s}\tilde{g}), i)\rangle = \langle b, \varphi(\tilde{s})\varphi(\tilde{g}), i\rangle = \langle b, \varphi(\tilde{s}), i\rangle\varphi(\tilde{g}) = f(\tilde{x})\varphi(\tilde{g})$$

and so the following diagram is commutative:

$$\begin{tikzcd} \widetilde{X}^* \arrow[r, "f\times f"] \arrow[d, "\widetilde{\tau}"'] & X^* \arrow[d, "\tau"] \\ \widetilde{G} \arrow[r, "\varphi"] & G, \end{tikzcd}$$

that is $\tau(f(\tilde{x}), f(\tilde{y})) = \varphi(\widetilde{\tau}(\tilde{x}, \tilde{y}))$. □

Now, let X be a space with categorical universal covering map $p : \widetilde{X} \longrightarrow X$. A necessary and sufficient condition for the existence of categorical universal coverings using open covers of X can be found in [8]. We can consider p as a $\dfrac{\pi_1(X, x_0)}{\pi_1^{sp}(X, x_0)}$-principal bundle and since the quotient map $q : \pi_1(X, x_0) \longrightarrow \dfrac{\pi_1(X, x_0)}{\pi_1^{sp}(X, x_0)}$ is a covering group, we would like to investigate the above results in this situation.

Theorem 3.5. *If X has a non-simply connected universal covering space, then it has no connected locally trivial $\pi_1(X, x_0)$-bundle.*

Proof. Let $r : \overline{X} \longrightarrow X$ be a connected locally trivial $\pi_1(X, x_0)$-bundle of X. The quotient map $q : \pi_1(X, x_0) \longrightarrow \dfrac{\pi_1(X, x_0)}{\pi_1^{sp}(X, x_0)}$ is a covering homomorphism (fundamental groups are supposed to be discrete) and also compatible with transition functions by the construction of transition functions [10]. Hence, Theorem 3.2 implies that $\overline{X}$ is a connected covering space of $\widetilde{X}$. But this is a contradiction, because $\widetilde{X}$ is a Spanier space and has no connected covering space [6, 8]. □

Remark 3.6. Note that the above theorem is also true for every topological group G, when $q : G \longrightarrow \dfrac{\pi_1(X, x_0)}{\pi_1^{sp}(X, x_0)}$ is a covering group compatible with the system of transition functions.

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A COVERING PROPERTY IN PRINCIPLE BUNDLES

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خاصیت پوششی در کلاف‌های اصلی

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فرض کنید $p : X \longrightarrow B$ یک G-کلاف اصلی به طور موضعی بدیهی و $\widetilde{p} : \widetilde{X} \longrightarrow B$ یک $\widetilde{G}$-کلاف اصلی به طور موضعی بدیهی باشند. در این مقاله، با استفاده از ساختار کلاف‌های اصلی بر حسب نگاشت‌های گذر، نشان خواهیم داد که $\widetilde{G}$ یک گروه پوششی از G است اگروتنهااگر $\widetilde{X}$ یک فضای پوششی از X باشد. سپس نتیجه‌گیری خواهیم نمود فضای X که فضای پوششی جهانی همبند ساده ندارد، برای هر $x_\circ \in X$ نمی‌تواند $\pi(X, x_\circ)$-کلاف اصلی به طور موضعی بدیهی و همبند داشته باشد.

کلمات کلیدی: کلاف اصلی، فضای پوششی، گروه پوششی.