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THE SPECTRAL DETERMINATION OF THE MULTICONE GRAPHS $K_w \bigtriangledown C$ WITH RESPECT TO THEIR SIGNLESS LAPLACIAN SPECTRA

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Abstract. The main aim of this study is to characterize new classes of multicone graphs which are determined by their signless Laplacian spectra. A multicone graph is defined to be the join of a clique and a regular graph. Let C and K_w denote the Clebsch graph and a complete graph on w vertices, respectively. In this paper, we show that the multicone graphs $K_w \bigtriangledown C$ are determined by their signless Laplacian spectrum.

1. Introduction

In the past decades, graphs that are determined by their spectrum have received much more and more attention, since they have been applied to several fields, such as randomized algorithms, combinatorial optimization problems and machine learning. An important part of spectral graph theory is devoted to determining whether given graphs or classes of graphs are determined by their spectra or not. So, finding and introducing any class of graphs which are determined by their spectra can be an interesting and important problem. Let $G = (V, E)$ be a simple graph with vertex set $V = V(G) = \{v_1, \dots, v_n\}$ and edge set $E = E(G) = \{e_1, \dots, e_m\}$. Denote by $d(v)$ the degree of vertex v. All graphs considered here are simple and undirected. All notions on graphs that are not defined here can be found in [10, 11, 14, 23, 28, 32]. A graph consisting of k disjoint copies of an

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arbitrary graph Γ will be denoted by $k\Gamma$. The join of two graphs G and H is a graph formed from disjoint copies of G and H by connecting each vertex of G to each vertex of H. We denote the join of two graphs G and H by $G \bigtriangledown H$. The complement of a graph G is denoted by $\overline{G}$. Let $A(G)$ be the $(0,1)$-adjacency matrix of graph G. The matrices $L(G) = D(G) - A(G)$ and $Q(G) = SL(G) = D(G) + A(G)$ are called the Laplacian matrix and the signless Laplacian matrix of G, respectively, where $D(G)$ denotes the degree matrix. Note that $D(G)$ is diagonal. Let $q_1 \geq q_2 \geq \cdots \geq q_n$ be the distinct eigenvalues of G with multiplicities $m_1,\ m_2,\ \cdots\ ,\ m_n$, respectively. The multi-set $\mathrm{Spec}_Q(G) = \{[q_1]^{m_1}, q_2]^{m_2}, \cdots, [q_n]^{m_n}\}$ of eigenvalues of $Q(G)$ is called the signless Laplacian spectrum of G. We say two graphs G and H are Q-cospectral if $\mathrm{Spec}_Q(G) = \mathrm{Spec}_Q(H)$. Up to now, only some graphs with special structures are shown to be *determined by their spectra* (DS, for short) (see [1–9, 12, 15–17, 20, 23, 25–27, 30, 31] and the references cited in them). Van Dam and Haemers [29] conjectured that almost all graphs are determined by their spectra. Nevertheless, the set of graphs that are known to be detrmined by their spectra is too small. So, discovering infinite classes of graphs that are determined by their spectra can be an interesting problem. About the background of the question "Which graphs are determined by their spectrum?", we refer to [29]. In [23, 24] the authors characterized new classes of multicone graphs which are determined by their signless Laplacian spectra. Abdian and Mirafzal [7] characterized new classes of multicone graphs which were DS with respect to their spectra. Abdian [1] characterized new classes of multicone graphs with respect to their adjacency spectra as well as their Laplacian spectra. Abdian also proposed four conjectures about adjacency spectrum of complement and signless Laplacian spectrum of these multicone graphs. In [2], the author shown that multicone graphs $K_w \bigtriangledown P_{17}$ and $K_w \bigtriangledown S$ are determined by their adjacency and their Laplacian spectra, where P_{17} and S denote the Paley graph of order 17 and the Schläfli graph, respectively. Also, this author conjectured that these multicone graphs are determined by their signless Laplacian spectra. In [3], the author proved that multicone graphs $K_w \bigtriangledown L(P)$ are determined by both their adjacency and Laplacian spectra, where $L(P)$ denotes the line graph of the Petersen graph. He also proposed three conjectures about the signless Laplacian spectrum and the complement spectrum of these multicone graphs. Abdian [**?**] characterized multicone graphs $K_w \bigtriangledown P$ with respect to adjacency spectra, Laplacian spectra and signless Laplacian spectra, where P denotes the Petersen graph. The authors [25] characterized another new classes of multicone graphs which are determined by their adjacency and Laplacian spectra as well as their complement with respect to adjacency spectra. For seeing some multicone graphs which have been characterized so far refer to [4,6,9,25–27]. In the papers [1–4,6,7,9,25,26] graphs have not been characterized with respect to their signless Laplacian spectra. In this work, we present some techniques for characterizing these graphs with respect to the signless Laplacian spectra and we show that the multicone graphs $K_w \bigtriangledown C$ are determined by their signless Laplacian spectrum.

2. Some definitions and preliminaries

Some useful established results about the spectrum are presented in this section, will play an important role throughout the paper.

Lemma 2.1 ([12]). *Let G be a graph with n vertices, m edges, t triangles and vertex degrees $d_1, d_2, \cdots, d_n$. Let $T_k = \sum_{i=1}^{n} (q_i(G))^k$, then*

$T_0 = n$, $T_1 = \sum_{i=1}^{n} d_i = 2m$, $T_2 = 2m + \sum_{i=1}^{n} d_i^2$ and $T_3 = 6t + 3\sum_{i=1}^{n} d_i^2 + \sum_{i=1}^{n} d_i^3$.

Lemma 2.2 ([15]). *In any graph the multiplicity of the eigenvalue 0 of the signless Laplacian is equal to the number of bipartite components.*

Lemma 2.3 ([23]). *Let G be an r-regular graph on n vertices and G is determined by its signless Laplacian spectrum. Let H be a graph Q-cospectral with $G \bigtriangledown K_m$. If $d_1(H) = d_2(H) = \cdots = d_m(H) = n+m-1$, then $H \cong K_m \bigtriangledown G$.*

Lemma 2.4 ([12,23]). *Let G be a connected graph of order $n(n > 1)$, and the minimum degree of G is δ. Then $q_n < \delta$.*

Lemma 2.5 ([23]). *For $i = 1, 2$, let G_i be an r_i-regular graph on n_i vertices. Then*

$$P_{Q(G_1 \bigtriangledown G_2)}(x) = \frac{P_{Q(G_1)}(x-n_2)P_{Q(G_2)}(x-n_1)}{(x-2r_1-n_2)(x-2r_2-n_1)} f(x),$$

where $f(x) = x^2 - (2(r_1+r_2) + (n_1+n_2))x + 2(2r_1r_2 + r_1n_1 + r_2n_2)$.

Lemma 2.6 ([12]). *Let G be a graph on n vertices with vertex degrees $d_1, d_2, \cdots, d_n$. Then min $\{d_i + d_j\} \leq q_1 \leq$ max $\{d_i + d_j\}$, where (i, j) runs over all pairs of adjacent vertices of G.*

Lemma 2.7 ([21]). *Let G be a graph of order $n > 2$ and $q_1(G) \geq q_2(G) \geq ... \geq q_n(G)$. Then $q_2(G) \leq n-2$. Moreover, $q_{k+1}(G) = n-2$ $(1 \leq k < n)$ if and only if $\overline{G}$ has either k balanced bipartite components or $k+1$ bipartite components.*

Lemma 2.8 ([12]). *Let G be a graph with maximum degree d_1 and second maximum degree d_2. Then $q_2(G) \geq d_2 - 1$. If $q_2(G) = d_2 - 1$, then $d_1 = d_2$.*

Remark 2.9. For further information about the Clebsch graph, one can refer to [13, 19]. Also, $\mathrm{Spec}_A(C) = \left\{[10]^1,\ [2]^5,\ [-2]^{10}\right\}$.

3. Main Results

In the following we always suppose that $\Delta = d_1 \geq d_2 \geq \cdots \geq d_n = \delta$ and $q_1 \geq q_2 \geq \cdots \geq q_n$.

Proposition 3.1. *The signless Laplacian spectrum of the multicone graph $K_1 \bigtriangledown C$ is:*

$$\left\{\left[\frac{37 \pm \sqrt{89}}{2}\right]^1,\ [13]^5,\ [9]^{10}\right\}.$$

Proof By Lemma 2.5 the result follows(see also Theorem 3.1 of [12]). □

Theorem 3.2. *The multicone graph $K_1 \bigtriangledown C$ is DS with respect to its signless Laplacian spectra.*

Proof First, it is easy and straightforward to see that there is no disconnected graph Q-cospectral with the multicone graph $K_1 \bigtriangledown C$. Otherwise, let $\mathrm{Spec}_Q(G) = \mathrm{Spec}_Q(K_1 \bigtriangledown C)$ and $G = H_1 \cup H_2$, where H_i $(i = 1, 2)$ are the subgraph of G. It is easy to check that any of H_i must have three signless Laplacian eigenvalues. It is well-known that a graph has one or two signless Laplacian eigenvalue(s) if and only if it is either isomorphic to a disjoint union of isolated vertices or a disjoint union of complete graphs on the same vertices, respectively. But, by the spectrum of G and $\mathrm{Spec}_Q(K_w) = \left\{[2w-2]^1,\ [w-2]^{w-1}\right\}$ this case (having three distinct signless Laplacian eigenvalues for any of subgraphs H_1 and H_2) cannot happen. So, any graph Q-cospectral with the multicone graph $K_1 \bigtriangledown C$, the cone of the C graph, is connected. By Lemma 2.6 it is clear that $2d_1 \geq q_1 \approx 23.21$. So, $d_1 = \Delta \geq 12$. Also, it follows from Lemma 2.4 that $\delta > q_{17} = 9$. This means that $\delta \geq 10$. By Lemma 2.8 $d_2 \leq \dfrac{37 - \sqrt{89}}{2} + 1 \approx 14.78$. Hence we can conclude that

$\delta = d_{17} \leq 10 \leq d_2 \leq 14$.

We consider the following cases:

Case 1. $d_2 = 10$.

So, $d_2 = d_3 = \cdots = d_{17} = \delta = 10$. Therefore, by Lemma 2.1 $d_1 + 160 = 192$. So, $d_1 = 30$, a contradiction, since $10 \leq d_1 \leq \Delta = 16$.

Case 2. $d_2 = 11$.

Let we have a vertices of degree 10 and $16 - a$ vertices of degree 11 between d_i for $2 \leq i \leq 17$. Therefore, $d_1 + 10a + (16 - a)11 = 192$. This implies that $d_1 - a = 16$ and so $a + 16 = d_1$. But $16 \geq d_1 \geq 10$ and so $a + 16 = d_1 \in \{10, 11, \ldots, 16\}$. It is clear that a can only be 0. This means that we have 16 vertices of degree 11 and so $d_1 = 16$. In this case, it follows from Lemma 2.3 that $G \cong K_1 \bigtriangledown C$.

Case 3. $d_2 = 12$.

Let we have a vertices of degree 10, b vertices of degree 11 and $c \geq 1$ vertices of degree 12 between d_i for $2 \leq i \leq 17$. So, by Lemma 2.1 we get:

$$\begin{cases} a + b + c = 16, \\ 4a + 5b + 6c + d_1 = 192, \\ 16a + 25b + 36c + d_1^2 = 2192. \end{cases}$$

By a simple calculating we get $a = \dfrac{-d_1^2 + 23d_1}{2} - 56$, $b = 112 + d_1^2 - 22d_1$ and $c = \dfrac{21d_1 - d_1^2}{2} - 40$. It is clear that $d_1 \in \{12, 13, 14, 15, 16\}$, $0 \leq a, b, c \leq 16$ and $0 \leq a + b + c = 16$. Now, by replacing d_1 in a, b or c we will have a contradiction.

Case 4. $d_2 = 13$.

Let we have a vertices of degree 10, b vertices of degree 11, c vertices of degree 12 and $d \geq 1$ vertices of degree 13 between d_i for $2 \leq i \leq 17$.

$$\begin{cases} a+b+c+d=16, \\ 10a+11b+12c+13d+d_1=192, \\ 100a+121b+144c+169d+d_1^2=2192. \end{cases}$$

By a simple calculating we get $a = \dfrac{-d_1^2+23d_1}{2} - 56 - d$, $b = 112 + d_1^2 - 22d_1 + 3d$ and $c = \dfrac{21d_1 - d_1^2}{2} - 40 - 3d$. In a similar manner of case 4 we have a contradiction.

Case 5. $d_2 = 14$.

Let we have a vertices of degree 10, b vertices of degree 11, c vertices of degree 12, d vertices of degree 13 and $e \geq 1$ vertices of degree 14 between d_i for $2 \leq i \leq 17$.

$$\begin{cases} a+b+c+d+e=16, \\ 10a+11b+12c+13d+14e+d_1=192, \\ 100a+121b+144c+169d+196e+d_1^2=2192. \end{cases}$$

By a simple calculating we get $a = \dfrac{-d_1^2+23d_1}{2} - 56 - d - 3e$, $b = 112 + d_1^2 - 22d_1 + 8e + 3d$ and $c = \dfrac{21d_1 - d_1^2}{2} - 40 - 3d - 6e$. In a similar manner of case 3 or 4 we receive to a contradiction.□

Proposition 3.3. *The signless Laplacian spectrum of the multicone graph $K_2 \bigtriangledown C$ is:*

$$\left\{[26]^1,\ [14]^6,\ [10]^{10},\ [16]^1\right\}.$$

Proof By Lemma 2.5 the result follows(see also Corollary 3.1 of [24]). □

Theorem 3.4. *The multicone graph $K_2 \bigtriangledown C$ is DS with respect to its signless Laplacian spectra.*

Proof Let G be Q-cospectral with a multicone graph $K_2 \bigtriangledown C$. By Lemma 2.6 we can conclude that $q_1(G) = 26$. So, $2d_1 \geq 26$. This means that $d_1 \geq 13$. On the other hand, it follows from Lemma 2.4 $\delta > q_{18} = \delta \geq 11$(it is straightforward to see that any graph Q-cospectral with the multicone graph $K_2 \bigtriangledown C$ is connected). By Lemma 2.7 of [18] $d_3 \leq 14 + \sqrt{2} \approx 15.2$ and so $11 \leq \delta \leq d_3 \leq 15$. Now, we consider the following cases:

Case 1. $d_3 = 11$.

Take $d_1 + d_2 = x$ and $d_1^2 + d_2^2 = y$ and note that $d_1 \geq 13$ and so $24 \leq x \leq 34$.

In this case $d_3 = d_4 = d_5 = \cdots = d_{18} = \delta = 11$. Therefore, $x + 176 = 226$ and so $x = 50$, a contradiction.

Case 2. $d_3 = 12$.

Assume that there are a vertices of degree 11 and $16 - a$ vertices of degree 12 between d_i for $3 \leq i \leq 18$. So, $x + 11a + (16-a)12 = 226$ and so $x = a + 34$. So, we must have $24 \leq x = a + 34 \leq 34$ or $-10 \leq a \leq 0$. It is clear that a can be only 0. This means that G has 12 vertices of degree 16 and $x = d_1 + d_2 = 34$. Therefore, $d_1 = d_2 = 17$ and by Lemma 2.3 the result follows.

Case 3. $d_3 = 13$.

Assume that there are a vertices of degree 11, b vertices of degree 12 and $c \geq 1$ vertices of degree 13 between d_i for $3 \leq i \leq 18$. So

$$\begin{cases} a + b + c = 16, \\ 11a + 12b + 13c = 226 - x, \\ 121a + 144b + 169c = 2882 - y. \end{cases}$$

One can easily check that $a = -102 + \dfrac{23x - y}{2}$, $b = 170 - 22x + y$ and $c = -52 + \dfrac{21x - y}{2}$. So, the summation of the x and y must be even.To put that another way,

$$\begin{cases} x = 26(d_1 = d_2 = 13), \\ y = 338. \end{cases}, \begin{cases} x = 30(d_1 = d_2 = 15), \\ y = 450. \end{cases},$$
$$\begin{cases} x = 34(d_1 = d_2 = 17), \\ y = 578. \end{cases}, \begin{cases} x = 28(d_1 = d_2 = 14), \\ y = 392. \end{cases},$$
$$\begin{cases} x = 32(d_1 = d_2 = 16), \\ y = 512. \end{cases}, \begin{cases} x = 28(d_1 = 15, d_2 = 13), \\ y = 394. \end{cases},$$
$$\begin{cases} x = 30(d_1 = 17, d_2 = 13), \\ y = 458. \end{cases}, \begin{cases} x = 32(d_1 = 17, d_2 = 15), \\ y = 514. \end{cases},$$

$$\begin{cases} x = 30(d_1 = 16, d_2 = 14), \\ y = 452. \end{cases}$$

It is clear that $0 \leq a, b, c = 16$. Now, by replacing any of the above cases we will have a contradiction.

Case 4. $d_3 = 14$.

Suppose that there are a vertices of degree 11, b vertices of degree 12, c vertices of degree 13 and $d \geq 1$ vertices of degree 14 between d_i for $4 \leq i \leq 11$. So

$$\begin{cases} a + b + c + d = 16, \\ 11a + 12b + 13c + 14d = 226 - x, \\ 121a + 144b + 169c + 196d = 2882 - y. \end{cases}$$

It is easy to see that $a = -102 - 3d + \dfrac{23x - y}{2}$, $b = 170 + 8d - 22x + y$ and $c = -52 - 6d + \dfrac{21x - y}{2}$. So, the summation of the x and y must be even, since a is a non-negative integer number. So,

$$\begin{cases} x = 30(d_1 = d_2 = 15), \\ y = 450. \end{cases}, \begin{cases} x = 34(d_1 = d_2 = 17), \\ y = 578. \end{cases},$$
$$\begin{cases} x = 28(d_1 = d_2 = 14), \\ y = 392. \end{cases}, \begin{cases} x = 32(d_1 = d_2 = 16), \\ y = 512. \end{cases},$$
$$\begin{cases} x = 32(d_1 = 17, d_2 = 15), \\ y = 514. \end{cases}, \begin{cases} x = 30(d_1 = 16, d_2 = 14), \\ y = 452. \end{cases}$$

It is clear that $0 \leq a, b, c, d = 16$ and $d \geq 1$. Now, by replacing any of the above cases we will have a contradiction.

Case 5. $d_3 = 15$.

Let there are a vertices of degree 11, b vertices of degree 12, c vertices of degree 13, d vertices of degree 14 and $e \geq 1$ vertices of degree 15 between d_i for $3 \leq i \leq 18$. So,

$$\begin{cases} a + b + c + d + e = 16, \\ 11a + 12b + 13c + 14d + 15e = 226 - x, \\ 121a + 144b + 169c + 196d + 225e = 2882 - y. \end{cases}$$

we get $a = -102 - 3d - 6e + \dfrac{23x - y}{2}$, $b = 170 + 8d + 15e - 22x + y$ and $c = -52 - 6d - 10e + \dfrac{21x - y}{2}$. Put simply,

$$\begin{cases} x = 30(d_1 = d_2 = 15), \\ y = 450. \end{cases}, \begin{cases} x = 34(d_1 = d_2 = 17), \\ y = 578. \end{cases},$$

$$\begin{cases} x = 32(d_1 = d_2 = 16), \\ y = 512. \end{cases}, \begin{cases} x = 32(d_1 = 17, d_2 = 15), \\ y = 514. \end{cases}$$

In a similar manner of Case 3 or 4 we will have a contradiction. $\square$

Now, we show that multicone graphs $K_w \bigtriangledown C$ are DS with respect to their signless Laplacian spectra. For proving this fact we need one lemma.

Lemma 3.5. *Let G be a graph of order n. If $n-2$ is one of the signless Laplacian eigenvalues of G with the multiplicity of at least 2 and $q_1(G) > n-2$, then G is the join of two graphs.*

Proof By Lemma 2.7 $\overline{G}$ has either at least 2 balanced bipartite components or at least 3 bipartite components. In other words, $\overline{G}$ is disconnected. Thus G is connected and it is the join of two graphs. $\square$

Remark 3.6. Note that in Lemma 3.5 the condition $q_1(G) > n-2$ is critical. For instance, consider the graph which is the disjoint union of two triangles. It is easy to see that the signless Laplacian spectrum of this graph is $\{[4]^2, [1]^4\}$, whereas this graph is not the join of any two graphs.

Theorem 3.7. *Multicone graphs $K_w \bigtriangledown C$ are DS with respect to their signless Laplacian spectra.*

Proof We solve the theorem by the mathematical induction on w. For $w = 1, 2$ this theorem was proved(see Theorems 3.2 and 3.4). Let the theorem be true for w; that is, if $\mathrm{Spec}_Q(H) = \mathrm{Spec}_Q(K_w \bigtriangledown C)$, then $H \cong K_w \bigtriangledown C$, where H is an arbitrary graph Q-spectral with a multicone graph $K_w \bigtriangledown C$(the inductive hypothesis). We show that it follows from $\mathrm{Spec}_Q(G) = \mathrm{Spec}_Q(K_{w+1} \bigtriangledown C)$ that $G \cong K_{w+1} \bigtriangledown C$. It is clear that G has one vertex and $16 + w$ edges more than H and $\mathrm{Spec}_Q(G) = \mathrm{Spec}_Q(K_1 \bigtriangledown H)$(by the inductive hypothesis $H \cong K_w \bigtriangledown C$). On the other hand, by Lemma 3.5 G and H are the join of two graphs. So, we must have $G \cong K_1 \bigtriangledown H$. Now, the inductive hypothesis follows the result. $\square$

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THE SPECTRAL DETERMINATION OF THE MULTICONE GRAPHS $K_w \bigtriangledown C$ WITH RESPECT TO THEIR SIGNLESS LAPLACIAN SPECTRA

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مشخصه‌سازی گراف‌های چندمخروطی $K_w \bigtriangledown C$ نسبت به طیف لاپلاسین بدون علامت آن‌ها

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هدف این مقاله مشخصه‌سازی گراف‌های چندمخروطی است که به وسیله طیف لاپلاسین بدون علامت خود تعیین می‌شوند. یک گراف چندمخروطی از پیوند یک گراف منظم با یک گراف کامل به‌دست می‌آید. در این مقاله نشان می‌دهیم که گراف‌های چند مخروطی $K_w \bigtriangledown C$ به وسیله طیف لاپلاسین بدون علامت خود به‌طور یکتا مشخص می‌شوند، جایی‌که K_w گراف کامل w راسی و C بیانگر گراف کلبسچ می‌باشد.

کلمات کلیدی: گراف چندمخروطی، طیف لاپلاسین بدون‌علامت، گراف کلبسچ.