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HYPERIDEALS IN M-POLYSYMMETRICAL HYPERRINGS

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Abstract. An M-polysymmetrical hyperring $(R, +, \cdot)$ is an algebraic system, where $(R, +)$ is an M-polysymmetrical hypergroup, $(R, \cdot)$ is a semigroup and $\cdot$ is bilaterally distributive over $+$. We introduce the concept of hyperideals of an M-polysymmetrical hyperring and by using this concept, we construct an ordinary quotient ring. Finally, the fundamental theorem of homomorphism is derived in the context of M-polysymmetrical hyperrings.

1. Introduction

The concept of a hyperstructures was first introduced by Marty at the 8^{th} international Congress of Scandinavian Mathematicians. Unfortunately Marty had a short life (1911-1940) and he died young, during the Second World War when his airplane was shot down over the baltic sea, while he was going on a mission to Finland. The hyperstructure theory had applications to several domains of theoretical and applied mathematics[2, 4]. Mittas in his paper[16], which has been announced in the French Academy of Sciences, has introduced a special type of hypergroup that he has named polysymmetrical. Also, in the same paper, Mittas has given certain fundamental properties of this hyperstructure. Staring from the above paper and having called Mittas' structure M-polysymmetrical hypergroup (in order to distinguish this polysymmetrical hypergroup from other types of polysymmetrical hypergroups) Yatras has proceeded to a profound analysis of this hypergroup [23] and

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its subhypergroups[24] and homomorphisms[22]. The hyperings have appeared as a new class of algebraic hyperstructures more general than that of hyperfields, introduced by krasner[12] in the theory of valued fields. Different types of hyperrings have been proposed[3, 5, 13, 19, 18]. Mittas and Yatras in [17] introduced M-polysymmetrical hyperrings (M-P.HRs). In [5], the isomorphism theorems of ring theory are derived in the context of Krasner hyperrings. Now, in this paper, we investigate the hyperideals of M-polysymmetrical hyperrings.

An outline of the paper is as follows. After the introduction, in Sections 2 and 3, we briefly present introductory concepts concerning the M-polysymmetrical hypergroups, M-polysymmetrical hyperrings, and we recall some basic theorems. In Section 4, we introduce the hyperideals of M-polysymmetrical hyperrings and show that every quotient M-polysymmetrical hyperring by any hyperideal is a ring. In Section 5, we give the isomorphism theorems in the context of M-polysymmetrical hyperrings. Finally, in Section 6, we consider fundamental relations.

2. Basic definitions and results

We recall the definition of M-polysymmetrical hypergroup of [23] as follows.

Definition 2.1. A non-empty set H is called an M-polysymmetrical hypergroup (M-P.H.) if it is endowed with a hyperoperation $+ : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ is the set of all non-empty subsets of H, that satisfies the following axioms:

(1) $+$ is associative, i.e, for every $x, y, z \in H$ we have $x + (y + z) = (x + y) + z$;
(2) $+$ is commutative, i.e, for every $x, y \in H$, $x + y = y + x$;
(3) there exists $0 \in H$ such that for every $x \in H$ we have $x \in x + 0$;
(4) for every $x \in H$ there exists $x' \in H$ such that $0 = x + x'$, (x' is an opposite or symmetrical of x, with regard to considered 0, and the set of all the opposites $S(x) = \{x' \mid 0 = x + x'\}$ is the symmetrical set of x);
(5) for every $x, y, z \in H$, $x' \in S(x)$, $y' \in S(y)$ and $z' \in S(z)$, $x \in y + z$ implies that $x' \in y' + z'$.

Note that in the above definition, if A and B are two non-empty subsets of H and $x \in H$, then

$$A + B = \bigcup_{\substack{a \in A \\ b \in B}} a + b, \ x + A = \{x\} + A \text{ and } A + x = A + \{x\}.$$

Theorem 2.2. [23] *Let E be a set and G its subset with the structure of an abelian group. Also let 0 be its neutral element and for each $x \in G$, $-x$ be its opposite. If*

(1) *there exist a partition R of E and mapping one-to-one of quotient-set E/R on G such as for every $x \in G$, $f^{-1}(x) = C_R(x)$, [where $C_R(x)$ is the class of E mod R that contains the element x] and*
(2) $C_R(0) = \{0\}$,

then the hyperoperation $x \oplus y = f^{-1}[f(C_R(x)) + f(C_R(y))]$ defined on E, through the group G gives in E the structure of a M-P.H of which the group of reduction $E/(0)$ coincides to E/R.

Lemma 2.3. [23]

(1) *S(0)=0 therefore 0+0=0.*
(2) *For every $x \in H$ we have $0 \in 0 + x \Rightarrow x = 0$; more generally for every $x, y \in H$, $y \in x + y \Rightarrow x = 0$.*
(3) *0 is unique, named zero (of H).*
(4) *For every $x, y, z, w \in H$ we have*
$$(x + y) \cap (z + w) \neq \phi \Rightarrow x + y = z + w.$$
(5) *For every $x, y, z, w \in H$ we have*
$$z \in x + y \Rightarrow x + y = 0 + z.$$
(6) *For every $x, y, z \in H$*
$$z \in x + y \Rightarrow (\forall x^{'} \in S(x)) \ [y \in z + x^{'}].$$

Theorem 2.4. [23] *The sets $C(x) = 0 + x$, where x traverse H, form a partition of H and we have $x + y = 0 + x + y = (0 + x) + (0 + y)$. Moreover, for every $x, y \in H$, $x + y$ is a class of the partition and the set $G = \{C(x) \mid x \in H\}$ of these classes is an abelian group according to the operation $C(x) + C(y)$.*

According to the induced hyperoperation into H, for every $x, y \in H$, we define $x/y = \{t \in H \mid x \in t + y\}$. So, $t \in x/y \Rightarrow x \in t + y \Rightarrow t \in x + y'$ for every $y' \in S(y)$. Also, $t \in x + y' \Rightarrow x \in t + y \Rightarrow t \in x : y$. Consequently, $x/y = x + y'$ and since $x + y' = x + y' + 0 = x + S(y)$, it follows that $x/y = x + S(y)$.

Note that every M-P.H. $(H, +)$ has subhypergroups, for instance $\{0\}$, which is also M-P.H. (trivial case). Let $h \subseteq H$ be a subhypergroup of H and let $x \in h$. Then, obviously, by virtue of reproductiveness $x + h = h$, there is $y \in h$ such that $x \in x + y$ and therefore $y = 0$. Thus, $0 \in h$. Then we have $0 + x \subseteq h$, i.e., $C(x) \subseteq h$, so the class mod 0 of H that contains x being contained in h. By virtue of reproductiveness of h, $0 \in x + y$. Consequently, there is a symmetric $x^{'} \in S(x)$ of x

in h. Thus we have $0 + x' = C(x') = S(x) \subseteq h$. Finally, it is clear that h satisfies the axiom (5) and so we have the following important Theorem.

Theorem 2.5. [23]

(1) *Every subhypergroup of an M-P.H. is an M-P.H. with the same zero.*
(2) *A non void subset h of the M-P.H. $(H, +)$ is a subhypergroup if and only if, for every $x, y \in h$, $x/y \subseteq h$ is valid.*
(3) *Consequently we have that every subhypergroup of an M-P.H. is an M-polysymmetrical subhypergroup (M-P.SH.) of the same M-P.H.*
(4) *A non void subset h of the M-P.H. $(H, +)$ is a subhypergroup if and only if, for every $x, y \in h$, we have $x + S(y) \subseteq h$.*
(5) *A non void subset h of H is an M-P.SH. if and only if it is stable under the hyperoperation of $(H, +)$ and if it contains an element x of H, it includes also its symmetric set $S(x)$.*

We choose, for every class C, mod 0, of H one element x_c as distinguished element of the class (axiom of choice), let it be $\overline{G}$ the set of this elements. Then we consider the mapping $f : G \to \overline{G}$ with $f(C) = x_c \in \overline{G}$. Obviously, it is one-to-one and by considering this map, we define the following operation on $\overline{G}$: $x \perp y = f[C(x) + C(y)]$ for every $x, y \in \overline{G}$. Clearly, the above operation is commutative and associative. On the other hand, since $C(0) = \{0\}$, it follows that $0 \in \overline{G}$, and $C(0) + C(x) = 0 + x = C(x)$ holds $0 \perp x = x$ for every $x \in \overline{G}$. Also, if x' is a distinguished element of the class $S(x)$, $x \in \overline{G}$, we have

$$S(x) = C(x') \text{ so } x \perp x' = f[C(x) + C(x')] = f[C(0)] = 0.$$

Finally, we observe that for every $x, y \in \overline{G}$ we have

$$f[C(x) + C(y)] = f(C(x)) \perp f(C(y)).$$

Consequently, we have the following theorem.

Theorem 2.6. [23] *For every M-polysymmetrical hypergroup $(H, +)$, there is a subset $\overline{G}$ of H with abelian group's structure (with the neutral element zero of the hypergroup) isomorphic to the group of reduction $H/0$. We call the group $(\overline{G}, \perp)$, the group of choice of $(H, +)$.*

3. M-polysymmetrical hyperrings

In [17], Mittas and Yatras introduced M-polysymmetrical hyperrings. We recall a non-empty set R is an M-polysymmetrical hyperring

(M-P.HR) if it is endowed with a hyperoperation $+ : R \times R \to \mathcal{P}^*(R)$ and an operation $\cdot : R \times R \to R$ that satisfies the following axioms:

(1) $(R, +)$ is a M-polysymmetrical hypergroup,
(2) $(R, \cdot)$ is a semigroup,
(3) the multiplication is bilaterally distributive over addition, i.e, for all $x, y, z \in R$:

$$x \cdot (y + z) = x \cdot y + x \cdot z \text{ and } (x + y) \cdot z = x \cdot z + y \cdot z.$$

If in (3) we have inclusions instead of equalities, then we say that the M-polysymmetrical hyperring is weak.

Example 3.1. [17] Let $(K, +, \cdot)$ be a commutative algebraically close field with characteristic p and $n \neq 0$ be a coprime to p number and ξ_n be the multiplicative group of the n-th roots of the unity of K. We define the following hyperoperation in K:

$$x \oplus y = \{z \in K : x^n + y^n = z^n\},$$

the $x \oplus y$ is a class modulo ξ_n in K and it holds that

$$x \oplus y = (x\xi_n) \oplus (y\xi_n).$$

It can easily be proved that K endowed with the above hyperoperation is a M-P.H. This M-P.H combined with the multiplication in K give the hyperstructure $(K, \oplus, \cdot)$ which is an M-P.HR.

The following theorem, which is described in [17], gives us a method to construct an M-P.HR.

Theorem 3.2. *Suppose that E is a set with the structure of a multiplicative semigroup whose has an absorbing element, having as a subset, a ring $(A, +, \cdot)$ such that multiplication is the restriction of the corresponding one of the semigroup $(E, \cdot)$ in A and the zero (0) of A is the absorbing element of the semigroup $(E, \cdot)$. Then, if*

(1) *there is a partition R of E having the property*

$$xC_R(y) = C_R(x)y = C_R(xy) \quad for\ every\ x, y \in E,$$

(2) *there is a bijective mapping of the quotient set E/R on A such that for every $x \in A$*

$$f^{-1}(x) = C_R(x),$$

where $C_R(x)$ is the class of E mod(R) that contains element x,
(3) $C_R(0) = \{0\}$.

the hyperoperation $x \oplus y = f^{-1}[f(C_R(x)) + f(C_R(y))]$ defined on E through the group $(A, +)$ of the operation $x \circ y = xy$ (that is the operation *of the semigroup and the ring) makes E an M-P.HR whose ring of reduction $E/(0)$ coincides with E/R.*

Example 3.3. Let $E = \{0, 1, a, b, c, d, e\}$ be a semigroup such that its multiplication is according to the following table:

$\cdot$	0	1	a	b	c	d	e
0	0	0	0	0	0	0	0
1	0	1	a	b	c	d	e
a	0	a	1	c	b	e	d
b	0	b	c	0	0	c	b
c	0	c	b	0	0	b	c
d	0	d	e	b	c	1	a
e	0	e	d	b	c	a	1

and $(A = \{0, 1, b, e\}, +, \cdot)$ is a ring according to the following tables:

$+$	0	1	b	e
0	0	1	b	e
1	1	b	e	0
b	b	e	0	a
e	e	0	1	b

$\cdot$	0	1	b	e
0	0	0	0	0
1	0	1	b	e
b	0	b	0	b
e	0	e	b	1

Now, if we get a partition $R = \{C_R(0) = \{0\}, C_R(1) = \{1, a\}, C_R(b) = \{b, c\}, C_R(e) = \{d, e\}\}$ of E, by using Theorem 3.2 we can construct E as an M-P.HR such that the addition is according to the following table

$+$	0	1	a	b	c	d	e
0	0	$\{1, a\}$	$\{1, a\}$	$\{b, c\}$	$\{b, c\}$	$\{d, e\}$	$\{d, e\}$
1	$\{1, a\}$	$\{b, c\}$	$\{b, c\}$	$\{d, e\}$	$\{d, e\}$	0	0
a	$\{1, a\}$	$\{b, c\}$	$\{b, c\}$	$\{d, e\}$	$\{d, e\}$	0	0
b	$\{b, c\}$	$\{d, e\}$	$\{d, e\}$	0	0	$\{1, a\}$	$\{1, a\}$
c	$\{b, c\}$	$\{d, e\}$	$\{d, e\}$	0	0	$\{1, a\}$	$\{1, a\}$
d	$\{d, e\}$	0	0	$\{1, a\}$	$\{1, a\}$	$\{b, c\}$	$\{b, c\}$
e	$\{d, e\}$	0	0	$\{1, a\}$	$\{1, a\}$	$\{b, c\}$	$\{b, c\}$.

Lemma 3.4. *Let $(S, \cdot)$ be a semigroup with an absorbing element 0 and $\{A_x\}_{x\in S}$ be a family of non-empty sets such that $A_0 = \{\bar{0}\}$ and for every $x, y \in S, A_x \cap A_y = \varnothing$. Fix $\bar{x} \in A_x$, for all $x \in S$ and set $K = \cup_{x\in S} A_x$, then for every $a, b \in K$ there is $x, y \in S$ such that $a \in A_x,\ b \in A_y$. We define*

$$a \odot b = \overline{x \cdot y}, \forall (a, b) \in A_x \times A_y.$$

Then $(K, \odot)$ is a semigroup with absorbing element $\bar{0}$.

Proof. Let $a, b \in K$, then since for all $x, y \in S$, $A_x \cap A_y = \phi$, there exists unique elements $x, y \in S$ where $a \in A_x$, $b \in A_y$. Thus there exists unique element $z = x \cdot y$ such that $a \odot b = \overline{x \cdot y} = \overline{z}$. So $(S, \odot)$ is well defined. Also $(S, \odot)$ is associative, because for all $a, b, c \in K$ there exists $x, y, z \in S$ such that $a \in A_x$, $b \in A_y$, $c \in A_z$ and so

$$(a \odot b) \odot c = \overline{x \cdot y} \odot c = \overline{(x \cdot y) \cdot z} = \overline{x \cdot (y \cdot z)} = a \odot \overline{y \cdot z} = a \odot (b \odot c).$$

Finally, for all $a \in K$ there exists $x \in S$ such that $a \in A_x$ and so $a \odot \overline{0} = \overline{x \cdot 0} = \overline{0}$. Similarly $\overline{0} \odot a = \overline{0}$. □

The following theorem states a method for construction an M-P.HR of an arbitrary ring.

Theorem 3.5. *Let $(R, +, \cdot)$ be an arbitrary ring and $\{A_x\}_{x \in R}$ be a family of sets such that $A_0 = \{\bar{0}\}$ and for every $x, y \in R$, $A_x \cap A_y = \varnothing$. Set $K = \cup_{x \in R} A_x$, then for every $a, b \in K$ there is $x, y \in R$ such that $a \in A_x$, $b \in A_y$. We define*

$$a \oplus b = A_{x+y},$$

then $(K, \oplus, \odot)$ is a weak M-P.HR, where $\odot$ is defined in Lemma 3.4. Moreover, if the sum of products is a singleton then $(K, \oplus, \odot)$ is an M-P.HR.

Proof. Following [23, Theorem 3.1], $(K, \oplus)$ is an M-P.H. Also, considering previous lemma $(K, \oplus)$ is semigroup. It only remains that show $(K, \oplus, \odot)$ is weakly distributive. For every $a, b, c \in K$ there exists $x, y, z \in S$ such that $a \in A_x$, $b \in A_y$, $c \in A_z$ and

$$a \odot (b \oplus c) = \overline{x \cdot (y + z)} = \overline{x \cdot y + x \cdot z} \subseteq \overline{x \cdot y} \oplus \overline{x \cdot z} = a \odot b \oplus a \odot c.$$

□

By using the above theorem we make the following example.

Example 3.6. Suppose that the ring $(R, +, \cdot)$ is defined according to the following tables:

$+$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$\cdot$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	2	2
3	0	0	2	2

we set

$$A_0 = \{0\} \quad A_1 = \{a, b\} \quad A_2 = \{c\} \quad A_3 = \{d, e\}$$
$$K = \{0, a, b, c, d, e\}$$

then $(K, \oplus, \odot)$ is made according to the following tables is an M-P.HR:

$\oplus$	0	a	b	c	d	e
0	0	$\{a,b\}$	$\{a,b\}$	c	$\{d,e\}$	$\{d,e\}$
a	$\{a,b\}$	0	0	$\{d,e\}$	c	c
b	$\{a,b\}$	0	0	$\{d,e\}$	c	c
c	c	$\{d,e\}$	$\{d,e\}$	0	$\{a,b\}$	$\{a,b\}$
d	$\{d,e\}$	c	c	$\{a,b\}$	0	0
e	$\{d,e\}$	c	c	$\{a,b\}$	0	0

and

$\odot$	0	a	b	c	d	e
0	0	0	0	0	0	0
a	0	0	0	0	0	0
b	0	0	0	0	0	0
c	0	0	0	c	c	c
d	0	0	0	c	c	c
e	0	0	0	c	c	c

4. Hyperideals in M-P.HRs

In this section, for the first time we introduce the concept of hyperideal of an M-P.HR and present some results in this respect. Moreover, by using this concept, we construct a quotient ring.

Definition 4.1. Let I be a non-empty subset of R. We call I is a left (right)[bi-] hyperideal of R if I is a subhypergroup of $(R, +)$ and $RI \subseteq I(IR \subseteq I)[RIR \subseteq I]$. A hyperideal is a left and right hyperideal.

Theorem 4.2. *I is a left hyperideal if and only if*

(1) *$x \in I$ implies $x' \in I$, for all $x' \in S(x)$;*
(2) *$x, y \in I$ implies $x + y \subseteq I$,*
(3) *$x \in I$ implies $rx \in I$, for all $r \in R$.*

Proof. By Theorem 2.5, we have I is a subhypergroup of $(R, +)$ if and only if (1) and (2) hold. So, by the definition of left hyperideal, the proof is completed. □

Remark 4.3. In the above theorem, $(1), (2)$ are equivalent with $x + S(y) \subseteq I$, that it means I is also a subhypergroup of R.

Corollary 4.4. *Let I be a hyperideal of an M-P.HR $(R, +, \cdot)$. Then, $(I, +, \cdot)$ is an M-P.HR.*

Example 4.5. Clearly (0) and R are hyperideals of R.

Example 4.6. Let $(E, +, \cdot)$ be the M-P.HR in Example 3.3. If we get $I = \{0, b, c\}$ then I is a hyperideal of E.

Example 4.7. Let $(K, \oplus, \odot)$ be the M-P.HR in Example 3.6. Set $I = \{0, c\}$ and $J = \{0, a, b\}$. Then I, J are two hyperideals of K.

Lemma 4.8. *Let $(R, +, \cdot)$ be an M-P.HR. If $\{I_j\}_{j\in J}$ is a family of hyperideals of R, then $\cap I_j$ is also a hyperideal of R.*

Proof. By Theorem 4.2, proof is straightforward. □

Definition 4.9. Let R be an M-P.HR and X a subset of R, then the smallest, in the sense of inclusion, hyperideal of R which contains X is called generated hyperideal by X and denoted by $< X >$. If $X = \phi$ then $< X >= \{0\}$.

By considering Definition 4.9 and using Lemma 4.8, we conclude that $< X >= \cap I_j$, where $\{I_j\}_{j\in J}$ is a family of hyperideals of R containing X.

Proposition 4.10. *If R is a commutative M-P.HR and X is a non-empty subset of R, then the following set*

$$Y = \{t \mid t \in \sum_{i=1}^{m} r_i x_i + \sum_{i=1}^{n} x_i : \; r_i \in R, \; x_i \in X \cup S(X)\}$$

is a hyperideal of R containing X, where $S(X) = \cup_{x\in X} S(x)$.

Proof. We consider $x \in \sum_{i=1}^{m} r_i x_i + \sum_{i=1}^{n} x_i, \; y \in \sum_{i=1}^{m} r_i y_i + \sum_{i=1}^{n} y_i$ with $r_i \in R, \; x_i, y_i \in X \cup S(X)$ then according to the definition of M-P.HR we have $y' \in \sum_{i=1}^{m} r_i y'_i + \sum_{i=1}^{n} y'_i$ for $y' \in S(y), \; y'_i \in S(y_i)$ and $x+y' \subseteq \sum_{i=1}^{m} r_i x_i + \sum_{i=1}^{n} x_i + \sum_{i=1}^{m} r_i y'_i + \sum_{i=1}^{n} y'_i = \sum_{i=1}^{2m} r_i z_i + \sum_{i=1}^{2n} z_i$ where $z_j \in X \cup S(X)$. Thus $x + y' \subseteq Y$, i. e, $x + S(y) \subseteq Y$.

Also for every $r, r_i \in R, \; x_i \in X \cup S(X)$ we have $r(\sum_{i=1}^{m} r_i x_i + \sum_{i=1}^{n} x_i) = \sum_{i=1}^{m} (r r_i) x_i + \sum_{i=1}^{n} r x_i = \sum_{i=1}^{m+n} s x_i \subseteq Y$ where $s \in R$. Thus Y is a hyperideal of R. Also (if in Y set $r_i = 0$) Y is clearly containing X. □

Corollary 4.11. *If R is commutative M-P.HR the hyperideal Y in the above proposition is equal to $< X >$, namely*

$$< X >= \{t \mid t \in \sum_{i=1}^{m} r_i x_i + \sum_{i=1}^{n} x_i : r_i \in R, \; x_i \in X \cup S(X)\}$$

because it is clear that $< X >\subseteq Y$ (because Y is one of the hyperideals containing X). Also $Y \subseteq< X >$ because we have $X \subseteq< X >$ and since a hyperideal is also an M-P.SH then $S(X) \subseteq< X >$, thus $X \cup S(X) \subseteq< X >$ and so for every $r_i \in R, \; x_i \in X \cup S(X)$ we have $\sum_{i=1}^{n} (r_i x_i + x_i) \subseteq< X >$, therefore $Y \subseteq< X >$. Consequently $Y =< X >$.

Corollary 4.12. *If R is a commutative and unitary M-P.HR and X is a non-empty subset of R then*

$$\langle X\rangle = \{t: \ t\in \sum_{i=1}^{n} r_i x_i: \ r_i \in R, \ x_i \in X\}.$$

If R is an arbitrary M-P.HR and X is a subset of R, then

$$\langle X\rangle = \{t: \ t\in \sum_{i=1}^{m} r_i x_i + \sum_{i=1}^{n} x_i s_i + \sum_{i=1}^{v} r_i x_i s_i + \sum_{i=1}^{w} x_i: \ r_i, s_i \in R\},$$

where $x_i \in X \cup S(X)$.

We remained that on an M-P.H H for every $x \in H$ the set $C_0(x) = 0 + x$ construct a partition on H, which is denoted by $mod(0)$ or (0), such that

$$x \equiv y \ mod(0) \ \Leftrightarrow \ C_0(x) = C_0(y) \ \Leftrightarrow \ 0 + x = 0 + y$$

and the set $H/(0) = \{C_0(x) = 0 + x : x \in H\}$ is an abelian group, which we call it group of reduction of H. For every $x \in H, \ x' \in S(x)$ we have $S(x) = C(x')$.

Similarly, if h is an M-P.SH of H then the set $C_h(x) = x+h$ construct a partition on H, which is denoted by $mod(h)$ such that

$$x \equiv y \ mod(h) \ \Leftrightarrow \ C_h(x) = C_h(y) \ \Leftrightarrow \ x + h = y + h$$

and the set of equivalence classes $H/(h) = \{C_h(x) : x \in H\}$ is an abelian group, which we call it group of reduction of H by h.

Theorem 4.13. *Let I be a hyperideal of an M-P.HR R. On the set $R/I = \{C_I(x) = x + I \mid x \in R\}$ of equivalence classes $mod(I)$ if we define*

$$C_I(x) + C_I(y) = C_I(z) \text{ for all } z \in x + y,$$
$$C_I(x) \cdot C_I(y) = C_I(xy),$$

then we have $(R/I, +, \cdot)$ as a ring which is called the ring of reduction of M-P.HR R by I and is denoted by $R(I)$.

Proof. The operation $+$ is well-defined because

(1) The set $\{C_I(z) : z \in x+y\}$ is a singleton, because $z \in x+y \ \Rightarrow$ $x + y = 0 + z$ and so

$$C_I(x) + C_I(y) = x + y + I = 0 + z + I = z + I = C_I(z).$$

(2) For every $x_1, y_1, x_2, y_2 \in R$ if $C_I(x_1) = C_I(x_2)$ and $C_I(y_1) = C_I(y_2)$ then $x_1 + x_2' \subseteq I$ and $y_1 + y_2' \subseteq I$ with $x_2' \in S(x_2), \ y_2' \in S(y_2)$ and therefore since $S(x_2' + y_2') = C_0((x_2 + y_2)')$ we have $x_1 + y_1 + (x_2 + y_2)' \subseteq I$ and so $x_1 + y_1 + I = x_2 + y_2 + I$ which means $C_I(x_1 + y_1) = C_I(x_2 + y_2)$.

Also the operation $\cdot$ is well-defined because if $C_I(x_1) = C_I(x_2)$ and $C_I(y_1) = C_I(y_2)$ then $x_1 + x_2' \subseteq I$ and $y_1 + y_2' \subseteq I$. Since I is a hyperideal we have

$$x_1y_1 + x_2'y_2 = x_1(y_1 + y_2') + (x_1 + x_2')y_2 \subseteq I \;\Rightarrow\; x_1y_1 + 0 + x_2'y_2 \subseteq I$$

Since $x_2'y_2 \in S(x_2y_2)$ and $S(x_2y_2) = 0 + x_2'y_2$ then $x_1y_1 + (x_2y_2)' \subseteq I$. □

Theorem 4.14. *Let I and J be two hyperideals of an M-P.HR R and $I \subseteq J$. Then, $C_I(J)$ is a hyperideal of the reduction of M-P.HR R.,*

Proof. Let $C_I(r) \in R(I)$ and $C_I(a) \in C_I(J)$ then by Theorem 4.13, $C_I(a)C_I(r) = C_I(ar) \in C_I(J)$, because J is a hyperideal and $ar \in J$. □

Corollary 4.15. *Let R be an M-P.HR. On the set $R/(0) = \{C_0(x) = 0 + x : x \in R\}$ of equivalence classes mod(0) if we define*

$$C_0(x) + C_0(y) = C_0(z) \text{ for all } z \in x + y$$
$$C_0(x) \cdot C_0(y) = C_0(xy),$$

then we have $(R/(0), +, \cdot)$ as a ring which is called the ring of reduction of M-P.HR R denoted by $R(0)$.

Proposition 4.16. *If the M-P.HR R is M-polysymmetrical hyperfield (M-P.HF) and I be a hyperideal of R then, the set R/I of the equivalence classes mod(I) is a field called field of reduction of M-P.HR R by I denoted by $F(I)$.*

Proposition 4.17. *If R is a commutative M-P.HR and I is a hyperideal of R, then R/I is also a commutative ring.*

Conversely we have the proposition:

Proposition 4.18. *If the cancellation law for multiplication holds in R then it holds in R/I as well.*

5. Homomorphisms

In this section, we give the definition of homomorphism between M-P.HRs and we present some its properties. Finally, we prove the fundamental theorem of homomorphisms.

Definition 5.1. Let R, R_1 be an M-P.HRs. A mapping φ from R into R_1 is said to be a normal homomorphism if for all $a, b \in R$

$$\varphi(a + b) = \varphi(a) + \varphi(b) \text{ and } \varphi(ab) = \varphi(a)\varphi(b)$$

Remark 5.2. $\varphi(0) = 0_1$, because $\varphi(0) = \varphi(0+0) = \varphi(0) + \varphi(0) \Rightarrow \varphi(0) + (\varphi(0))' = \varphi(0) + \varphi(0) + (\varphi(0))'$, where $(\varphi(0))' \in S(\varphi(0)) \Rightarrow 0_1 = \varphi(0) + 0_1$, therefore by [24], $\varphi(0) = 0_1$.

Regarding the above definition we have the following propositions.

Proposition 5.3. (1) *$\varphi(C_0(x)) = C_0(\varphi(x))$, thus $\varphi(S(x)) = S(\varphi(x))$.*
(2) *$\varphi(C_I(x)) = C_{\varphi(I)}(\varphi(x))$, which I is a hyperideal of R.*

Proof. (1) We have $\varphi(C_0(x)) = \varphi(0 + x) = \varphi(0) + \varphi(x) = 0_1 + \varphi(x) = C_{0_1}(\varphi(x))$. Since $\varphi(x') \in S(\varphi(x))$, where $x' \in S(x)$, $(\varphi(x))' \in S(\varphi(x))$ (because $0_1 = \varphi(0) = \varphi(x + x') = \varphi(x) + \varphi(x') \Rightarrow \varphi(x') \in S(\varphi(x))$), we have $\varphi(S(x)) = \varphi(C_0(x')) = \varphi(0 + x') = 0_1 + \varphi(x') = C_{0_1}(\varphi(x')) = S(\varphi(x))$.
(2) We have $\varphi(C_I(x)) = \varphi(x + I) = \varphi(x) + \varphi(I) = C_{\varphi(I)}(\varphi(x))$. □

Proposition 5.4. (1) *The homomorphic image $\varphi(I)$ of every hyperideal I of R is a hyperideal of $\varphi(R)$ (and if φ is onto, a hyperideal of R_1),*
(2) *The image inverse $\varphi^{-1}(I_1)$ of every hyperideal I_1 of $\varphi(R)$ is a hyperideal of R and the kernel $\aleph_{(\varphi)} = \varphi^{-1}(0_1) \subseteq \varphi^{-1}(I_1)$.*

Proof. (1) Let $a_1 \in \varphi(I)$ then there is $a \in I$ such that $a_1 = \varphi(a) \Rightarrow a_1 + \varphi(I) = \varphi(a) + \varphi(I) = \varphi(a + I) = \varphi(I)$. Also for all $r_1 \in \varphi(R)$, $a_1 \in \varphi(I)$ there is $r \in R$, $a \in I$ such that $r_1 = \varphi(r)$, $a_1 = \varphi(a) \Rightarrow r_1a_1 = \varphi(r)\varphi(a) = \varphi(ra) \in \varphi(I)$. If φ is onto and $r_1 \in R_1$ there is $r \in R$ such that $r_1 = \varphi(r)$, also for every $a_1 \in \varphi(I)$ there is $a \in I$ such that $a_1 = \varphi(a)$ so $r_1a_1 = \varphi(r)\varphi(a) = \varphi(ra) \in \varphi(I)$.
(2) Let $x, y \in \varphi^{-1}(I_1)$ then there is $x_1, y_1 \in I_1$ such that $x \in \varphi^{-1}(x_1)$, $y \in \varphi^{-1}(y_1))$ so $\varphi(x + S(y)) = \varphi(x) + \varphi(S(y)) = \varphi(x) + S(\varphi(y)) \subseteq \varphi(\varphi^{-1}(x_1)) + S(\varphi(\varphi^{-1}(y_1))) = x_1 + S(y_1) \subseteq I_1$. Also let $r \in R$, $a \in \varphi^{-1}(I_1)$ then there is $r_1 \in R_1$, $a_1 \in I_1$ such that $r = \varphi^{-1}(r_1)$, $a = \varphi^{-1}(a_1) \Rightarrow ra = \varphi^{-1}(r_1)\varphi^{-1}(a_1)$, so $\varphi(ra) = \varphi(\varphi^{-1}(r_1))\varphi(\varphi^{-1}(a_1)) = r_1a_1 \in I_1$, therefore $ra \in \varphi^{-1}(I_1)$.

□

Corollary 5.5. *The kernel $\aleph_{(\varphi)}$ of the homomorphism φ, is a hyperideal of R.*

Proposition 5.6. *Let φ be a normal homomorphism from R into R_1 and $\aleph_{(\varphi)} = \{0\}$, then*

$$\varphi(x) \equiv \varphi(y)\ mod(0_1) \Leftrightarrow x \equiv y\ mod(0),$$

conversely if φ is one to one then $\aleph_{(\varphi)} = \{0\}$.

Proof. Let $x, y \in R$ be such that $\varphi(x) \equiv \varphi(y)\ mod(0_1)$, then $0_1+\varphi(x) = 0_1 + \phi(y)$, so $\varphi(x) + \varphi(y') = \varphi(y) + \varphi(y')$, where $y' \in S(y)$. It follows that $\varphi(x + y') = \varphi(y + y') = \varphi(0) = 0_1$. Thus, if $\aleph_{(\varphi)} = \{0\}$ then $x + y' = 0 \ \Rightarrow\ 0 + x = 0 + y \ \Rightarrow\ x \equiv y\ mod(0)$. From right to left is similarly proved.

For the converse, let $x \in \aleph_{(\varphi)}$, then $\varphi(x) = 0_1 = \varphi(0)$ and since φ is one to one, $x = 0$. Thus, $\aleph_{(\varphi)} = \{0\}$. □

In the following we state the isomorphism theorems in the context of M-polysymmetrical hyperrings.

Theorem 5.7. *Let φ be a normal homomorphism from R into R_1, then $R/\aleph_{(\varphi)} \cong \varphi(R)/(0_1)$ (where $R/\aleph_{(\varphi)}$ is the reduced ring of R by $\aleph_{(\varphi)}$ and $\varphi(R)/(0_1)$ is the reduced ring of $\varphi(R)$).*

Proof. We define the mapping

$$\psi : R/\aleph_{(\varphi)} \to \varphi(R)/(0_1)$$

by setting $\psi(C_{\aleph_{(\varphi)}}(x)) = C_{0_1}(\varphi(x))$ for all $C_{\aleph_{(\varphi)}}(x) \in R/\aleph_{(\varphi)},\ C_{0_1}(\varphi(x)) \in \varphi(R)/(0_1)$.

(1) We first prove that ψ is well-defined and one to one,

$$\begin{array}{rcl} C_{\aleph_{(\varphi)}}(x) = C_{\aleph_{(\varphi)}}(y) & \Leftrightarrow & x + \aleph_{(\varphi)} = y + \aleph_{(\varphi)} \\ & \Leftrightarrow & x + y' \subseteq \aleph_{(\varphi)} \\ & \Leftrightarrow & \varphi(x + y') = 0_1 \\ & \Leftrightarrow & \varphi(x) + \varphi(y') = 0_1 \\ & \Leftrightarrow & 0_1 + \varphi(x) = 0_1 + \varphi(y) \\ & \Leftrightarrow & C_{0_1}(\varphi(x)) = C_{0_1}(\varphi(y)). \end{array}$$

(2) ψ is a homomorphism because

$$\begin{array}{rcl} \psi(C_{\aleph_{(\varphi)}}(x) + C_{\aleph_{(\varphi)}}(y)) & = & \psi[(x + \aleph_{(\varphi)}) + (y + \aleph_{(\varphi)})] \\ & = & \{\psi(z + \aleph_{(\varphi)}) :\ z \in x + y\} \\ & = & \{\varphi(z) + 0_1 :\ z \in x + y\} \\ & = & \varphi(x + y) + 0_1 \\ & = & \varphi(x) + \varphi(y) + 0_1 \\ & = & (\varphi(x) + 0_1) + (\varphi(y) + 0_1) \\ & = & C_{0_1}(\varphi(x)) + C_{0_1}(\varphi(y)) \\ & = & \psi(C_{\aleph_{(\varphi)}}(x)) + \psi(C_{\aleph_{(\varphi)}}(x)), \end{array}$$

also

$$\begin{array}{rcl} \psi(C_{\aleph_{(\varphi)}}(x) \cdot C_{\aleph_{(\varphi)}}(y)) & = & \psi[(x + \aleph_{(\varphi)}) \cdot (y + \aleph_{(\varphi)})] \\ & = & \psi[(xy + \aleph_{(\varphi)}) = \varphi(xy) + 0_1 \\ & = & \varphi(x)\varphi(y) + 0_1 \\ & = & (\varphi(x) + 0_1) \cdot (\varphi(y) + 0_1) \\ & = & \psi(C_{\aleph_{(\varphi)}}(x)) \cdot \psi(C_{\aleph_{(\varphi)}}(y)). \end{array}$$

(3) ψ is clearly onto.

$\square$

Example 5.8. Let $R_1 = \{0,1,2,3,4\}$ be an M-polysymmetrical hyperring by the following tables:

$+$	0	1	2	3	4
0	0	$\{1,2\}$	$\{1,2\}$	3	4
1	$\{1,2\}$	0	0	3	3
2	$\{1,2\}$	0	0	4	3
3	3	4	4	0	$\{1,2\}$
4	4	3	3	$\{1,2\}$	0

,

$\cdot$	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	3	3
4	0	0	0	3	3

Also, let $(K, \oplus, \odot)$ be the M-polysymmetrical hyperring studied in Example 3.6. Define $\varphi : K \to R_1$ by $\varphi(0) = 0, \varphi(a) = 1, \varphi(b) = 2, \varphi(c) = 3, \varphi(d) = 4, \varphi(e) = 4$. Then it is not difficult to see that $\aleph_{(\varphi)} = \{0\}$. Also, $\varphi(K) = R_1$. So $K/\aleph_{(\varphi)} = K/(0)$. It is easy to see that $K/\aleph_{(\varphi)} \cong R$ and $R_1/(0_1) \cong R$, when R is a ring defined according to the following tables:

$+$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

,

$\cdot$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	2	2
3	0	0	2	2

Therefore, we have $K/\aleph_{(\varphi)} \cong R_1/(0_1)$.

Corollary 5.9. *If the normal homomorphism $\varphi : R \to R_1$ is onto, then $R/\aleph_{(\varphi)} \cong R_1/(0_1)$.*

Corollary 5.10. *If $\aleph_{(\varphi)} = \{0\}$, then the ring of reduction $R/(0)$ and $\varphi(R)/(0_1)$ are isomorphic.*

Lemma 5.11. *If R is an M-P.HR and I, J are hyperideals of R, then $I + J$ is a hyperideal of R.*

Lemma 5.12. *If R is an M-P.HR and I, J are hyperideals of R such that $I \subseteq J$, then I is a hyperideal of J.*

Proof. By Theorem 4.2 and Corollary 4.4, the proof is straightforward. $\square$

Theorem 5.13. *If R is an M-P.HR and I, J are hyperideals of R, then the reduced ring $I/(I \cap J)$ is isomorphic to reduced ring $(I + J)/J$.*

Proof. We define the mapping

$$\varphi : I/(I \cap J) \to (I + J)/J$$

by setting $\varphi(C_{I\cap J}(a)) = C_J(a)$ for all $a \in I$, it is proved φ is a normal isomorphism and so $I/(I \cap J) \cong (I + J)/J$. □

Lemma 5.14. *If R is an M-P.HR and I, J are hyperideals of R such that $I \subseteq J$, then J/I is an ideal of R/I.*

Theorem 5.15. *If R is an M-P.HR and I, J are hyperideals of R such that $I \subseteq J$, then $(R/I)/(J/I) \cong R/J$.*

Proof. We define the mapping

$$\varphi : R/I \to R/J$$

by setting $\varphi(C_I(x)) = C_J(x)$ for all $x \in R$, it is proved φ is an onto normal homomorphism such that $\aleph_{(\varphi)} = J/I$ and so by using Corollary 5.9 the proof is completed. □

6. Fundamental relation on M-P.HRs

The notion of fundamental relation on hypergroups was introduced by Koskas [11], and then studied by Corsini [1], Freni [7, 8, 9] and Gutan [10], Vougiouklis [20, 21], Davvaz et. al. [6]. In [7], Freni firstly proved that the relation β is transitive in every hypergroup. The relations γ and γ^* were firstly introduced and analyzed by Freni [8]. He proved that the relation γ on hypergroup is transitive and $\gamma = \gamma^*$. Also, Freni [9] determined a family $P_\sigma(H)$ of subsets of a hypergroup H such that the geometric space $(H, P_\sigma(H))$ is strongly transitive. The letter γ already has been used for the corresponding fundamental relation on hyperrings by Vougiouklis [20]. Thus, there is a confusion on the symbolism. In 1990, Vougiouklis at the fourth AHA congress [20], introduced the concept of fundamental relation γ on a hyperring, and then it studied by himself and many authors, for example see [5, 6, 14, 21]. In this section, we use Γ instead of γ for hyperrings.

Recently, Mirvakili and Davvaz [15] proved that the relation Γ on every hyperfield is an equivalence relation and $\Gamma = \Gamma^*$. In [6], Davvaz and Vougiouklis introduced a new strongly regular equivalence relation on a hyperring such that the set of quotients is an ordinary commutative ring and later some properties of relation α are studied[14].

Definition 6.1. [20] Let R be a hyperring. We define the relation Γ as follows :
$x \, \Gamma \, y \;\Leftrightarrow\; \exists n \in \mathbb{N}, \exists k_i \in \mathbb{N}, \exists (x_{i1}, \ldots, x_{ik_i}) \in R^{k_i}, 1 \leq i \leq n$, such that $\{x, y\} \subseteq \sum_{i=1}^{n} \left(\prod_{j=1}^{k_i} x_{ij} \right)$.

Definition 6.2. [6] Let R be a hyperring. We consider the relation α as follows:
$x \,\alpha\, y \iff \exists n \in \mathbb{N}$, $\exists (k_1, \ldots, k_n) \in \mathbb{N}^n$, $\exists \sigma \in \mathbb{S}_n$ and $[\exists (x_{i1}, \ldots, x_{ik_i}) \in R^{k_i}$, $\exists \sigma_i \in \mathbb{S}_{k_i}$, $(i = 1, \ldots, n)]$ such that $x \in \sum_{i=1}^{n} \left(\prod_{j=1}^{k_i} x_{ij} \right)$ and $y \in \sum_{i=1}^{n} A_{\sigma(i)}$, where $A_i = \displaystyle\prod_{j=1}^{k_i} x_{i\sigma_i(j)}$.

The relation α and Γ are reflexive and symmetric. Let α^* and Γ^* be the transitive closure of α and Γ. Then we recall the following theorem:

Theorem 6.3. [20, Theorem 1][6, Theorem 4] *Let $(R, +, \cdot)$ be a hyperring.*

(1) *Γ^* is the smallest equivalence relation on R such that the quotient R/Γ^* is a ring.*
(2) *α^* is the smallest equivalence relation on R such that the quotient R/α^* is a commutative ring.*

Theorem 6.4. *$\Gamma^* = mod(0)$ and so $R/\Gamma^* \cong R(0)$.*

Proof. It easy to see that $\Gamma^*(x) = C_0(x)$ for all $x \in R$. Hence, $\Gamma^* = mod(0)$. □

Theorem 6.5. *In every R, $\beta_+ = \beta_+^* = \Gamma = \Gamma^*$, and so Γ is an equivalence relation.*

Proof. It is clear that if $x\beta_+ y$, then $x\Gamma y$. Now, if $x\Gamma y$, then there exist $n \in \mathbb{N}$, $k_i \in \mathbb{N}$, and $(x_{i1}, \ldots, x_{ik_i}) \in R^{k_i}$, where $1 \leq i \leq n$ such that $\{x, y\} \subseteq \sum_{i=1}^{n} \left(\prod_{j=1}^{k_i} x_{ij} \right)$. But $z_i = \prod_{j=1}^{k_i} x_{ij}$ is singleton and so $\{x, y\} \subseteq \sum_{i=1}^{n} u_i$. This means $x\beta_+ y$. Freni [8] proved that in any hypergroup $(R, +)$, $\beta_+^* = \beta_+$ and this completes the proof. □

Theorem 6.6. *$R/\alpha^* \cong R(0)/\gamma_{\cdot}^*$.*

Proof. By Theorem 6.4, $R/\Gamma^* \cong R(0)$. Since $+$ is commutative, it follows that

$$R/\alpha^* \cong R/\Gamma^*/\gamma_{\cdot}^* \cong R(0)/\gamma_{\cdot}^*.$$

□

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HYPERIDEALS IN M-POLYSYMMETRICAL HYPERRINGS

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ابرایده‌آل‌های ابرحلقه‌های M-چندمتقارن

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یک ابرحلقه M-چندمتقارن $(R,+,\cdot)$ یک دستگاه جبری است که در آن $(R,+)$ ابرگروه M-چندمتقارن، $(R,\cdot)$ نیم‌گروه و عمل ضرب روی ابرعمل جمع خاصیت توزیع‌پذیری دوطرفه دارد. در این مقاله، ما مفهوم ابرایده‌آل‌ها در یک ابرحلقه M-چندمتقارن را مطالعه می‌کنیم و با استفاده از این مفهوم حلقه خارج‌قسمتی را به‌دست می‌آوریم. در نهایت، قضیه اساسی همریختی برای ابرحلقه‌های M-چندمتقارن بیان می‌شود.

کلمات کلیدی: ابرحلقه M-چندمتقارن، ابرایده‌آل، حلقه کاهش‌یافته.