

Prediction of the deformation modulus of rock masses using Artificial Neural Networks and Regression methods

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Abstract

Static deformation modulus is recognized as one of the most important parameters governing the behavior of rock masses. Predictive models for the mechanical properties of rock masses have been used in rock engineering because direct measurement of the properties is difficult due to time and cost constraints. Using empirical methods, the deformation modulus is estimated indirectly from classification systems. This paper presents the results of the application of Artificial Neural Networks (ANN) technique and Regression models to estimate the deformation modulus of rock masses. A database, including 224 actual measured deformation modulus, Uniaxial Compressive Strengths of the rock (UCS), and Rock Mass Rating (RMR) was established. Data were collected from different projects. To predict E_m by regression, a nonlinear regression method was used. This model showed the coefficient correlation of 0.751 and mean absolute percentage error (MAPE) of 9.911%. Also a three-layer ANN was found to be optimum, with an architecture of two neurons in the input layer, four neurons in the hidden layer and one neuron in the output layer. The correlation coefficient determined for deformation modulus predicted by the ANN was 0.786 and the quantity of MAPE was 6.324%. With respect to the results obtained from the two models, the ANN technique was shown to be better than the regression model because of its higher accuracy.

Keywords: Rock mass modulus, Neural Networks, Regression method, discontinuity.

1. Introduction

The design process of geo-structures built in and on rock strata requires proper input parameter representing the in-situ rock mass characteristics, such as the joint frequency, weathering state, joint conditions and rock mass strength. The design parameter, however, is easily tested in laboratory experiments only on small intact rock specimens. Among the many design properties, the deformation modulus of in-situ rock mass is a crucial parameter and has a vital importance for the design and successful execution of rock

engineering projects, because the deformation modulus is the best representative parameter of the pre-failure mechanical behavior of the rock material and of a rock mass. The modulus of deformation is also very important in the interpretation of monitored deformation around underground opening.

There are several methods to determine the deformation modulus of rock mass directly, by field or in-situ tests; like pressure meter [1], dilatometer [2], plate jacking [3], plate loading [4], radial jacking, flat jack, cable jack, and geophysical methods [5-7]. Although in-situ techniques are the best methods to determine deformability modulus of rock masses, they are time-consuming, expensive and can only be performed when the exploration space are excavated. This constraint forced the investigators to develop an empirical equation for indirect estimation of the deformation modulus of rock masses based on other rock mass properties that can be easily determined at low cost such as

RMR, the tunneling quality index (Q), GSI, etc. Many researchers developed some empirical equations to estimate rock mass deformation modulus, but the suggested equations are based on a limited data set with a low correlation coefficient.

This paper attempts to produce a reliable empirical equation to estimate the deformation modulus of different rock types with specific rock mass conditions using rock mass classification systems and intact rock properties. To do this, Artificial Neural Networks (ANN) and regression modeling are applied on 224 data each of which contains RMR, UCS and measured deformation modulus.

2. Review of previous researches

The first correlation between RMR and rock mass deformation modulus was proposed by Bieniawski, as [8]:

$$E_m(GPa) = 2RMR - 100 \quad \text{(for RMR} \ge 50) \tag{1}$$

Serafim-Pereira then proposed the following expression based on RMR system [9],

$$E_m(GPa) = 10^{(RMR-10)/40}$$
 (for RMR < 50) (2)

Nicholson and Bieniawski presented the following relation considering RMR the elasticity modulus of the intact rock E [10]:

$$E_m/E = 0.01 \times (0.0028.RMR^2 + 0.9^{\frac{RMR}{22.82}})$$
(3)

Mitri et al. also obtained a formula including elasticity modulus of intact rock materials and RMR [11]:

$$E_m(GPa) = E[0.5(1 - \cos(\pi . RMR/100))]$$
(4)

where, E is the elasticity modulus of intact rock material in GPa.

Galera et al. derived an empirical formula based on Serafim-Pereira work considering elasticity modulus of intact rock materials [2]:

$$E_m/E = e^{(RMR - 100)/36}$$
(5)

Grimstad and Barton presented a new empirical model based on Q system. This equation is valid for Q greater than 1 [12]:

$$E_m(GPa) = 25\log Q \tag{6}$$

Hoek and Brown found the following relation for the E_m based on the GSI which is valid for rock with σ_c less than 100 MPa [13]:

$$E_m(GPa) = \sqrt{\frac{\sigma_c}{100}} \times 10^{\frac{GSI-10}{40}}$$
(7)

where, σ_c is the compressive strength of rock in MPa.

Later, Hoek et al. empirically estimated E_m based on GSI and D (Disturbance factor) in the following form [14]:

$$E_{m}(GPa) = (1 - \frac{D}{2}) \times \sqrt{\frac{\sigma_{c}}{100}} \times 10^{\frac{GSI-10}{40}}$$

$$\sigma_{c} \le 100 \text{ MPa}$$

$$E_{m}(GPa) = (1 - \frac{D}{2}) \times 10^{\frac{GSI-10}{40}}$$

$$\sigma_{c} > 100 \text{ MPa}$$
(8)

Hoek and Diecrich also suggested the following formulas based on GSI [15]:

$$E_m(MPa) = 100 \left(\frac{1 - D/2}{1 + e^{((75 + 25D - GSI)/11)}} \right)$$
(10)

$$E_m(MPa) = E\left(0.02 + \frac{1 - D/2}{1 + e^{((75 + 25D - GSI)/11)}}\right)$$
(11)

Palmstrom recommended that GSI is useful for weak rock mass with RMR<20 [16].

Gardner derived the following expression based on RQD and elasticity modulus of intact rock [17]:

$$E_m / E = 0.0231 \times RQD - 1.32 (\ge 0.15)$$
(12)

Zhang and Einstein proposed the following relations based on RQD system [18]:

$$E_m / E = 0.2 \times 10^{0.0186 RQD - 1.91}$$
 (Lower bound) (13)

$$E_m / E = 1.8 \times 10^{0.0186 RQD - 1.91}$$
 (Upper bound) (14)

$$E_m / E = 10^{0.0186 RQD - 1.91}$$
 (Mean) (15)

Palmstrom and Singh presented the following equations based on RMi classification system [2]:

$$E_m = 5.6 \,\mathrm{RMi}^{0.375}$$
 (for RMi>0.1) (16)

$$E_m = 7 \,\mathrm{RMi}^{0.4}$$
 (for 1 < $R\mathrm{Mi}$ < 30) (17)

Gokceoglu et al. presented a useful formula for predicting E_m based on weathering degree of rock (WD) [19]. Weathering is the gradual destruction of rock under surface conditions. Weathering may involve physical processes (mechanical weathering) or chemical activity (chemical weathering) or the actions of living things (organic weathering). In principle, it is very simple to quantify the influence of weathering on a geotechnical parameter. A simple comparison of a rock mass parameter (for example intact rock strength, spacing or conditions of discontinuities, etc.) in an exposure in which different degrees of weathering are present in the same unit should give the quantitative reduction values. Therefore the WD factor can be obtained as:

$$WD = \frac{\text{weathered rock mass parameter}}{\text{fresh rock mass parameter}}$$
(18)

and the deformation modulus of rock mass can be estimated by the following relation:

$$E_m(GPa) = 0.0308 \times \left[\frac{(\frac{E}{UCS}) \times (1 + \frac{RQD}{100})}{WD}\right]$$
(19)

where, E is the elasticity modulus of intact rock material in GPa and WD is the weathering degree.

Sonmez et al. presented other formulas to predict E_m as [20]:

$$E_m(GPa) = 0.3166 \times RMR - 8.9542$$
 (20)

$$E_m(GPa) = 0.00001 \times RMR^{3.176}$$
(21)

$$E_m(GPa) = 10.038 \times \ln(GSI) - 31.078$$
 (22)

$$E_m(GPa) = 0.1451 \times e^{0.0654GSI}$$
(23)

Mohammadi and Rahmannejad estimated E_m by the following formula [21]:

$$E_m(GPa) = 0.0003RMR^3 - 0.0193RMR^2 + 0.3157RMR + 3.4064$$
(24)

In order to present a new model, a complete database was derived from bibliography. Then two methods are used:the first one is curve fitting and regression analysis; and the other one is Artificial Neural Networks (ANN).

3. Database information

In the present study, 224 data sets were collected. Each data set contains the parameters such as the deformation modulus, RMR and UCS. All of these data have been collected from bibliography and library studies that measured in different spots in the world such as:

- ✓ The collected data sets from road and railway construction sites in Korea [1].
- ✓ Data set measured by PMT (pressure meter tests) and performed at eight field sites which included six rock types such as Granite, Gneiss, Andesite, Tuff, Sandstone and shale. The studying areas were mainly located in Cheonla-do and some areas were in Chungcheong-do and Kyungsang-do, Korea [22].
- ✓ Data set provided from two dam sites; namely Deriner (Artvin) dam site and Ermenek (Karaman) dam site in Turkey. The Deriner (Artvin) dam site is mainly covered with grey and pinky Quartz-Diorite and the other dam site, Ermenek (Karaman) is covered with light Limestone [4].
- ✓ Data set derived partially from bibliography [2], [3], [5], [6] and [7].

The results of statistical analysis of these data are illustrated in Table 1 and Figure 1.

Table 1. Statistical parameter	rs of			
database				

	E _m (GPa)	RMR	UCS (MPa)	
Min	0.0003	15.73	9.99	
Max	45.62	94.28	259.28	

4. Multiple Nonlinear Regressions

4.1. Theory of Regression Analysis

Regression analysis is usually used to analyze the problems where several parameters may affect their results and some multivariable function may arise and the relations may be linear or nonlinear [23]. In this study various nonlinear models like polynomial, exponential and power polynomial have been used to analyze the output parameter (y). In these models the function y is related to the variable x with some constant a, b, c, d and etc. The variable x is chosen to be the input parameters where affecting the y results.

Table 2 shows the nonlinear regression model and related formula usually used for the analysis of y prediction.



Figure 1. The histograms and statistical evaluations of the data used in this research

Table 2. Nonlinear regression models and the	heir
related formula [26]	

Regression model	Equation
Polynomial of order two Polynomial of order three Exponential Power	$\begin{array}{l} Y=ax^2+bx+c\\ Y=ax^3+bx^2+cx+d\\ Y=ab^x\\ Y=ax^b \end{array}$

In order to check the result of regression analysis, coefficient correlation (R), Mean Absolute

Percentage Error (MAPE), Mean Square Error (MSE) etc can be used. In this research, R and

MAPE are applied to check the results of regression and ANN modeling.

The quantity R, called the correlation coefficient, measures the strength and the direction of a relationship between two or more variables. The mathematical formula for computing R is [23]:

$$R = \frac{n \sum x_i y_i - (\sum x_i) (\sum y_i)}{\sqrt{n (\sum x_i^2) - (\sum x_i)^2} \sqrt{n (\sum y_i^2) - (\sum y_i)^2}}$$
(25)

where, x_i is the input parameter, y_i is the output parameter and n is the number of data.

MAPE is the measure of accuracy in statistics. It usually expresses accuracy as a percentage, and is defined by the following formula [23]:

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{A_i - F_i}{A_i} \right|$$
(26)

where, A_t is the actual value and F_t is the forecast value of the output variable.

4.2. New correlation between RMR and E_m

In this model the geotechnical parameters of UCS and RMR are used to predict the value of E_m . At first, a sensitivity analysis is performed to determine the critical parameter that has more effect on E_m . Sensitivity analysis is a technique used to determine how different values of an independent variable (like UCS and RMR) will impact a particular dependent variable (like E_m) under a given set of assumptions. Therefore, the Em-RMR and Em-UCS graphs are drawn that can be seen in Figure 2 As is clear from this figure, E_m is more sensitive to the RMR than UCS.



Figure 2. Sensitivity analysis in data sets: relation between Em and UCS and RMR

With these data, several correlations have been investigated to estimate E_m and the following equation has been derived:

$$E_m = 1.473 \times 10^{-3} \times RMR^{1.616} \times UCS^{0.492} \quad (27)$$

where, E_m is in GPa and UCS is in MPa. UCS is considered twice in the suggested equation, one time in RMR and one time as a variable; this is because of the importance of intact rock properties on the deformation modulus of rock mass.

Figure 3 shows a three dimensional model obtained based on the equation (26) with a coefficient correlation of 0.751 and MAPE of 9.911%.

In order to investigate the effect of data classification on the obtained coefficient correlation, the range of input parameters (UCS and RMR) is broken into two and three intervals in such a way that the number of data in each group is equal. Then the correlation was investigated within each interval. Table 3 and 4 show the results of input data grouping.

As shown in Tables 3 and 4, data classification does not improve the correlation coefficient.

5. Artificial Neural Network (ANN)

5.1. Theory of ANN

ANNs are simplified mathematical models inspired by the biological structure and functioning of the brain. In other words, to be able to decide and act under uncertainty or even deal with situations having limited previous experience [24].

ANNs are mathematic models consisting of interconnected processing nodes (neurons) under a pre-specified topology (layers). Usually the neurons operate in parallel layers. A typical network topology consists of the input layer, one or more hidden layers and the output layer as shown in Figure 4.

In the topology shown in Figure 4, each neuron of the input layer $(X_1, X_2 \text{ and } X_3)$, sends out its weighted signal to the Y neuron found in the hidden layer [25]. The combined input signal in the Y neuron has the following form:

$$Y_{in} = \sum_{i=1}^{n} w_i . x_i$$
 (28)

where, x_i is the signal of the *i*th input neuron,

 W_i the weighting factor of the *i* th neuron.

The input signal (Y_{in}) is introduced to the activation function of the Y neuron and signaled to the neurons of the output layer, Z_1 and Z_2 following the general form:

 $y = f(Y_{in})$ (Transfer function) taking into account the weighting of the connection links, namely v_1 and v_2 . The transfer functions are designed to map a neuron, or layer, net output to its actual output. They are simple linear or nonlinear step functions. The type of these transfer functions depends on the purpose of the neural network [26-27].

Table 3. Obtained models to predict Em based on RMR classification				
No. of interval	RMR range	Obtained equation	R	MAPE (%)
2	0 <rmr<50< th=""><th>E_m=-8.979+0.175RMR+0.266UCS-1.176UCS²- 2.508UCS³+2.369UCS⁴-5.265UCS⁵</th><th>0.517</th><th>95.72</th></rmr<50<>	E_m =-8.979+0.175RMR+0.266UCS-1.176UCS ² - 2.508UCS ³ +2.369UCS ⁴ -5.265UCS ⁵	0.517	95.72
2	50 <rmr<100< th=""><th>E_m=50453.751-3338.159RMR+87.907RMR²- 1.152RMR³+0.008RMR⁴-0.0002RMR⁵+0.07UCS</th><th>0.623</th><th>39.16</th></rmr<100<>	E _m =50453.751-3338.159RMR+87.907RMR ² - 1.152RMR ³ +0.008RMR ⁴ -0.0002RMR ⁵ +0.07UCS	0.623	39.16
	0 <rmr<40< th=""><th>$\begin{split} & E_m \!\!=\!\! -8.571 \!+\! 0.171 RMR \!\!+\! 0.275 UCS \!\!-\! 0.002 UCS^2 \!\!-\! 0.0002 UCS^3 \!\!+\! 2.069 \!\times\! 10^{-3} UCS^4 \!\!-\! 4.756 \!\times\! 10^{-10} UCS^5 \end{split}$</th><th>0.616</th><th>44.36</th></rmr<40<>	$ \begin{split} & E_m \!\!=\!\! -8.571 \!+\! 0.171 RMR \!\!+\! 0.275 UCS \!\!-\! 0.002 UCS^2 \!\!-\! 0.0002 UCS^3 \!\!+\! 2.069 \!\times\! 10^{-3} UCS^4 \!\!-\! 4.756 \!\times\! 10^{-10} UCS^5 \end{split} $	0.616	44.36
3	40 <rmr<70< td=""><td>$E_{m} = -115.806 + 3.079 RMR - 0.02 RMR^{2} + 1.165 UCS - 0.02 UCS^{2} + 0.0001 UCS^{3} - 2.57 \times 10^{-7} UCS^{4}$</td><td>0.532</td><td>67.25</td></rmr<70<>	$E_{m} = -115.806 + 3.079 RMR - 0.02 RMR^{2} + 1.165 UCS - 0.02 UCS^{2} + 0.0001 UCS^{3} - 2.57 \times 10^{-7} UCS^{4}$	0.532	67.25
	70 <rmr<100< td=""><td>E_m=418572.901-26415.844RMR+664.866RMR²- 8.343RMR³+0.052RMR⁴-0.0001RMR⁵+0.062UCS</td><td>0.621</td><td>24.18</td></rmr<100<>	E _m =418572.901-26415.844RMR+664.866RMR ² - 8.343RMR ³ +0.052RMR ⁴ -0.0001RMR ⁵ +0.062UCS	0.621	24.18

|--|

Table 4. Obtained models to predict Em based on UCS classification

No. of interval	UCS range	Obtained equation	R	MAPE (%)
2	0 <ucs<120< th=""><th>E_m=1.916+3.377×10⁻³RMR+4.787×10⁻³UCS- 5.071×10⁻⁴RMR²-1.237UCS²×10⁻³+ 3.763RMR×UCS</th><th>0.592</th><th>59.45</th></ucs<120<>	E _m =1.916+3.377×10 ⁻³ RMR+4.787×10 ⁻³ UCS- 5.071×10 ⁻⁴ RMR ² -1.237UCS ² ×10 ⁻³ + 3.763RMR×UCS	0.592	59.45
2	120 <ucs<170< td=""><td>E_m=724.845+0.241RMR+0.001RMR²- 16.407UCS+0.134UCS²-0.0005UCS³+6.111×10⁻ ⁷UCS⁴</td><td>0.723</td><td>75.54</td></ucs<170<>	E_m =724.845+0.241RMR+0.001RMR ² - 16.407UCS+0.134UCS ² -0.0005UCS ³ +6.111×10 ⁻ ⁷ UCS ⁴	0.723	75.54
	0 <ucs<100< td=""><td>E_m=-2.889+0.164RMR+0.06UCS-0.003RMR²- 0.002UCS²+0.005RMR.UCS</td><td>0.581</td><td>64.14</td></ucs<100<>	E _m =-2.889+0.164RMR+0.06UCS-0.003RMR ² - 0.002UCS ² +0.005RMR.UCS	0.581	64.14
3	100 <ucs<140< td=""><td>E_m=-2461.141-199.785RMR+6.365RMR²- 9.935RMR³-0.0008RMR⁴- 0.000002RMR⁵+0.07UCS</td><td>0.635</td><td>42.25</td></ucs<140<>	E _m =-2461.141-199.785RMR+6.365RMR ² - 9.935RMR ³ -0.0008RMR ⁴ - 0.000002RMR ⁵ +0.07UCS	0.635	42.25
	140 <ucs<170< td=""><td>E_m=-240.728+1.799RMR- 0.03RMR²+0.002RMR³+3UCS- 0.014UCS²+0.00002UCS³</td><td>0.708</td><td>93.33</td></ucs<170<>	E_m =-240.728+1.799RMR- 0.03RMR ² +0.002RMR ³ +3UCS- 0.014UCS ² +0.00002UCS ³	0.708	93.33



Figure 3. Three dimensional model to predict Em based on the equation (26)



Figure 4. Schematic diagram of Neural Network

First, ANN processes the large input data to training itself. After the training stage, network predicted new result of new data. This results in a predicted output pattern. Any particular network can be defined using three fundamental components: a transfer function, network architecture and a learning law.

5.2. Input data set

The first and most important stage for predicting E_m in ANN technique is data collection. The data was chosen in neural network must have a good correlation with E_m . In this research, these data was randomized by ANN and placed into training, validation and test subsets.

In the present study, 224 data sets were collected. From these, 65% of the data were chosen for training, 15% for validation and 20% for the final test.

5.3. ANN topology

An appropriate architecture was obtained from feed-forward back propagation. A three-layer network with logarithmic sigmoid transfer function neurons in the hidden layer, and Levenberg-Marquardt (LM) algorithm corresponding to Em in the output layer, was chosen.

As there is no direct method to identify the number of hidden layers and number of neurons in each hidden layer, several network topologies were examined for this work. Also the LM algorithm was chosen to train the ANNs because it is known to be the fastest method for training moderate-sized feed-forward neural networks. LM algorithm is a modification of Newton's method for nonlinear optimization. This algorithm does not utilize second derivatives unlike Newton's method (in the Hessian computation, the second derivative component is ignored assuming it is small). This method is based on the concept of quadratic approximation of error function in a local region. Note that if the error function is truly quadratic in nature, Newton's method finds the minimum solution in a single iteration. Therefore, the success of this technique depends upon how the error function resembles a quadratic function. If the quadratic approximation is not appropriate, the algorithm may diverge. Searching of an optimal solution using this method requires calculation of the inverse of the Hessian matrix, which should be positive definite. Newton's method does not always guarantee the positive definiteness of Hessian matrix. LM introduces a regularization term into the Hessian matrix so that the positive definiteness of the Hessian matrix is guaranteed.

5.4. Testing and validating the model

After training the network, testing and validation of the ANN model was done with new data sets that were not used during the training process. The MAPE and R between the predicted and measured E_m were taken as the performance measures.

The prediction was based on the input data sets. Several models were built, and the quality of the results obtained for some of them is shown in Table 5. The correlation coefficients and mean squared errors for the different models are presented in this table. As can be seen, the best model has the MAPE equal to 6.324% and the correlation coefficient equal to 0.786 for test data sets. This model is optimum model with a 2-4-1 architecture that shown in Figure 5.

 Table 5. Comparison between some of suggested models in ANN method

models in Aitit method				
No.	Model	R	MAPE (%)	
1	2-13-1	0.413	25.048	
2	2-9-1	0.573	17.794	
3	2-6-1	0.639	9.834	
4	2-4-1	0.786	6.324	
5	2-13-9-1	0.514	21.434	
6	2-10-9-1	0.239	57.943	
7	2-13-6-1	0.394	34.754	
8	2-4-4-1	0.439	27.646	



Figure 5. Suggested ANN model for predicting Em

Figure 6 illustrates the comparison of the real value of E_m using insitu tests and measured value of E_m by two models, regression and ANN.

This graph shows a good agreement between measured values of E_m using regression and ANN methods and real ones. Therefore these two indirect methods can be reliably used to predict rock mass modulus.



Figure 6. Comparison of real and measured values obtained by two models

6. Conclusions

The analysis for indirect estimation of deformation modulus of rock masses was investigated using regression and ANN methods and the following conclusions can be drawn:

- 1) RMR and UCS are both incorporated in order to estimate E_m . RMR reflects the discontinuities situation within rock masses and UCS reflects the intact rock properties and also rock type. Incorporation of UCS in to the regression and ANN modeling leads to improving the correlation coefficient.
- 2) RMR has more effect on E_m rather than UCS.
- 3) The proposed non linear regression model has an acceptable correlation coefficient, so that it can be used to estimate the rock mass deformation modulus. This relation is based on data from different parts of the world with different lithology.
- 4) Data classification does not improve the correlation coefficient in the regression analysis method.
- 5) The results obtained from this research show that an ANN is a useful tool to predict deformation modulus of rock masses. However, the relationship among

the inputs and outputs is very complex. The optimum ANN architecture was found to be two neurons in the input layer: one hidden layer with 4 neurons; and one neuron in the output layer.

6) As can be seen, the value of correlation coefficient in ANN is a little greater than that of the regression models.

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