Journal of Algebraic Systems
Vol. 9, No 1, (2021), pp 137-149

LEFT ABSORBING HYPER K-ALGEBRAS

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ABSTRACT. In the present manuscript, we introduce a type of hyper K-algebra which is called left absorbing hyper K-algebra and investigate some of the related properties. We also show that set of all types of positive implicative and commutative hyper K-ideal form a distributive latttice and study their diagrams when positive implicative and commutative hyper K-ideal are a hyper K-ideal and the hyper K-algebra is left absorbing.

1. INTRODUCTION

The concept of BCK-algebra that is a generalization of set difference and propositional calculi was established by Imai and Iséki [4] in 1966. In [5], Jun et al. applied the hyper structures BCK-algebra. In 1934, Marty [7] introduced for the first time the hyper structure theory in the 8th congress of Scandinavian Mathematicians proceedings. In [3], Borzooei et al. introduced the generalization of BCK-algebra and hyper BCK-algebra, called hyper K-algebra. They studied properties of hyper K-algebra. In [9], Roodbari et al. defined 27 different types of positive implicative and 9 different types of commutative hyper K-ideal. In [2], Borzooei et al. studied lattices structures on ideals of a BCK-algebras. In this article, we introduce left absorbing hyper K-algebra and investigate some related properties. Moreover, We show that all types of positive implicative and commutative hyper K-ideals

DOI: 10.22044/jas.2020.8717.1423.
MSC(2010): 06F35.
Keywords: Hyper K-algebra, Hyper K-ideal, Positive implicative hyper K-ideal, Commutative hyper K-ideal.
Received: 21 July 2019, Accepted: 30 October 2020.
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that defined in [9], form a distributive latttice and study their diagrams when the hyper K-agebra is left absorbing. Section 2, concerns definitions and theorems that are needed in the sequel. Section 3, we give definition of left absorbing hyper K-algebras and then we investigate some properties of them.

2. Preliminaries

In this section, we give concerns definitions and theorems that are needed in the sequel.

Definition 2.1. [3] Let H be a nonempty set and " $\circ$ " be a hyper operation on H, that $\circ$ is a function from $H\times H$ to $P^*(H)=P(H)\backslash\{\emptyset\}$. Then H is called a *hyper K-algebra* iff it contains a constant "0" and satisfies the following axioms:

(HK1): $(x\circ z)\circ(y\circ z)<x\circ y$,
(HK2): $(x\circ y)\circ z=(x\circ z)\circ y$,
(HK3): $x<x$
(HK4): $x<y, y<x\Longrightarrow x=y$,
(HK5): $0<x$

for all $x,y,z\in H$, where $x<y$ means $0\in x\circ y$ and for every $A,B\subseteq H, A<B$ is defined by $\exists a\in A$, $\exists b\in B$ such that $a<b$. If $A,B\subseteq H$, then $A\circ B:=\bigcup_{a\in A,b\in B} a\circ b$.

Theorem 2.2. [3] *Let $(H,\circ,0)$ be a hyper K-algebra. Then for all $x,y,z\in H$ and for all nonempty subsets A,B and C of H the following statements hold:*

$(i)\, x\circ y<z\Leftrightarrow x\circ z<y$,
$(ii)\, (x\circ\, z)\circ(x\circ y)<y\circ z$,
$(iii)\, x\circ\, (x\circ y)<y$,
$(iv)\, x\circ\, y<x$,
$(v)\, A\subseteq B\Rightarrow A<B$,
$(vi)\, x\in x\circ 0$,
$(vii)\, (A\circ\, C)\circ(A\circ B)<B\circ C$,
$(viii)\, A\circ\, B<C\Leftrightarrow A\circ C<B$.

Definition 2.3. [11] Let H_1 and H_2 be two hyper K-algebras. A mapping $f:H_1\to H_2$ is said to be a homomorphism if

(1) $f(0)=0$,
(2) $f(x\circ y)=f(x)\circ f(y), \forall x,y\in H_1$.

Theorem 2.4. [11] *Let $(H_1,\circ_1,0)$ and $(H_2,\circ_2,0)$ be two hyper K-algebras and $H=H_1\times H_2$. Then $(H,\circ,0)$ where $(a_1,b_1)\circ(a_2,b_2)=$*

$(a_1 \circ_1 a_2, b_1 \circ_2 b_2)$ for all $(a_1, b_1), (a_2, b_2) \in H$ is a hyper K-algebra, and it is called the hyper K-product of H_1 and H_2.

Definition 2.5. [10] A hyper K-algebra $(H, \circ, 0)$ is called simple if for all distinct elements $a, b \in H \setminus \{0\}$, $a \not< b$ and $b \not< a$, otherwise is called normal.

Theorem 2.6. [10] *Let $(H, \circ, 0)$ be a simple hyper K-algebra. Then for all $x \in H$, $x \circ 0 = \{x\}$.*

Definition 2.7. [1] Let $(H, \circ, 0)$ be a hyper K-algebra. Then $(H, \circ, 0)$ is called:
(i) weak implicative, if for all $x, y \in H$, $x < x \circ (y \circ x)$,
(ii) implicative, if for all $x, y \in H$, $x \in x \circ (y \circ x)$.

Definition 2.8. [3, 11] Let I be nonempty subset of a hyper K-algebra such that $0 \in I$. Then I is said to be a hyper K-ideal (weak hyper K-ideal) of H if $x \circ y < I(x \circ y \subseteq I)$ and $y \in I$ imply $x \in I$ for all $x, y \in H$.

Theorem 2.9. [9] *Let I be a hyper K-ideal of hyper K-algebra $(H, \circ, 0)$ and A, B be nonempty subsets of H, then $A \circ B < I$ iff $A \circ B \cap I \neq \emptyset$.*

Notation 2.10. *Let A and I be two nonempty sets, we set $AR_1I := A \subseteq I$, $AR_2I := A \cap I \neq \emptyset$ and $AR_3I := A < I$.*

Definition 2.11. [9] Let I be a nonempty subset of a hyper K-algebra $(H, \circ, 0)$ such that $0 \in I$. Then I is called a positive implicative hyper K-ideal of type (i, j, k) of H and we write $I - PIHKI(i, j, k)$, if $(x \circ y) \circ zR_iI$ and $y \circ zR_jI$ imply that $x \circ zR_kI$ for all $x, y, z \in H, i, j, k \in \{1, 2, 3\}$.

Definition 2.12. [9] Let I be a nonempty subset of a hyper K-algebra $(H, \circ, 0)$ such that $0 \in I$. Then I is called a commutative hyper K-ideal of type (i, j) and we write $I - CHKI(i, j)$, if $(x \circ y) \circ zR_iI$ and $z \in I$ imply that $x \circ (y \circ (y \circ x))R_jI$ for all $x, y, z \in H, i, j \in \{1, 2, 3\}$.

Definition 2.13. [1] Let I be a nonempty subset of a hyper K-algebra $(H, \circ, 0)$ such that $0 \in I$. Then I is called an implicative (weak implicative) hyper K-ideal if $(x \circ z) \circ (y \circ x) < I$ $((x \circ z) \circ (y \circ x) \subseteq I)$ and $z \in I$ imply $x \in I$, for all $x, y, z \in H$.

Theorem 2.14. [1] *Let I be a hyper K-ideal of hyper K-algebra H. Then I is an (weak) implicative hyper K-ideal if and only if $(x \circ (y \circ x) \subseteq I)x \circ (y \circ x) < I$ implies that $x \in I$, for any $x, y \in H$.*

Definition 2.15. [6] Let ρ be a relation defined on a set X. Then converse of ρ (denoted by $\bar{\rho}$) is defined by $a \, \bar{\rho} \, b \Leftrightarrow b \, \rho \, a$, $a, b \in X$.

Definition 2.16. [6] If (X, ρ) be a partially ordered set (poset) then the poset $(\bar{X}, \bar{\rho})$, where $\bar{X} = X$ and $\bar{\rho}$ is converse of ρ is called dual of X.

Definition 2.17. [6] Let $(L, \leq)$ be a partially ordered set. Then L is called a chain if every two members are comparable, i.e. $x \leq y$ or $y \leq x$ for all $x, y \in L$, and it is said to be a lattice if for every $a, b \in L$, Sup$\{a, b\}$ and Inf $\{a, b\}$ exist in L, in this case, we write $Sup\{a, b\} = a \vee b$ and $Inf\{a, b\} = a \wedge b$.

Definition 2.18. [6] A lattice L is called a distributive lattice if $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ for all $a, b, c \in L$.

Theorem 2.19. [6] *A chain is a distributive lattice.*

Theorem 2.20. [6] *Two lattices L and M are distributive lattices iff $L \times M$ is distributive lattice.*

3. **Left absorbing hyper K-algebras**

In this section we define the concept of left absorbing hyper K-algebras. Also, some related properties are investigated.

Definition 3.1. Let H be a nonempty set and "$\circ$" a hyper operation on H. Then "$\circ$" is called a left absorbing hyper operation if $x \in x \circ y$ for all $x, y \in H$.

Theorem 3.2. *Let H containig 0 be a set and "$\circ$" a left absorbing hyper operation on H. Then $(H, \circ, 0)$ is hyper K-algebra iff satisfies the following axioms:*

(1) $(x \circ y) \circ z = (x \circ z) \circ y$,
(2) $x < x$,
(3) $x < y$, $y < x \Longrightarrow x = y$.

for all $x, y, z \in H$.

Proof. Let H be a hyper K-algebra, it is clear (1), (2) and (3) hold. Conversely, since " $\circ$ " is a left absorbing hyper operation on H, we have $x \circ y \subseteq (x \circ z) \circ (y \circ z)$ then $(x \circ z) \circ (y \circ z) < x \circ y$, also $0 \in 0 \circ x$, for all $x, y, z \in H$, so $(HK1)$ and $(HK5)$ hold and $(H, \circ, 0)$ is a hyper K-algebra. □

The following examples show that properties (1) and (2) in the above theorem are independent from each other.

Example 3.3. Let $H = \{0, 1, 2\}$ and consider the following Cayley tables:

$\circ_1$	0	1	2
0	{0}	{0}	{0}
1	{1,2}	{1,2}	{0,1}
2	{2}	{2}	{0,2}

$\circ_2$	0	1	2
0	{0}	{0}	{0}
1	{1,2}	{0,1}	{1,2}
2	{2}	{2}	{0,2}

Hyper operations $\circ_1$ and $\circ_2$ are left absorbing on H. In $(H, \circ_1, 0)$ the properties 1 and 3 hold, but $1 \not< 1$ and in $(H, \circ_2, 0)$ the properties 2 and 3 hold but $(1 \circ_2 0) \circ_2 2 \neq (1 \circ_2 2) \circ_2 0$.

Definition 3.4. The hyper K-algebra which has been introduced in theorem 3.2 is called a left absorbing hyper K-algebra.

Example 3.5. Let $H = \{0, 1, 2\}$ and consider the following Cayley tables:

$\circ_1$	0	1	2
0	{0,1}	{0}	{0}
1	{1}	{0,1}	{1}
2	{1,2}	{0,2}	{0,2}

$\circ_2$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{1}
2	{2}	{0,1}	{0,1,2}

Then $(H, \circ_1, 0)$ is a left absorbing hyper K-algebra, but $(H, \circ_2, 0)$ is not a left absorbing hyper K-algebra, since $2 \notin 2 \circ_2 1$.

Theorem 3.6. *Let $(H_1, \circ_1, 0)$ and $(H_2, \circ_2, 0)$ be a left absorbing hyper K-algebra and a hyper K-algebra respectively, and $f : H_1 \to H_2$ be an onto homomorphism. Then $(H_2, \circ_2, 0)$ is a left absorbing hyper K-algebra.*

Proof. Let $t, s \in H_2$. Since f is an onto homomorphism, there exist $x, y \in H_1$ that $f(x) = t$ and $f(y) = s$. Since $(H_1, \circ_1, 0)$ is a left absorbing hyper K-algebra, we have $x \in x \circ_1 y$. So $f(x) \in f(x \circ_1 y) = f(x) \circ_2 f(y) = t \circ_2 s$ and $(H_2, \circ_2, 0)$ is a left absorbing hyper K-algebra. □

Theorem 3.7. *Let H_1 and H_2 be two left absorbing hyper K-algebras. Then $H = H_1 \times H_2$ is a left absorbing hyper K-algebra.*

Proof. By Theorem 2.4, H is a hyper K-algebra. Since $x_1 \in x_1 \circ_1 x_2$ and $y_1 \in y_1 \circ_2 y_2$ we have $(x_1, y_1) \in (x_1 \circ_1 x_2, y_1 \circ_2 y_2) = (x_1, y_1) \circ (x_2, y_2)$ and the proof is complete. □

Theorem 3.8. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra. Then the hyper operation $\circ$ is order preserving, i.e. if $y < z$ then $x \circ y < x \circ z$ and $y \circ x < z \circ x$, for all $x, y, z \in H$. Also if $B < C$ then $A \circ B < A \circ C$ and $B \circ A < C \circ A$, for all subsets A, B and C of H.*

Proof. Since $x \in x \circ t$ for all $x, t \in H$ we get $x \circ y < x \circ z$. Also $y < z$ implies $y \circ x < z \circ x$, since $y \in y \circ x$ and $z \in z \circ x$. The proof of the other cases are similar. □

Theorem 3.9. *Let $(H, \circ, 0)$ be a simple hyper K-algebra. Then H is a left absorbing hyper K-algebra.*

Proof. Let $x, y \in H$. By Theorem 2.2 (iv), we have $x \circ y < x$, so there exist $a \in x \circ y$ where $a < x$. Since H is simple we get $a = 0$ or $a = x$. If $a = 0$ then $0 \in x \circ y$ which is a contradiction to the simplicity of H. Thus $a = x$ and $x \in x \circ y$. □

Theorem 3.10. *Let $(H, \circ, 0)$ be a simple left absorbing hyper K-algebra. Then for all $x \in H$, $0 \circ 0 \subseteq x \circ x$.*

Proof. By (HK2), (HK3) and Theorem 2.6, we have $0 \circ 0 \subseteq (x \circ x) \circ 0 = (x \circ 0) \circ x = x \circ x$. □

In Example 3.5, $(H, \circ_1, 0)$ is a left absorbing hyper K-algebra but it is not simple and $0 \circ_1 0 \nsubseteq 2 \circ_1 2$.
It is clear that any implicative hyper K-algebra is weak implicative hyper K-algebra but the converse is not true. For example, $(H, \circ_2, 0)$ in Example 3.5, is weak implicative hyper K-algebra but it is not implicative hyper K-algebra. The following theorem shows that these concepts are equivalent when the hyper K-algebra is left absorbing.

Theorem 3.11. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra. Then H is implicative hyper K-algebra.*

Proof. Since $x \in x \circ (y \circ x)$, by definition 2.7(iii), H is implicative. □

Theorem 3.12. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra and $0 \in I \subseteq H$. Then I is a weak hyper K-ideal.*

Proof. Let $x \circ y \subseteq I$ and $y \in I$. Since H is a left absorbing hyper K-algebra we have $x \in I$. □

The left absorbing condition in Theorem 3.12 is necessary, since $I = \{0, 1\}$ is not weak hyper K-ideal of $(H, \circ_2, 0)$ in Example 3.5, because $2 \circ_2 1 \subseteq I$ and $2 \notin I$. Even, under condition of Theorem 3.12, I may not be hyper K-ideal of H. Because $I = \{0, 1\}$ is not a hyper K-ideal of $(H, \circ_1, 0)$ in Example 3.5, since $2 \circ_1 1 < I$ and $2 \notin I$. Now, we want to study the relationship between all types of positive implicative and commutative hyper K-ideals. We show that all types of these two hyper K-ideals form a distributive lattice. Also, we investigate these relationships in a left absorbing hyper K-algebra.

3.1. **Lattice of $I - PIHKI(i, j, k)$ and left absorbing hyper K-algebras.**

Theorem 3.13. *Let A and I be two nonempty subsets of a hyper K-algebra H. Then AR_iI imply AR_jI iff $i \leq j$ where $i, j, k \in \{1, 2, 3\}$.*

Proof. Since $A \subseteq I \Rightarrow A \cap I \neq \emptyset \Rightarrow A < I$, by notation 2.10, we have $AR_iI \Rightarrow AR_jI$ iff $i \leq j$. □

Theorem 3.14. *Let H be a hyper K-algebra and L be a set of $I - PIHKI(i, j, k)$ on H, such that I is fixed and $i, j, k \in \{1, 2, 3\}$. Then $(L, \sqsubseteq)$ is a distributive lattice where $(i, j, k) \sqsubseteq (i', j', k')$ iff $i \geq i'$, $j \geq j'$ and $k \leq k'$.*

Proof. Let $L = (\{(i, j, k)|i, j, k \in \{1, 2, 3\}\}, \subseteq)$ and $L_1 = (\{1, 2, 3\}, \leq)$ where $\leq$ is usual order and L_2 is dual of L_1. Then it is clear that L_1 and L_2 are chains, so $(L, \sqsubseteq)$ is isomorphic to $L_2 \times L_2 \times L_1$ and by Theorems 2.19 and 2.20, $(L, \sqsubseteq)$ is a distributive lattice. □

The diagram of the lattice introduced in Theorem 3.14 is as Figure 1 (for simplicity, we use ijk instead of $I - PIHKI(i, j, k)$), if I be a hyper K-ideal of H, then by Theorems 3.15, 3.17, 3.18 and 3.19 in Ref. [9], AR_2I is equivalent to AR_3I and in this case, its diagram is as Figure 2. In the following diagrams, any two comparable elements are joined by lines and non-comparable elements are not joined. Moreover, in such a way that if $ijk \leq i'j'k'$ then ijk lies left $i'j'k'$ in the Figure 1.

Theorem 3.15. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra and $0 \in I \subseteq H$. Then I is a $I - PIHKI(1, j, k)$ where $j, k \in \{1, 2, 3\}$.*

Proof. Let $(x \circ y) \circ z \subseteq I$. Since H is left absorbing hyper K-algebra we have, $x \circ z \subseteq (x \circ y) \circ z \subseteq I$ and by Theorem 3.13 the proof is complete. □

The following example shows that in Theorem 3.15, the left absorbing condition of H is necessary.

Example 3.16. Consider a hyper K-algebra $H = \{0, 1, 2\}$ with Cayley table as follows. Then $(H, \circ, 0)$ is not left absorbing and $I = \{0, 1\}$ is not a $I - PIHKI(1, 1, 3)$. Since $(2 \circ 1) \circ 0 = \{1\} \subseteq I$ and $1 \circ 0 \subseteq I$ but $2 \circ 0 \not< I$.

If H be a left absorbing hyper K-algebera then all $I - PIHKI(1, j, k)$ where $j, k \in \{1, 2, 3\}$ are equivalent to each other and the diagram of

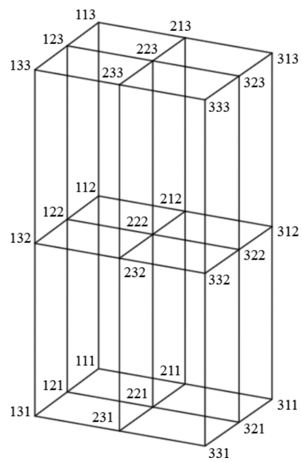

FIGURE 1. Diagram of $I - PIHKI(i, j, k)$

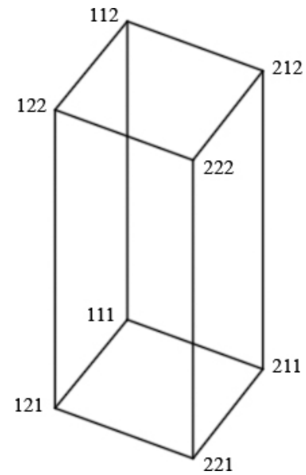

FIGURE 2. Diagram of $I - PIHKI(i, j, k)$, when I is a hyper K-ideal

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{0,1}
2	{2}	{1}	{0,1,2}

$I - PIHKI(i, j, k)$, is as Figure 3. when I is a hyper K-ideal, its diagram is as Figure 4.

3.2. Lattice of $I - CHKI(i, j)$ and left absorbing hyper K-algebras.

Theorem 3.17. *Let H be a hyper K-algebra and L' be a set of $I - CHKI(i, j)$ on H, such that I is fixed and $i, j \in \{1, 2, 3\}$. Then $(L', \preceq)$ is a distributive lattice where $(i, j) \preceq (i', j')$ iff $i \geq i'$ and $j \leq j'$.*

Proof. The proof is similar to the proof of Theorem 3.14. □

The diagram of the lattice introduced in Theorem 3.17 is as Figure 5 and if I is a hyper K-ideal of H, then its diagram is as Figure 6.

Theorem 3.18. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra. Then every nonempty subset of H containing 0 is a $I - CHKI(1, j)$; $j \in \{2, 3\}$.*

Proof. By according to Figure 5, it is sufficient to prove the theorem for type $(1, 2)$. Let $0 \in I \subseteq H$ and $(x \circ y) \circ z \subseteq I$, $z \in I$. Since H is left absorbing hyper K-algebra, we have $x \in I$. So $x \circ (y \circ (y \circ x)) \cap I \neq \emptyset$. □

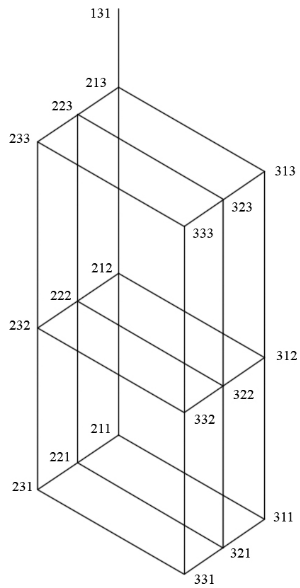

FIGURE 3. Diagram of $I - PIHKI(i, j, k)$, when H is left absorbing

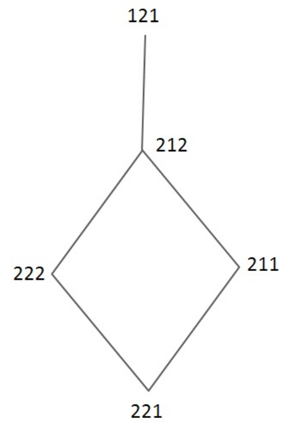

FIGURE 4. Diagram of $I - PIHKI(i, j, k)$, when H is left absorbing and I a hyper K-ideal

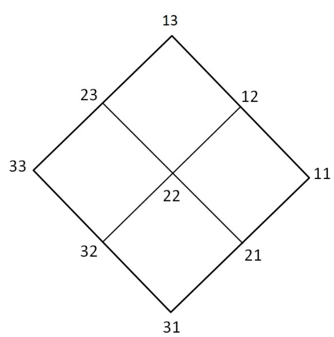

FIGURE 5. Diagram of $I - CHKI(i, j)$

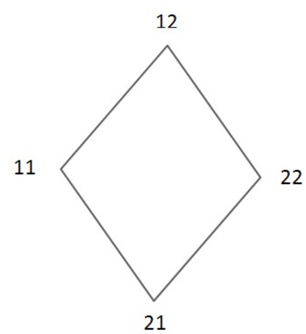

FIGURE 6. Diagram of $I - CHKI(i, j)$, when I is a hyper K-ideal

The following example shows that in Theorem 3.18, the left absorbing condition of H is necessary.

Example 3.19. In Example 3.16, $(H, \circ, 0)$ is not a left absorbing hyper K-algebra and $I = \{0, 1\}$ is not a $I - CHKI(1, 3)$. Since $(2 \circ 0) \circ 1 = \{1\} \subseteq I$ but $2 \circ (0 \circ (0 \circ 2)) = \{2\} \nsubseteq I$.

By considering Theorem 3.18 and Figure 5, we see that $I-CHKI(1,j)$ where $j \in \{2,3\}$ are equivalent to each other, so Figure 5 changes to Figure 7 and when I is a hyper K-ideal its diagram is as Figure 8.

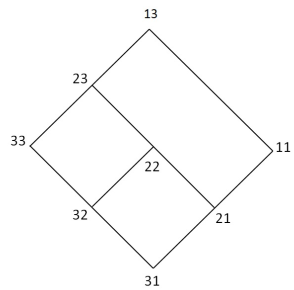

FIGURE 7. Diagram of $I-CHKI(i,j)$, when H is left absorbing

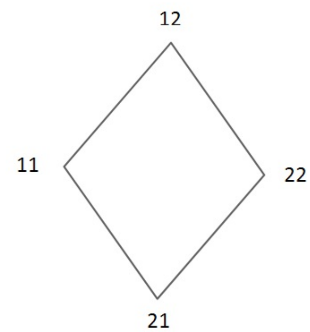

FIGURE 8. Diagram of $I-CHKI(i,j)$, when H is left absorbing and I a hyper K-ideal

Theorem 3.20. *Let $(H,\circ,0)$ be a left absorbing hyper K-algebra. Then $I=\{0\}$ is a $I-CHKI(i,j)$; $i,j \in \{2,3\}$.*

Proof. Considering Figure 5, it is sufficient to prove the theorem for type $(3,2)$. Let $(x \circ y) \circ 0 < I = \{0\}$, by Definition 2.1, there exists $a \in x \circ y$ such that $a \circ 0 < \{0\}$. So $a \in a \circ 0 = \{0\}$ and $0 \in x \circ y$. Since H is left absorbing, we have $x \circ y \subseteq x \circ (y \circ (y \circ x))$ and $0 \in (x \circ (y \circ (y \circ x))) \cap I \neq \emptyset$. Thus $x \circ (y \circ (y \circ x)) \cap I \neq \emptyset$ and $I = \{0\}$ is a $I-CHKI(3,2)$. □

The following example shows that in Theorem 3.20, the left absorbing condition of H is necessary.

Example 3.21. In the following hyper K-algebra, we see that $H = \{0,1,2,3\}$ is not left absorbing and $I=\{0\}$ is not a $I-CHKI(2,3)$. Since $(3 \circ 2) \circ 0 = \{0,1\} \cap I \neq \emptyset$ but $3 \circ (2 \circ (2 \circ 3)) = \{3\} \nless I$.

$\circ$	0	1	2	3
0	{0,1}	{0,1}	{0,1}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{1,2}	{0,1}	{2}
3	{3}	{3}	{0,1}	{0,1}

Theorem 3.22. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra and $I \subseteq H$ be a $I - CHKI(i, 1)$; $i \in \{2, 3\}$. Then I is a weak hyper K-ideal of H.*

Proof. Let $x \circ y \subseteq I$ and $y \in I$. Since H is left absorbing, we have $x \circ y \subseteq (x \circ y) \circ 0$, by Theorem 3.13, $(x \circ y) \circ 0 R_i I$ where $i \in \{2, 3\}$, by assumption we have $x \circ (y \circ (y \circ x)) \subseteq I$. Since H is left absorbing, we get $x \in I$ and I is a weak hyper K-ideal. □

Theorem 3.23. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra and $I \subseteq H$ be a $I - CHKI(3, 1)$. Then I is a hyper K-ideal of H.*

Proof. Let $x \circ y < I$ and $y \in I$. Since H is left absorbing, we have $x \circ y \subseteq (x \circ y) \circ 0$, so $(x \circ y) \circ 0 < I$. By assumption of theorem, we have $x \in x \circ (y \circ (y \circ x)) \subseteq I$ and the proof is complete. □

The following example shows that the converse of the above theorem is not true in general.

Example 3.24. Consider $H = \{0, 1, 2\}$. Then $(H, \circ, 0)$ is a left absorbing hyper K-algebra. It could be easly seen that $I = \{0, 1\}$ is a hyper K-ideal of H, but is not $I - CHKI(3, 1)$. Because, $(1 \circ 0) \circ 0 = \{1, 2\} < I$ and $1 \circ (0 \circ (0 \circ 1)) = \{1, 2\} \nsubseteq I$

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1,2}	{0,1}	{0,1}
2	{2}	{2}	{0,1,2}

Theorem 3.25. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra. Then the only implicative hyper K-ideal of H is H.*

Proof. Let $I \subseteq H$ be an implicative hyper K-ideal of H and $x \in H$. Since H is left absorbing, we have $x \in x \circ x$ and so $0 \in x \circ x \subseteq x \circ (x \circ x)$ and consequently $x \circ (x \circ x) < I$. By assumption $x \in I$ and $H \subseteq I$, so $I = H$. □

The following table shows that the converse of the above theorem is not true in general.

Example 3.26. The following table shows a hyper K-algebra structure on $H = \{0, 1, 2\}$, but not left absorbing hyper K-algebra. $I = \{0\}$, $\{0, 1\}$ and $\{0, 2\}$ are not Since $2 \circ (2 \circ 2) < \{0, 1\}$ but $2 \notin \{0, 1\}$. So $I = \{0, 1\}$ is not an implicative hyper K-ideal of H. Similarly, $I = \{0\}$ and $I = \{0, 2\}$ are not an implicative hyper K-ideal of H. Consequently, $I = H$ is the only implicative hyper K-ideal of H.

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0,2}	{2}
2	{2}	{0,1}	{0,1}

Theorem 3.27. *Let $(H, \circ, 0)$ be a left absorbing hyper K-algebra. Then every nonempty subset of H containing 0 is a weak implicative hyper K-ideal of H.*

Proof. Let $0 \in I \subseteq H$ and $x \circ (y \circ x) \subseteq I$. Since H is left absorbing hyper K-algebra we have $x \in x \circ (y \circ x) \subseteq I$. So $x \in I$ and the proof is complete. □

Example 3.28. Let $H = \{0, 1, 2\}$ and consider the following table. We see that H is not left absorbing hyper K-algebra and $I = \{0, 2\}$ is not a weak implicative hyper K-ideal of H. Since $(1 \circ 2) \circ (2 \circ 1) = \{0\} \subseteq I$ but $1 \notin I$.

$\circ$	0	1	2
0	{0}	{0}	{0}
1	{1}	{0}	{0}
2	{2}	{1}	{0,1}

OPEN PROBLEM: Under what suitable condition a left absorbing hyper operation satisfies axiom (HK2)?

Conclusion. In this study, authors reduced the conditions necessary to be hyper K-algebra of a hyper operation by introducing left absorbing hyper K-algebras and proved the theorems related to them. Also it was showed that the types of positive implicative and commutative hyper K-ideals form a distributive lattice. Theorems 3.18 and 3.20 is proved by using the figures of these lattices.

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LEFT ABSORBING HYPER K-ALGEBRAS

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ابر K-جبرهای چپ جاذب

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چکیده: در این مقاله، به معرفی یک نوع از ابر K-جبرها که به آن ابر K-جبرهای چپ جاذب گوییم، می پردازیم. سپس برخی از خواص این دسته را بررسی می کنیم. هم چنین نشان می دهیم که انواع ابر K-ایده ال های استلزامی مثبت و جابه جایی یک مشبکه توزیع پذیر را تشکیل می دهند و نمودارهای آن ها را در حالتی که ابر K-جبر چپ جاذب و ابر K-ایده ال های استلزامی مثبت و جابه جایی، یک ابر K-ایده ال باشند، مطالعه می کنیم.

کلمات کلیدی: ابر K-جبر، ابر K-ایده ال، ابر K-ایده ال استلزامی مثبت، ابر K-ایده ال جابه جایی.