

## Application of spectrum-volume fractal modeling for detection of mineralized zones

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### Abstract

The main goal of this research work was to detect the different Cu mineralized zones in the Sungun porphyry deposit in NW Iran using the Spectrum-Volume (S-V) fractal modeling based on the sub-surface data for this deposit. This operation was carried out on an estimated Cu block model based on a Fast Fourier Transformation (FFT) using the C++ and MATLAB programming. The S-V log-log plot was generated and six Cu populations were distinguished. Based on the S-V log-log plot obtained, different mineralized zones were detected in the Sungun deposit. Copper mineralized zones in the porphyry and skarn types commenced from 0.12% and 1.3%, respectively. A supergene enrichment zone began from 0.82%; it was located in the eastern part of this deposit. The enriched skarn zones were situated in the eastern and SE parts of the Sungun deposit that overlapped the intersection of cretaceous limestones and porphyry stock. Overlapping between the resulting zones derived via the S-V fractal model and geological zones and evidences were calculated using the logratio matrix, which indicated that the S-V fractal model had proper results for detection of the mineralized zones.

**Keywords:** *Spectrum-Volume Fractal Model, C++ Programming, Cu Mineralized Zones, Sungun.*

### 1. Introduction

Separation of ore mineralized zones is essential in mineral exploration and mining engineering. Detection of enriched supergene from hypogene zone in different ore deposits has an important role for this purpose [1-4]. The conventional methods available for delineation of ore mineralized zones are the geological-based descriptive techniques such as mineralography-petrography, fluid inclusions, and stable/radiogenic isotopes [5].

Mathematical models have been used for recognition of mineralized zones since 2000, especially fractal analysis, which was submitted by Mandelbrot (1983) [6]. Fractal geometry is a source of many methods such as Number-Size (N-S) [6], Concentration-Area (C-A) [7], Concentration-Perimeter (C-P) [8], Spectrum-Area (S-A) [9], Concentration-Distance

(C-D) [10], Concentration-Volume (C-V) [11], and Spectrum-Volume (S-V) [12].

Transformation of ore grade data to frequency time series could represent noises in the geochemical data and remove these from the final geochemical map. In the recent years, Fast Fourier Transformation (FFT) and Decomposed Wavelet Transportation (DWT) have been used for the interpretation of geochemical data [13-16]. In this work, various Cu mineralized zones were defined based on the S-V fractal model [12] and a C++ and MATLAB programming. In addition, the copper mineralized zones obtained by the S-V model were correlated by the logratio matrix with the geological zones including supergene enrichment, hypogene, and skarns [19, 20].

## 2. Methodology

The major application of FFT is signal processing as a time-series. In other words, discrete and continuous functions of spatial data (e.g. geochemical data) can be converted to frequency in the time-series data type using Discrete Fourier transformation (DFT) [17-18]. This transformation causes a decrease in the computation time for signal processing in computerized coding. Cheng et al. (1999) [9] established the S-A fractal method for separation of the geophysical/geochemical anomalies from the background based on a 2D FFT. In addition, Afzal et al. (2012) [12] proposed the S-V fractal modeling for detection of the Cu mineralized zones in porphyry deposits with an Iranian case study based on a 3D FFT in the following form:

$$F(\omega) = \sum_{i=1}^{nfx} \sum_{j=1}^{nfy} \sum_{k=1}^{nfz} m3d(i, j, k) e^{-j\vec{r} \cdot \vec{k}} \quad (1)$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \quad (2)$$

$$\vec{k} = k_1\hat{x} + k_2\hat{y} + k_3\hat{z} \quad (3)$$

$$k_1 = \frac{kx}{\Delta x} \quad k_2 = \frac{ky}{\Delta y} \quad k_3 = \frac{kz}{\Delta z} \quad (4)$$

$\vec{k}$  and  $\vec{r}$  are vectors for wave values and spatial positions for ore grades, respectively. For 3D FFT, a block model as a prototype should be generated as a 3D matrix. For this purpose, an estimated block model is used. The original borehole data for ore element concentrations was interpolated using the geostatistical estimation/simulation methods. Distribution of ore grade (Cu in this scenario) is modeled utilizing geostatistical estimation like the kriging methods for this matrix.  $\vec{k}$  varies between 0 and  $2\pi$ :

$$K_1x, K_2y, K_3z \in \{0 \text{ to } 2\pi\} \quad (5)$$

$\Delta x$ ,  $\Delta y$ , and  $\Delta z$  are differences between maximum and minimum in the  $x$ ,  $y$ , and  $z$  dimensions, respectively. Based on calculations, a MATLAB code was constructed for 3D FFT. Application of C++ with MATLAB is proper for a better

visualization and a decrease in the computation time.

Running of the MATLAB code is slow and time-consuming. In order to solve this problem, C++ programming was carried out using the OpenMp tool. This instrument is applied for parallel processing due to the decrease in the computation time [21]. This tool divides the loop into more than 10 parts (Threads), as depicted in Figure 1. Moreover, all Threads are run simultaneously [22]. This operation can be used to decrease time in C++ programming in comparison with the MATLAB code. Finally, the S-A log-log plot was built up for delineation of the copper mineralized zones. According to this log-log plot, high- and low-pass filters were used to distinguish between different populations. Moreover, this data was returned to ore grades by Inverse Fast Fourier Transformation (IFFT).

## 3. Case study

The Sungun porphyry deposit is one of the major Iranian copper mines, which is located in the NE of Tabriz city (NW of Iran; Figure 2). This deposit is situated in the Uremia-Dokhtar magmatic arc as a main Iranian metallogenic belt. The Sungun porphyry stock intruded into the Sungun anticline and Cretaceous limestones and Eocene andesitic-trachytic rocks, as depicted in Figure 2 [23, 24]. Based on this intrusion, a skarn deposit has been explored and mined since 120 years ago. However, alteration zones occur as a ring structure from potassic in the center to propylitic in the margins. The main mineralized zones include the supergene enrichment, hypogene, and skarn zones due to ore mineralogy [24]. The supergene enrichment and skarn zones were extended in the eastern and SE parts of this deposit according to the geological evidences [23, 25]. The major Cu minerals consist of chalcopyrite, chalcocite, covellite, and bornite. The skarn zone occurs in the intersection of Cretaceous limestones and porphyry stock in the eastern, especially SE, part of the Sungun deposit.

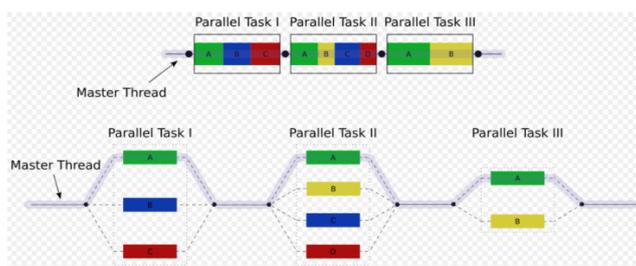


Figure 1. Threads in OpenMp [22].

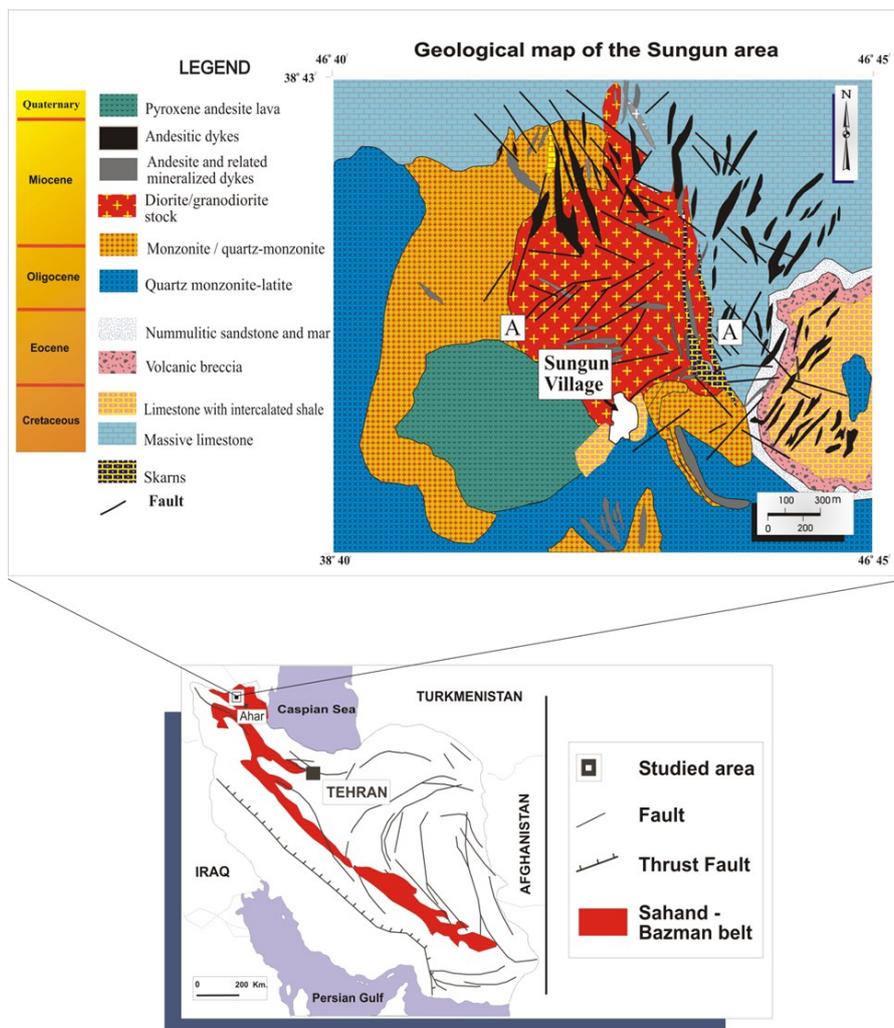


Figure 2. Sungun deposit in geological map of Iran (modified from Shahabpour and Doorandish, 2008; Stocklin, 1977) [26, 27] and geological map of Sungun deposit (modified from Mehrpartou, 1993; Afzal et al. 2011) [3, 23].

#### 4. Discussion

##### 4.1. Dataset

From 138 drillcores in the deposit, 38000 chip samples were collected at 2-m intervals. These samples were analyzed by XRF and ICP-MS for various elements related to Cu mineralization.

##### 4.2. S-V fractal modeling

At this stage, a block model with 482160 cells was created for the Sungun deposit. The cell sizes were 25 m, 25 m, and 12.5 m for the X, Y, and Z dimensions, indicated by Xstep, Ystep, and Zstep in the MATLAB and C++ codes. Distribution of Cu in this block model was estimated by ordinary kriging based on 38000 core samples by Datamine studio [3]. Moreover, the porphyry stock and skarn zones were modeled by the RockWorks 15 software package due to ore minerals, as depicted in Figure 3. The sub-surface data was used for this geological modeling, which included collar coordinates of drillcores, azimuth and dip (orientation), lithology, alteration,

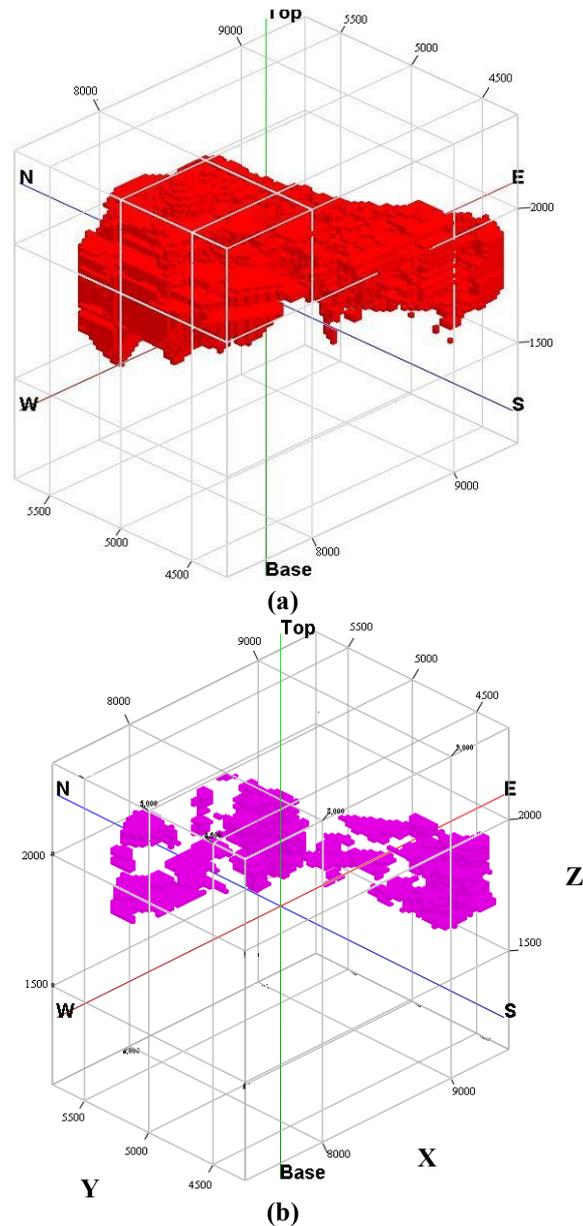
mineralogy, and zonation using the “lithoblending” algorithm. The estimated Cu block model was input for the C++ code, and power spectra for all cells were calculated. Then the S-V log-log plot was constructed for Cu, which represented six populations, as depicted in Figure 4. The first (right hand) and last (left hand) populations were noise and background populations in this log-log plot, respectively (Figure 4). High- and low-pass filters were separated from the other populations, and this data was returned to ore grades by Inverse Fast Fourier Transformation (IFFT). A band-pass filter is a device that passes power spectrum (PS) within a certain range and rejects (attenuates) PS outside that range. The function used to correct the edge effect is a type of guarded window, which is common in the application of FFT and IFFT. The results obtained by the S-V fractal modeling showed that copper mineralization started from 0.12% in porphyry stock, as illustrated in Figure 5. Furthermore, the main Cu mineralization began

form 0.46% as a major hypogene zone, which was extended in most parts of the Sungun deposit. In addition, the supergene enrichment zone contains Cu value  $\geq 0.82\%$ , located in the central and eastern parts of this deposit. At last, the skarn

zone happened with Cu values  $\geq 1.3\%$ , located in the SE part of the deposit (Figure 5). The Cu threshold values for the mineralized zones are indicated in Table 1.

**Table 1. Cu threshold values for mineralized zones.**

Mineralized zones	Hypogene	Supergene enrichment	Skarn
Cu threshold value (%)	0.46	0.82	1.3



**Figure 3. Porphyry stock (a) and skarn zone (b) 3D models in Sungun deposit based on geological data (coordinate is UTM system).**

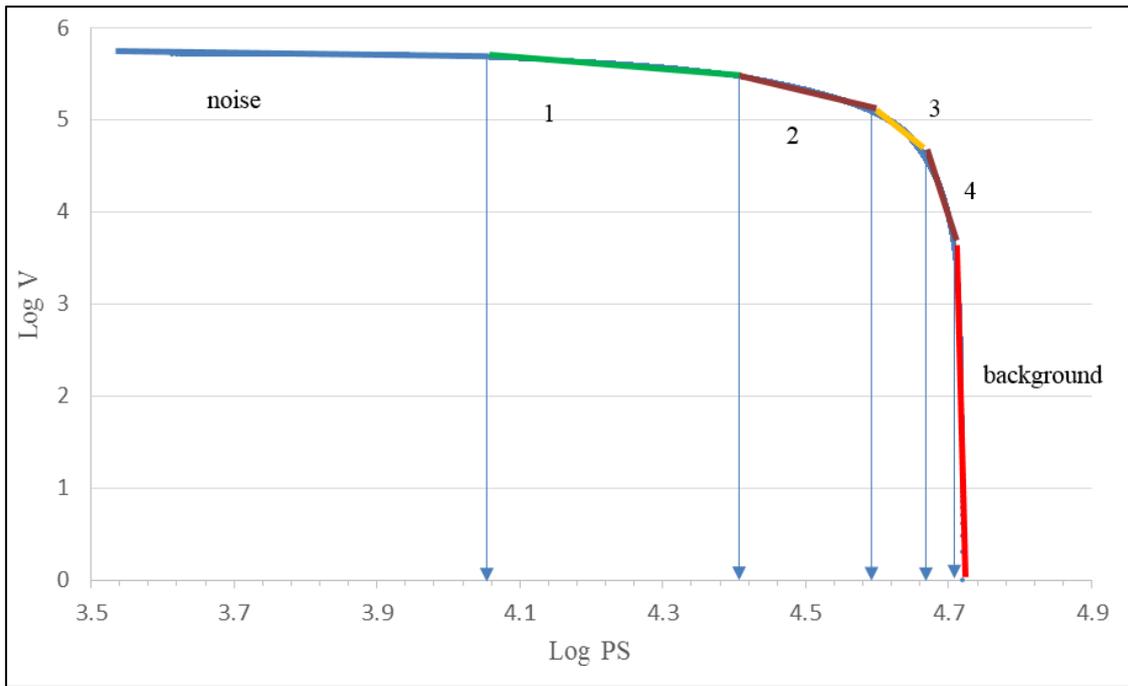


Figure 4. S-A log-log plot for Cu.

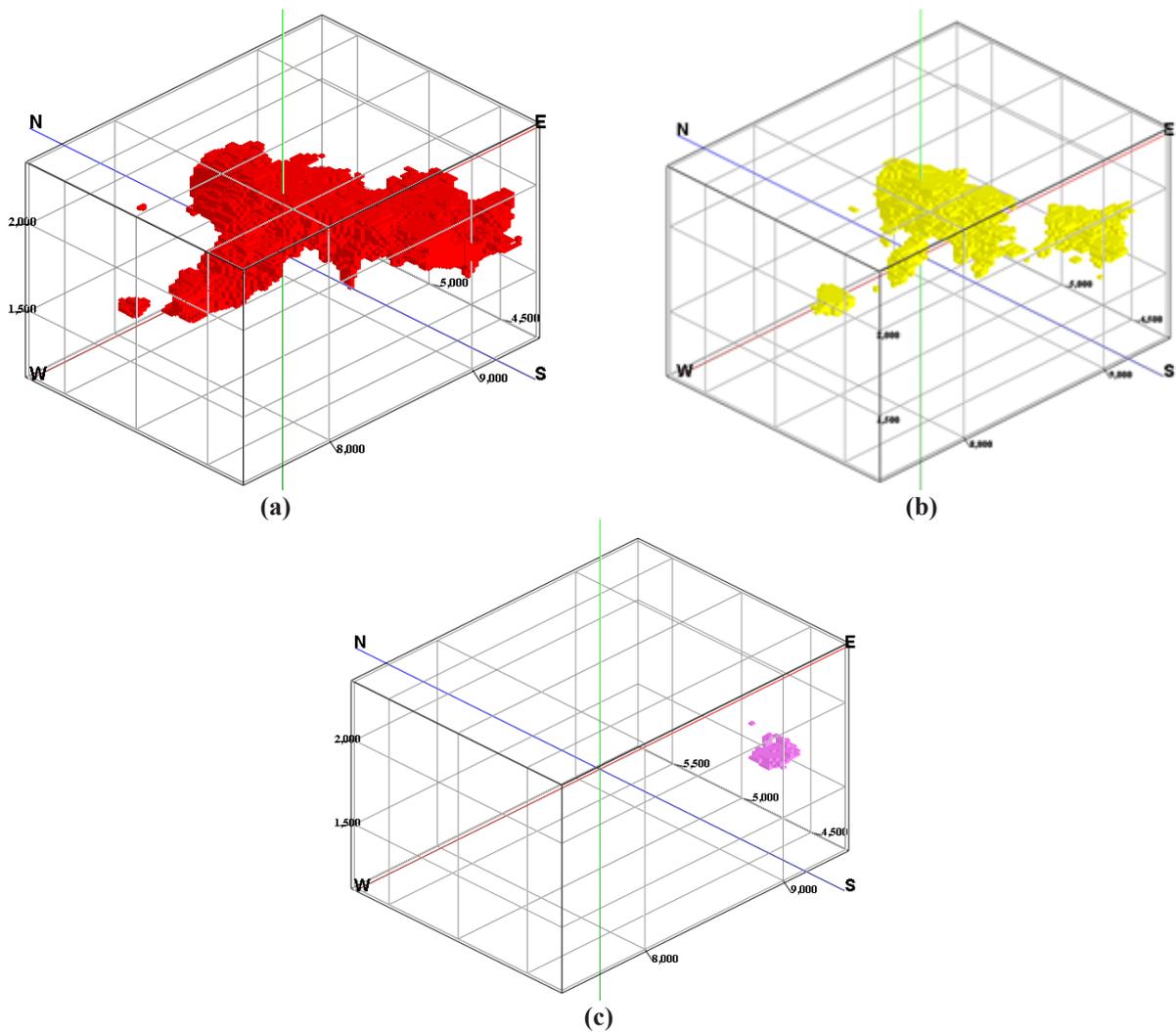


Figure 5. Copper mineralized zones based on S-V modeling: a) major hypogene; b) supergene enrichment; c) skarn.

### 4.3. Correlation between geological and fractal models

In this section, mineralized zones derived via the S-V and geological models were correlated to each other by the logratio matrix, which was established by Carranza (2011) [20]. This is a 2 × 2 matrix for calculation of overlapping between two binary models including mathematical and geological data based on the Overall Accuracy (OA), as depicted in Table 2. Utilizing the geological models (Figure 3), an intersection operation between a fractal mineralized zone model and different zones in the geological ore model (Figure 5) was performed to obtain the

number of voxels corresponding to each of the four classes of overlapped zones, as shown in Table 2.

This operation was carried out for comparison between the geological and fractal mineralized zones. The OAs are higher than 0.9 (90%), which represent a proper accuracy of the S-V C++ code (Table 3). High values for OAs show that there is a proper association between the geological zones with results derived via the S-V fractal modeling, especially for the skarn zone. Thus there is a good correlation between the fractal and geological mineralized zones, as depicted in Table 3.

**Table 2. Logratio matrix for calculation of overlapping between fractal and geological zones.**

		Geological model	
		Inside zone	Outside zone
Fractal model	Inside zone	True positive (A)	False positive (B)
	Outside zone	False negative (C)	True negative (D)
		Overall accuracy = (A+D)/(A+B+C+D)	

**Table 3. Comparison between mineralized zones between geological and fractal models.**

		Geological hypogene zone	
		Inside zone	Outside zone
Major hypogene zone via S-V model	Inside zone	34782	15678
	Outside zone	22978	408722
		Overall accuracy = 0.9198	
		Geological supergene enrichment zone	
		Inside zone	Outside zone
Supergene enrichment zone via S-V model	Inside zone	8203	1639
	Outside zone	3564	468754
		Overall accuracy = 0.9892	
		Geological skarn zone	
		Inside zone	Outside zone
Skarn zone via S-V model	Inside zone	190	257
	Outside zone	4433	477280
		Overall accuracy = 0.9902	

### 5. Conclusions

The results derived from this work indicate that the S-V model could be effectively applicable for detecting different mineralization zones in porphyry copper deposits, especially the supergene enrichment, hypogene, and skarn zones. In addition, this fractal model can interpret different stages of mineralization. The S-V model is a proper fractal modeling procedure, which uses the relationship between volumes containing and power spectrum values that enclose the volumes for power spectrum values of Cu associated with various zones, and satisfies power-law relationships. Furthermore, the S-V model is applicable to the power spectrum of ore element in other porphyry deposit types including Mo, Au,

and W, for which the spatial patterns for concentration values satisfy a multifractal model. In this work, the S-V fractal modeling delineated different mineralized zones. The thresholds for the supergene enrichment, hypogene, and skarn zones were detected as 0.82%, 0.46%, and 1.3%, respectively. Using the C++ programming synchronic MATLAB code decreased the time for the S-V fractal modeling for huge datasets such as Sungun mine. Furthermore, the results obtained by this code have a good accuracy based on the geological mineralized zones. In addition, this code can be used for huge deposits such as porphyry deposits. The results obtained by the logratio matrix shows that there is a good correlation between the mineralized zones obtained by the fractal and geological models. The

results obtained by the S-V fractal model show that the supergene enrichment zone extend in the central and eastern parts of the studied deposit. Moreover, skarn zone occur in the SE part of the Sungun deposit. Finally, C++ programming can be used for generation of fractal modeling based on the FFT process in different fields of geosciences, especially geochemical exploration.

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## استفاده از مدل سازی فرکتالی طیف توان - حجم برای شناخت زون های کانه‌زایی

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### چکیده:

هدف از این پژوهش، جدایش زون‌های کانه‌زایی مس در کانسار مس پورفیری سونگون واقع در شمال غربی ایران با استفاده از روش فرکتالی طیف‌توان- حجم و بر اساس داده‌های عمقی است. این فرآیند بر اساس مدل بلوکی تخمین خورده و تبدیل عیارها با تبدیل فوریه با استفاده از برنامه‌نویسی در دو نرم‌افزار MATLAB و C++ است. با توجه به منحنی لگاریتمی طیف توان- حجم، شش جامعه در این کانسار مشخص شدند. کانه‌زایی در تیپ‌های پورفیری و اسکارن از عیارهای ۰/۱۲ و ۱/۳ درصد آغاز می‌شود. همچنین زون سوپرژن از عیار ۰/۸۲ درصد آغاز می‌شود که در بخش شرقی کانسار قرار دارد. زون اسکارن در جنوب شرقی کانسار و در تقاطع با آهک‌های کرتاسه و توده پورفیری قرار دارد. برای صحت‌سنجی کار، میزان همپوشانی بین زون‌های زمین‌شناسی و زون‌های حاصل از فرکتال با استفاده از ماتریس logratio همپوشانی انجام گرفت که میزان بسیار مناسبی از همپوشانی را که مؤید دقت مناسب این روش است را نشان داد.

**کلمات کلیدی:** مدل فرکتالی طیف توان- حجم، برنامه‌نویسی C++، زون‌های کانه‌زایی مس، سونگون.

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