Journal of Algebraic Systems
Vol. 7, No. 2, (2020), pp 217-226

EQUALIZERS IN THE CATEGORIES FUZZ AND TOPFUZZ

GH. MIRHOSSEINKHANI* AND N. NAZARI

ABSTRACT. It is well known that the categories **Fuzz** of fuzzes and **TopFuzz** of topological fuzzes are both complete and cocomplete, and some categorical properties of them were introduced by many authors. In this paper, we introduce the structure of equalizers in these categories. In particular, we show that every regular monomorphism is an injective map, but monomorphisms need not be injective, in general.

1. INTRODUCTION AND PRELIMINARIES

In 1992, Wang introduced the theory of topological molecular lattices as a generalization of ordinary topological spaces, fuzzy topological spaces and L-fuzzy topological spaces in terms of closed elements, molecules, remote neighborhoods and generalized order-homomorphisms [9]. Then, many authors characterized some topological notions in such spaces, such as convergence theories of molecular nets or ideals, separation axioms and other notions.

Topological fuzzes are an important special class of topological molecular lattices. A fuzz is a pair $(F, ')$ consisting of a completely distributive complete lattice F and an order-reversing involution $' : F \to F$, that is, $x \leq y$ if and only if $y' \leq x'$ and $x'' = x$ for all $x, y \in F$. A topological fuzz or a fuzzy topological space in the sense of Hutton, is a triple $(F, ', \tau)$ such that $(F, ')$ is a fuzz and $\tau \subseteq F$ is a topology, i.e.,

MSC(2010): Primary: 06DXX; Secondary: 06F30, 18A30.
Keywords: Fuzz, topological fuzz, molecular lattice, equalizer.
Received: 10 July 2018, Accepted: 24 February 2019.
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it is closed under finite meets, arbitrary joins and $0, 1 \in \tau$, where 0 and 1 are the smallest and the greatest elements of F, respectively [5, 11].

The category of all fuzzes with their homomorphisms is denoted by **Fuzz**, and the category of all topological fuzzes with their homomorphisms is denoted by **TopFuzz**. It is well known that these categories are both complete and cocomplete, and some categorical properties of them were introduced by many authors [3, 4, 5, 6, 7]. Also, the category **Fuzz** is cartesian closed [11], but since the category **TopFuzz** has the category **Top** of topological spaces as a reflective and coreflective full subcategory, it follows that it is not cartesian closed [8]. In this paper, we introduce the structure of equalizers in these categories. In particular, we show that every regular monomorphism is an injective map, but monomorphisms need not be injective, in general.

Let us first recall some definitions and properties of fuzzes and topological fuzzes.

Definition 1.1. [9] An element a of a lattice F is called coprime, if $a \leq b \vee c$ implies $a \leq b$ or $a \leq c$, for every $b, c \in F$, and it is called completely coprime if, for every $S \subseteq F$, $a \leq \vee S$ implies $a \leq s$ for some $s \in S$.

Throughout this paper, we denote by $M(F)$ and $\overline{M}(F)$ the set of all nonzero coprime elements and nonzero completely coprime elements of a complete lattice F, respectively. Nonzero coprime elements are also called molecules. If F is a completely distributive complete lattice, then F is $\vee$-generated by the set $M(F)$, i.e., every element of F is a join of some elements of $M(F)$. Thus, a completely distributive complete lattice is called a molecular lattice.

Let $f : F \to G$ be a mapping between molecular lattices such that preserves arbitrary joins. Then f has a right adjoint and denoted by $\hat{f}$. Moreover, $\hat{f}(y) = \bigvee\{x \in F \mid f(x) \leq y\}$ for every $y \in G$.

Definition 1.2. [9] A mapping $f : F \to G$ between molecular lattices is called a generalized order-homomorphism (briefly, **GOH**) if f preserves arbitrary joins and its right adjoint $\hat{f}$ preserves arbitrary joins and arbitrary meets.

Definition 1.3. [10] A mapping $f : (F, ') \to (G, ')$ between fuzzes is called an order-homomorphism or a fuzz map in this paper if f preserves arbitrary joins and its right adjoint $\hat{f}$ preserves $'$.

It is easy to show that every fuzz map is a **GOH**, but the converse is not true, in general [10].

Definition 1.4. [5, 11] A fuzz map $f : (F, ', \tau) \to (G, ', \eta)$ between topological fuzzes is said to be continuous if $b \in \eta$ implies $\hat{f}(b) \in \tau$.

Lemma 1.5. [9] *If $f : F \to G$ is a* **GOH***, then f preserves the coprime elements.*

In the following, we recall some definitions and properties of categorical structures [1].

Definition 1.6. Let $A \overset{f}{\underset{g}{\rightrightarrows}} B$ be a pair of morphisms. A morphism $E \xrightarrow{e} A$ is called an equalizer of f and g provided that $f \circ e = g \circ e$; and for any morphism $E' \xrightarrow{e'} A$ with $f \circ e' = g \circ e'$, there exists a unique morphism $E' \xrightarrow{\bar{e}} E$ such that $e' = e \circ \bar{e}$.

Definition 1.7. A morphism $E \xrightarrow{e} A$ is called a regular monomorphism provided that it is an equalizer of some pair of morphisms.

Definition 1.8. A square

$$\begin{tikzcd} P \arrow[r, "\bar{g}"] \arrow[d, "\bar{f}"'] & B \arrow[d, "f"] \\ A \arrow[r, "g"'] & C \end{tikzcd}$$

is called a pullback square provided it commutes and that for any commuting square of the form

$$\begin{tikzcd} Q \arrow[r, "g'"] \arrow[d, "f'"'] & B \arrow[d, "f"] \\ A \arrow[r, "g"'] & C \end{tikzcd}$$

there exists a unique morphism $Q \xrightarrow{k} P$ such that $\bar{f} \circ k = f'$ and $\bar{g} \circ k = g'$.

Theorem 1.9. *Let $A \xrightarrow{f} C \xleftarrow{g} B$ be a pair of morphisms. If $A \xleftarrow{p_1} A \times B \xrightarrow{p_2} B$ is a product of A and B, and $E \xrightarrow{e} A \times B$ is an equalizer of $A \times B \overset{f \circ p_1}{\underset{g \circ p_2}{\rightrightarrows}} C$, then*

$$\begin{tikzcd} E \arrow[r, "p_1 \circ e"] \arrow[d, "p_2 \circ e"'] & A \arrow[d, "f"] \\ B \arrow[r, "g"'] & C \end{tikzcd}$$

is a pullback square.

2. Main results

In this section, we give the structure of equalizers in **Fuzz** and **TopFuzz**. Let $(F, ')$ be a fuzz. Then for every complete join sublattice E of F, i.e., $E \subseteq F$ and it is closed under arbitrary joins, we can define an order-reversing $c : E \to E$ defined by $x^c = \hat{e}(x')$, where $\hat{e}$ is the right adjoint of the inclusion map $e : E \to F$.

Definition 2.1. An element m of a fuzz $(F, ')$ is called $'$-coprime if, for every $x \in F$, either $m \leq x$ or $m \leq x'$ but not both.

We denote by $\widetilde{M}(F)$ the set of all $'$-coprime elements of F. It is easy to show that $\widetilde{M}(F) \subseteq \overline{M}(F) \subseteq M(F)$. For any topological space (X, τ), suppose $\mathcal{P}(X)$ denote the powerset of X and the order-reversing involution on $\mathcal{P}(X)$ is the subset complement. Then $(\mathcal{P}(X), \tau)$ is a topological fuzz and $\widetilde{M}(\mathcal{P}(X)) = \overline{M}(\mathcal{P}(X)) = M(\mathcal{P}(X)) = \{\{x\} \mid x \in X\}$. In general, $\widetilde{M}(F)$ is a join generating base for a fuzz $(F, ')$ if and only if F is fuzz isomorphic to $\mathcal{P}(\widetilde{M}(F))$, moreover $\widetilde{M}(F) = \overline{M}(F) = M(F)$.

For a fuzz map f, since f and $\hat{f}$ preserve arbitrary joins, we have $\hat{f}(0) = \hat{f}(\vee\phi) = \vee\phi = 0$ and similarly, $f(0) = 0$.

Lemma 2.2. *Let $f : F \to G$ be a fuzz map. Then the following statements hold:*

1. *$f \circ \hat{f} \leq id$, $\hat{f} \circ f \geq id$, $f \circ \hat{f} \circ f = f$ and $\hat{f} \circ f \circ \hat{f} = \hat{f}$, where id denote the identity map.*
2. *$\hat{f}$ is unique, i.e., if $g \circ f \geq id$ and $f \circ g \leq id$, then $g = \hat{f}$.*
3. *$\hat{f}(1) = 1$. Also, $f(a) = 0$ if and only if $a = 0$.*

Proof. For parts (1) and (2) see [2]. Since $f(1) \leq 1$, we have $1 \leq \hat{f}(f(1)) \leq \hat{f}(1)$ and hence $\hat{f}(1) = 1$. Now, let $f(a) = 0$. Then $a \leq \hat{f}(f(a)) = \hat{f}(0) = 0$ and hence $a = 0$. $\square$

Lemma 2.3. *Let $f : F \to G$ be a map between fuzzes such that preserves all joins.*

1. *If f is a fuzz map, then f preserves $'$-coprime elements.*
2. *If $\widetilde{M}(F)$ is a join generating base for F, then f is a fuzz map if and only if f preserves $'$-coprime elements.*

Proof. (1): Let $m \in \widetilde{M}(F)$ and $y \in G$. Then either $m \leq \hat{f}(y)$ or $m \leq (\hat{f}(y))' = \hat{f}(y')$. Thus either $f(m) \leq y$ or $f(m) \leq y'$. If $f(m) \leq y \wedge y'$, then $m \leq \hat{f}(y \wedge y') = \hat{f}(y) \wedge \hat{f}(y')$, which is a contradiction. Thus $f(m)$ is a $'$-coprime element.

(2): Let $y \in G$, $m \in \widetilde{M}(F)$ and f preserves $'$-coprime elements. Then

$$m \leq \hat{f}(y') \Leftrightarrow f(m) \leq y' \Leftrightarrow f(m) \nleq y \Leftrightarrow m \nleq \hat{f}(y) \Leftrightarrow m \leq (\hat{f}(y))'.$$

Thus $\hat{f}(y') = (\hat{f}(y))'$. Conversely, by part (1), the result holds. □

By Lemma 2.3, we have the following result.

Corollary 2.4. *$f : \{0,1\} \to F$ is a fuzz map if and only if $f(1) \in \widetilde{M}(F)$.*

Definition 2.5. Let $(F, ')$ be a fuzz and E be a complete join sublattice of F. Then we say that (E, c) is a subfuzz of F if (E, c) is a fuzz and the inclusion map $e : E \to F$ is a fuzz map.

Example 2.6. Let $F = \{0, x, y, 1\}$, where x and y are incomparable, $x' = y$ and $y' = x$. Then F is a fuzz and $E = \{0, x\}$ is a subfuzz of F. If $A = \{0, 1\}$, then A is a fuzz but it is not a subfuzz of F, because the inclusion $e : A \to F$ is not a fuzz map.

Definition 2.7. Let $(F, ', \mu)$ be a topological fuzz and (E, c) a subfuzz of F. If $\delta = \hat{e}(\mu)$, then (E, c, δ) is also a topological fuzz which is called a topological subfuzz of F.

Now, we present a characterization of subfuzzes. Let F be a fuzz. In the following we consider a mapping $J : F \to F$ which satisfies the following conditions:

(S1) $J(a) \leq a$ for all $a \in F$;
(S2) $J \circ J = J$;
(S3) J preserves arbitrary joins;
(S4) $J((J(a))') = J(a')$ for all $a \in F$.

Lemma 2.8. *Let $(F, ')$ be a fuzz and E be a complete join sublattice of F. Then (E, c) is a subfuzz of F if and only if there exists a mapping $J : F \to F$ which satisfies the conditions $(S1) - (S4)$, such that $E = ImJ$ and $a^c = J(a')$ for each $a \in E$.*

Proof. Let (E, c) be a subfuzz of F. Since the inclusion map $e : E \to F$ is a fuzz map, if we define a mapping $J : F \to F$ by $J(a) = \hat{e}(a)$ for each $a \in F$, then the result holds.

Conversely, let $J : F \to F$ be a mapping which satisfies the conditions $(S1) - (S4)$, such that $E = ImJ$ and $a^c = J(a')$ for each $a \in E$. Since J preserves joins, it follows that E is a complete join sublattice of F and hence is a complete lattice. Since J preserves order, c is an order-reversing. Now, we show that c is an involution on E. Let $x \in E$. Then we have

$$x^{cc} = (J(x))^{cc} = (J(x'))^c = J(x'') = J(x) = x.$$

The infimum in E is as following:

$$\sqcap_{i\in I} J(a_i) = (\vee_{i\in I}(J(a_i))^c)^c = (\vee_{i\in I} J(a_i'))^c$$
$$= (J(\vee_{i\in I} a_i'))^c = J(\vee_{i\in I} a_i')' = J(\wedge_{i\in I} a_i).$$

On the other hand, we have

$$\vee_{i\in I}\sqcap_{j\in J} J(a_{ij}) = J(\vee_{i\in I}\wedge_{j\in J} a_{ij}) = J(\wedge_{j\in J}\vee_{i\in I} a_{ij}) = \sqcap_{j\in J}\vee_{i\in I} J(a_{ij}).$$

Thus E is a completely distributive lattice and hence (E, c) is a fuzz. Finally, we show that $e : E \to F$ is a fuzz map. Let $x \in F$. Then we have

$$\hat{e}(x) = \vee\{J(a) \mid J(a) \leq x\} \Rightarrow \hat{e}(x) = J(\hat{e}(x)) \leq x \Rightarrow \hat{e}(x) \leq J(x).$$

Since $J(x) \leq x$, it follows that $\hat{e}(x) = J(x)$. Thus, for any $x \in F$ we have:

$$\hat{e}(x') = J(x') = J((J(x))') = J((\hat{e}(x))') = (\hat{e}(x))^c.$$

□

Lemma 2.9. *Let $(F, ')$ be a fuzz and E be a complete join sublattice of F. Let $\mathcal{S}$ be the collection of all mappings $J : F \to F$ which satisfy the conditions $(S1)-(S4)$ and $ImJ \subseteq E$. Then $\mathcal{S}$ with respect to pointwise order has a maximal element γ such that for each $J \in \mathcal{S}$, $ImJ \subseteq Im\gamma \subseteq E$.*

Proof. Consider $\gamma : F \to F$ given by $\gamma(a) = \vee\{J(a) \mid J \in \mathcal{S}\}$ for every $a \in F$. Then we have:

(1) $\gamma(a) \leq a$;
(2) $\gamma(\gamma(a)) = \gamma(\bigvee_{J\in\mathcal{S}} J(a)) = \bigvee_{I\in\mathcal{S}}\bigvee_{J\in\mathcal{S}} I(J(a))$
$= \bigvee_{I\in\mathcal{S}} I(a) = \gamma(a)$;
(3) $\gamma(\bigvee_{i\in I} a_i) = \bigvee_{J\in\mathcal{S}} J(\bigvee_{i\in I} a_i) = \bigvee_{J\in\mathcal{S}}\bigvee_{i\in I} J(a_i)$
$= \bigvee_{i\in I}\bigvee_{J\in\mathcal{S}} J(a_i) = \bigvee_{i\in I}\gamma(a_i)$;
(4) $\gamma((\gamma(a))') = \bigvee_{J\in\mathcal{S}} J((\gamma(a))') = \bigvee_{J\in\mathcal{S}} J((\bigvee_{I\in\mathcal{S}} I(a))')$
$= \bigvee_{J\in\mathcal{S}} J(\bigwedge_{I\in\mathcal{S}}(I(a))') = \bigvee_{J\in\mathcal{S}}\sqcap_{I\in\mathcal{S}} J((I(a))')$
$= \bigvee_{J\in\mathcal{S}} J(a') = \gamma(a')$.

It is easy to see that for every $J \in \mathcal{S}$, $J \leq \gamma$ and $Im\gamma \subseteq E$. On the other hand, we have $J(a) = J(J(a)) \leq \gamma(J(a)) \leq J(a)$. Thus $J(a) = \gamma(J(a))$, and hence $ImJ \subseteq Im\gamma$. □

By Lemmas 2.8 and 2.9, we have the following result.

Corollary 2.10. *Let $(F, ')$ be a fuzz and A be a complete join sublattice of F. Let $\mathcal{S}$ be the collection of all subfuzzes B of F such that $B \subseteq A$. Then $\mathcal{S}$ has a maximal element.*

Theorem 2.11. *The equalizer of $F \underset{g}{\overset{f}{\rightrightarrows}} G$ in* **Fuzz** *is the pair (E, e), where E is the maximal subfuzz of F such that $E \subseteq E_{fg} := \{x \in F \mid f(x) = g(x)\}$ and $e : E \to F$ is the inclusion map.*

Proof. Let $h : N \to F$ be a fuzz map such that $f \circ h = g \circ h$. Consider a mapping $J : F \to F$ given by $J(a) = h \circ \hat{h}(a)$ for every $a \in F$. Since $\hat{h} \circ h \circ \hat{h} = \hat{h}$, it is easy to show that J satisfies the conditions $(S1) - (S4)$. Thus $Imh = ImJ \subseteq E_{fg}$ and hence $h(x) \in E$. Now, we define $\alpha : N \to E$ by $\alpha(x) = h(x)$. Then $e \circ \alpha = h$ and for every $x \in E$ we have

$$\hat{\alpha}(x^c) = \hat{\alpha}(\hat{e}(x)^c) = \hat{\alpha} \circ \hat{e}(x') = \hat{h}(x') = (\hat{h}(x))' = (\hat{\alpha}(x))'.$$

Thus α is a fuzz map. Finally, it is easy to check that α is unique. □

Similar to the proof of Theorem 2.11, we have the following result.

Theorem 2.12. *$e : (E, \delta) \to (F, \tau)$ is an equalizer of $(F, \tau) \underset{g}{\overset{f}{\rightrightarrows}} (G, \eta)$ in* **TopFuzz** *if and only if e is an equalizer in* **Fuzz** *and $\delta = \{\hat{e}(a) \mid a \in \tau\}$.*

Corollary 2.13. *If $\widetilde{M}(F)$ is a join generating base for F, then the equalizer of $F \underset{g}{\overset{f}{\rightrightarrows}} G$ in* **Fuzz** *is the pair (E, e), where E is the complete join sublattice generated by the set $\widetilde{M}_{fg} := \{m \in \widetilde{M}(F) \mid f(x) = g(x)\}$ and $e : E \to F$ is the inclusion map.*

Proof. Consider $\gamma : F \to F$ given by $\gamma(a) = \vee\{m \in \widetilde{M}_{fg} \mid m \leq a\}$ for every $a \in F$. It is easy to show that the map γ satisfies the conditions $(S1) - (S4)$ and $Im\gamma = E$. On the other hand, for every mapping $J : F \to F$ which satisfies the conditions $(S1) - (S4)$ and $ImJ \subseteq E_{fg}$, we have $J(a) = J(\vee\{m \in \widetilde{M}(F) \mid m \leq a\}) = \vee\{J(m) \mid m \in \widetilde{M}(F), m \leq a\}$. Since $J(m) \leq m$, so for every $m \in \widetilde{M}(F)$, either $J(m) = m$ or $J(m) = 0$. Thus $ImJ \subseteq Im\gamma$, which shows that E is a maximal subfuzz of F such that $E \subseteq E_{fg}$, as desired. □

Remark 2.14. [8] The category **Top** is a reflective and coreflective full subcategory of **TopFuzz** via the embedding power functor $\mathcal{P}$: **TOP** $\to$ **TopFuzz** defined by $\mathcal{P}(X, \tau) = (\mathcal{P}(X), \tau)$.

Example 2.15. Let $X \underset{g}{\overset{f}{\rightrightarrows}} Y$ be two arbitrary continuous functions. Then the equalizer of $\mathcal{P}(X) \underset{\mathcal{P}(g)}{\overset{\mathcal{P}(f)}{\rightrightarrows}} \mathcal{P}(Y)$ in **TopFuzz** is the pair (E, e), where E is the complete join sublattice generated by the set $\{\{x\} \mid x \in X, f(x) = g(x)\}$ and $e : E \to \mathcal{P}(X)$ is the inclusion map. Thus $E = \mathcal{P}(E_{fg})$, where $E_{fg} := \{x \in X \mid f(x) = g(x)\}$ is the equalizer of f and g in the category **Top**. This of course amounts to the familiar fact that the reflector $\mathcal{P}$ preserves limits.

Recall that an extra order $\lhd$ on a complete lattice L is defined by $a \lhd b$ if, for every subset $S \subseteq L$, $b \leq \vee S$ implies $a \leq s$ for some $s \in S$ [7].

Let $\{F_i\}_{i \in I}$ be a family of fuzzes. In [11], the product of $\{F_i\}_{i \in I}$ in **Fuzz** was given by $\otimes_{i \in I} F_i = \{A \subset \prod_{i \in I} F_i \mid A = \downarrow A$; and for every $x \in A$ there exists $y \in A$ such that $x_i \lhd y_i$ for every $i \in I\}$, where $\downarrow A$ is the lower set of A. Now, by Theorem 1.9, we have the following result.

Corollary 2.16. *Let $F \xrightarrow{f} L \xleftarrow{g} G$ be a pair of fuzz maps. If $F \xleftarrow{p_1} F \otimes G \xrightarrow{p_2} G$ is the product of F and G, and $E \xrightarrow{e} F \otimes G$ is the equalizer of $F \otimes G \underset{g \circ p_2}{\overset{f \circ p_1}{\rightrightarrows}} L$, then the square*

$$\begin{tikzcd} E \arrow[r, "p_1 \circ e"] \arrow[d, "p_2 \circ e"'] & F \arrow[d, "f"] \\ G \arrow[r, "g"'] & L \end{tikzcd}$$

is a pullback in **Fuzz**.

By Theorems 2.11 and 2.12, we have every regular monomorphism in **Fuzz** and **TopFuzz** is an embedding map, but the following example shows that monomorphisms need not be injective, in general.

Example 2.17. A monomorphism in **Fuzz** and **TopFuzz** need not be an injective map. For instance, consider $F = \{0, x, y, 1\}$, where x and y are incomparable, $x' = x$, $y' = y$ and $G = \{0, a, 1\}$, where $0 < a < 1$, $a' = a$. Let the mapping $f : F \to G$ defined by: $f(0) = 0$, $f(x) = a$, $f(1) = f(y) = 1$. Then f is a **Fuzz**-morphism, but it is not injective. Now, we show that f is monomorphism. Let $L \underset{s}{\overset{r}{\rightrightarrows}} F$ such that $r \neq s$. Then there exists a $m \in M(L)$ such that $r(m) \neq s(m)$. Since r and s preserve coprime elements, it follows that $r(m)$ and $s(m) \in M(F) = \{x, y\}$. Without loss of generality, let $r(m) = x$ and $s(m) = y$. Then $f(r(m)) = a \neq f(s(m)) = 1$, this implies that $f \circ r \neq f \circ s$. Also, if $\tau_1 = \tau_2 = \{0, 1\}$, then f is a **TopFuzz**-morphism, and hence the result holds for **TopFuzz**.

Theorem 2.18. *If $f : F \to G$ is a monomorphism in* **Fuzz** *and* **TopFuzz**, *then the restriction mapping $f\mid_{\widetilde{M}(F)} : \widetilde{M}(F) \to \widetilde{M}(G)$ is injective.*

Proof. Let $m_1, m_2 \in \widetilde{M}(F)$ and $f(m_1) = f(m_2)$. Consider $r, s : \{0,1\} \to F$ given by $r(0) = s(0) = 0$, $r(1) = m_1$, $s(1) = m_2$. By Corollary 2.4, r and s are fuzz maps and continuous with respect to the discrete topology on $\{0,1\}$. Now, we have $f \circ r = f \circ s$ and hence by hypothesis $r = s$. Thus $m_1 = r(1) = s(1) = m_2$. □

3. Conclusions

It is well known that the categories **Fuzz** of fuzzes with their homomorphisms and **TopFuzz** of topological fuzzes with their homomorphisms are both complete and cocomplete, and some categorical properties of them were introduced by many authors. In this paper, we have presented the structure of equalizes in these categories. For this, we have defined some concepts as subfuzz and topological subfuzz and shown that equalizers are embedding and continuous embedding fuzz maps. Thus, every regular monomorphism in these categories is an injective map, but we have shown that monomorphisms need not be injective, in general.

Acknowledgments

We are very grateful to the referee(s) for the careful reading of the paper and for the useful comments.

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Journal of Algebraic Systems

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معادل‌سازها در رسته‌های فاز و تاپ‌فاز

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به عنوان یک نتیجه شناخته‌شده می‌دانیم که رسته فاز شامل فازی‌ها و رسته تاپ‌فاز شامل فازی‌های توپولوژیک، رسته‌هایی تام و هم-تام هستند و بعضی خواص رسته‌ای آن‌ها توسط پژوهشگران زیادی معرفی شده است. در این مقاله، ما به معرفی ساختار معادل‌سازها در این رسته‌ها می‌پردازیم. همچنین نشان می‌دهیم که هر تکریختی منظمی یک نگاشت یک‌به‌یک است اما تکریختی‌ها لزوماً توابع یک‌به‌یک نیستند.

کلمات کلیدی: فاز، تاپ فاز، معادل‌ساز.