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DISTANCE LAPLACIAN SPECTRUM OF THE COMMUTING GRAPHS OF FINITE CA-GROUPS

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ABSTRACT. The commuting graph of a finite group G, $\mathcal{C}(G)$, is a simple graph with vertex set G in which two vertices x and y are adjacent if and only if $xy = yx$. The aim of this paper is to compute the distance Laplacian spectrum and the distance Laplacian energy of the commuting graph of finite CA-groups.

1. Basic Concepts and Notations

We start by definition of some basic concepts and technical terms that are used freely throughout the paper. Let Γ be an undirected graph with vertex set $V(\Gamma) = \{v_1, \ldots, v_n\}$. The *adjacency matrix* $A(\Gamma)$ and the *Laplacian matrix* $L(\Gamma)$ are two important n by n matrices associated to the graph Γ. The adjacency matrix of Γ is defined as $A(\Gamma) = (a_{ij})$, where $a_{ij} = 1$ if and only if v_i and v_j are adjacent in Γ. The *Laplacian matrix* $L(\Gamma)$ is defined as $L(\Gamma) = A(\Gamma) + Deg(\Gamma)$, where, $Deg(\Gamma)$ is the diagonal matrix of Γ in which entries are degrees of vertices in Γ, see [7] for details.

Suppose Γ is a connected graph. The distance $d_\Gamma(v_i, v_j)$, $i \neq j$, is defined as the length of a shortest path connecting v_i and v_j. Note that $d(v_i, v_i) = 0$, where $1 \leq i \leq n$. The distance matrix $D = D(\Gamma)$ is an $n \times n$ matrix such that its (i, j)-th entry is $d_{ij} = d_\Gamma(v_i, v_j)$. The eigenvalues of $D(\Gamma)$ are called the $D-eigenvalues$ of Γ and the multi set of all such quantities together with their multiplicities is called the

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$D-$spectrum of Γ denoted by $DSpec(\Gamma)$. We refer to an interesting review paper of Aouchiche and Hansen [3] for the recent results about $D-$eigebvalues and $D-$spectrum of graphs.

Follow paper [2], the *distance Laplacian matrix* $D^L = D^L(\Gamma)$ of a connected graph Γ is defined as $D^L = Tr - D$, where $Tr(\Gamma)$ is the diagonal matrix whose diagonal entries are the transmissions in Γ and $Tr_\Gamma(v_i) = \sum_{j=1}^{n} d(v_i, v_j)$. Since the matrix D^L is real and symmetric, all of its eigenvalues can be written in the form $\lambda_1^L \geq \lambda_2^L \geq \cdots \geq \lambda_n^L = 0$.

Theorem 1.1. [4] *Let Γ be a connected graph on n vertices with diameter 2 and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n-1} > \lambda_n = 0$ be the Laplacian spectrum of Γ. Then the distance Laplacian spectrum of Γ is $2n - \lambda_{n-1} \geq 2n - \lambda_{n-2} \geq \cdots \geq 2n - \lambda_1 > \lambda_n^L = 0$. Moreover, for every $i \in \{1, 2, \ldots, n\}$ the eigenspaces corresponding to λ_i and to $2n - \lambda_{n-i}$ are the same.*

Aouchiche and Hansen [5] studied the distance Laplacian eigenvalues of connected graphs with a given number of vertices and a fixed chromatic number. They presented some lower bounds on the distance Laplacian spectral radius in terms of n and χ, where χ is used for the chromatic number of the graph under consideration.

Let G be a connected graph of order n. The *distance Laplacian energy* of G, $LED(G)$, is a new graph parameter that was introduced by Gutman et al. [14] as $LE_D(\Gamma) = \sum_{i=1}^{n} |\lambda_i^l - \frac{1}{n}\sum_{i=1}^{n} Tr(v_i)|$. The *commuting graph* of a finite group G, $\mathcal{C}(G)$, is a graph whose vertices are all elements of G and two distinct vertices x and y are adjacent if and only if $xy = yx$. If $\emptyset \neq A \subseteq G$ then the induced subgraph of $\mathcal{C}(G)$ with vertex set A is denoted by $\mathcal{C}(G, A)$. The group G is defined to be an abelian central group, CA-group, if the centralizer of non-central elements of G is abelian.

In the present paper, the characteristic polynomial of distance Laplacian matrix of the commuting graph of a given finite CA-group will be computed. As a consequence, the distance Laplacian energy of this graph is calculated for certain finite CA-groups.

Mirzargar and Ashrafi [11] proved that the commuting graph $\mathcal{C}(G, G \setminus Z(G))$ is a union of complete graphs if and only if G is an CA-group. Authors in [10] computed the automorphism of commuting graph of a finite CA-group.

Suppose Γ_1 and Γ_2 are two simple graphs with disjoint vertex sets. The graph union $\Gamma_1 \cup \Gamma_2$ has $V(\Gamma_1) \cup V(\Gamma_2)$ as vertex set and two vertices x and y are adjacent in $\Gamma_1 \cup \Gamma_2$ if and only if $(\{x, y\} \subseteq V(\Gamma_1)$ and x, y are adjacent in $\Gamma_1)$ or $(\{x, y\} \subseteq V(\Gamma_2)$ and x, y are adjacent in

Γ_2). The join $\Gamma_1 + \Gamma_2$ is a simple graph with vertex set $V(\Gamma_1) \cup V(\Gamma_2)$ in which two vertices x and y are adjacent in $\Gamma_1 + \Gamma_2$ if and only if ($x, y \in V(\Gamma_1)$ and $xy \in E(\Gamma_1)$) or ($x, y \in V(\Gamma_2)$ and $xy \in E(\Gamma_2)$) or ($x \in V(\Gamma_1)$ and $y \in V(\Gamma_1)$).

For the sake of completeness we mention here a result of [6] which is crucial throughout this paper.

Lemma 1.2. [6, Lemma 3.1] *Let G be a CA-group and $\Gamma = \mathcal{C}(G)$. Then $\Gamma = C_{m_0} + (C_{m_1} \cup C_{m_2} \cup \cdots \cup C_{m_s})$, where C_{m_0} is the induced subgraph of Γ by $Z(G)$ and $C_{m_i}, 1 \leq i \leq s$, are components of the graph $\mathcal{C}(G, G \setminus Z(G))$.*

The aim of this paper is to compute the Laplacian spectrum of the commuting graph of $CA-$groups. In certain cases, the energy of such graphs will be computed.

2. Finite CA-Groups

In this section, the characteristic polynomial of distance Laplacian matrix of a finite non-abelian CA-groups will be computed.

Theorem 2.1. *Let G be a CA-group and $C_{m_0} + (C_{m_1} \cup C_{m_2} \cup \cdots \cup C_{m_s})$. The determinant of the matrix $\mathcal{D}^L(\Gamma) - \lambda I_n$ is computed as follows:*

$$\lambda\Big(\lambda - \sum_{i=0}^{s}(m_i)\Big)^{m_0} \prod_{j=1}^{s}\Big(\lambda - 2\sum_{i=0}^{s}(m_i) + m_j + m_0\Big)^{m_j - 1}\Big(\lambda - 2\sum_{i=0}^{s}(m_i) + m_0\Big)^{s-1}.$$

Proof. Suppose u and v are distinct vertices of Γ. Then $d(u, v) = 2$ if and only if u and v are not adjacent in Γ. If $v \in V(C_{m_k})$, for some k such that $1 \leq k \leq s$, then,

$$\begin{aligned}
Tr_\Gamma(v) &= \sum_{u \in V(\Gamma)} d(v, u) \\
&= \sum_{j=0}^{s} \sum_{u \in V(C_{m_j})} d(v, u) \\
&= \sum_{j=1, j \neq k}^{s} \sum_{u \in V(C_{m_j})} d(v, u) + \sum_{u \in V(C_{m_k})} d(v, u) + \sum_{u \in V(C_{m_0})} d(v, u) \\
&= 2 \sum_{j=1, j \neq k}^{s} (m_j) + m_k - 1 + m_0 \\
&= 2|G| - m_k - m_0 - 1.
\end{aligned}$$

Set $O = \mathcal{D}^L(\Gamma) - \lambda I_n$. It is easy to see that, if $v \in V(C_{m_0})$ then $Tr_\Gamma(v) = |G| - 1$. Note that for $0 \leq k \leq s$, C_{m_k} is a complete graph of order m_k. Define $l_k = Tr_\Gamma(v)$, where $v \in V(C_k)$. Therefore,

$$O = \begin{vmatrix} (l_0-\lambda)I_{m_0} - \mathcal{D}(K_{m_0}) & -J_{m_0\times m_1} & \cdots & -J_{m_0\times m_s} \\ -J_{m_1\times m_0} & (l_1-\lambda)I_{m_1} - \mathcal{D}(K_{m_1}) & \cdots & -2J_{m_1\times m_s} \\ \vdots & \vdots & \ddots & \vdots \\ -J_{m_s\times m_0} & -2J_{m_s\times m_1} & \cdots & (l_s-\lambda)I_{m_s} - \mathcal{D}(K_{m_s}) \end{vmatrix}$$

$$= (\lambda - \sum_{i=0}^{s}(m_i))^{m_0} \prod_{j=1}^{s}(\lambda - (l_j+1))^{m_j-1}$$

$$\times \begin{vmatrix} m_0(s+1)-\lambda & -m_1 & -m_2 & \cdots & -m_s \\ -m_0 & l_1 - m_1 + 1 - \lambda & -2m_2 & \cdots & -2m_s \\ -m_0 & -2m_1 & l_2 - m_2 + 1 - \lambda & \cdots & -2m_s \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -m_0 & -2m_1 & -2m_2 & \cdots & l_s - m_s + 1 - \lambda \end{vmatrix}$$

$$= \lambda(\lambda - \sum_{i=0}^{s}(m_i))^{m_0} \prod_{j=1}^{s}(\lambda - (l_j+1))^{m_j-1}(\lambda - 2\sum_{j=1}^{s}(m_j) + m_0)^{s-1}.$$

This completes our argument. □

Theorem 2.1 implies that the largest distance Laplacian eigenvalue is $2|G| - |Z(G)|$. As an example, we assume that $\Gamma = K_2 + (K_1 \cup K_3 \cup K_4)$. A simple program by Gap [13] shows that $Spec_{D^L}(\Gamma) = \{0^1, 10^2, 14^3, 15^2, 18^2\}$.

Corollary 2.2. *If G is a CA-group, then the Laplacian spectrum of $\mathcal{C}(G)$ is computed as follows:*

$$Spec_L(\mathcal{C}(G)) = \{0^1, m_0^{s-1}, (m_s+m_0)^{m_s-1}, \ldots, (m_1+m_0)^{m_1-1}, |G|^{m_0}\}.$$

Proof. Apply Theorem 2.1 to deduce that

$$\begin{aligned} Spec_{D^L}(\mathcal{C}(G)) &= \{0^1, |G|^{m_0}, (2|G| - m_1 - m_0)^{m_1-1}, \cdots, \\ &\quad (2|G| - m_s - m_0)^{m_s-1}, (2|G| - m_0)^{s-1}\}. \end{aligned}$$

Now the proof follows from Theorem 1.1. □

Corollary 2.3. *If $\Delta = K_p + (rK_s \cup K_d)$, then the distance Laplacian characteristic polynomial of Δ is as follows:*

$$\begin{aligned} det(\mathcal{D}^L(\Delta) - \lambda I_p) &= \lambda(\lambda - (rs+d+p))^p(\lambda - (2rs - s + 2d + p))^{r(s-1)} \\ &\times (\lambda - (2rs+d+p))^{d-1}(\lambda - (2rs + 2d + p))^r. \end{aligned}$$

3. Examples

In this section, we apply our results given Section 2 to compute the distance Laplacian energy of the commuting graph of $\mathcal{C}(D_{2n})$, $\mathcal{C}(SD_{8n})$, $\mathcal{C}(T_{4n})$ and two other groups denoted by $U_{n,m}$ and V_{8n}. These groups can be presented as follows:

$$\begin{aligned}
D_{2n} &= \langle a, b \mid a^n = b^2 = e, b^{-1}ab = a^{-1}\rangle, \\
SD_{8n} &= \langle a, b \mid a^{4n} = b^2 = 1, b^{-1}ab = a^{2n-1}\rangle, \\
T_{4n} &= \langle a, b \mid a^{2n} = 1, a^n = b^2, bab^{-1} = a^{-1}\rangle, \\
U_{n,m} &= \langle a, b \mid a^{2n} = b^m = 1, aba^{-1} = b^{-1}ba\rangle, \\
V_{8n} &= \langle a, b \mid a^{2n} = b^4 = 1, ba = a^{-1}b^{-1}, b^{-1}a = a^{-1}b\rangle.
\end{aligned}$$

Example 3.1. For computation of the distance Laplacian energy of the commuting graph $\mathcal{C}(D_{2n})$, we note that,

$$\mathcal{C}(D_{2n}) = \begin{cases} K_2 + (\frac{n}{2}K_2 \cup K_{n-2}) & 2 \mid n \\ K_1 + (nK_1 \cup K_{n-1}) & 2 \nmid n \end{cases}.$$

We now apply Theorem 2.1 to compute the distance Laplacian characteristic polynomial of $\mathcal{C}(D_{2n})$, when $n > 2$. To do this, we note that,

$$det(\mathcal{D}^L(\mathcal{C}(D_{2n})) - \lambda I_n) = \begin{cases} \lambda(\lambda - 2n)(\lambda - 3n)^{n-2}(\lambda - 4n + 1)^n & 2 \nmid n \\ \lambda(\lambda - 2n)^2(\lambda - 4n + 4)^{\frac{n}{2}}(\lambda - 3n)^{n-3}(\lambda - 4n + 2)^{\frac{n}{2}} & 2 \mid n \end{cases},$$

and

$$\sum_{i=1}^{2n} Tr(v_i) = \sum_{i=1}^{2n} \delta_i^l = \begin{cases} 7n^2 - 8n & 2 \mid n \\ 7n^2 - 5n & 2 \nmid n \end{cases}.$$

Thus,

$$LE_D(\Gamma) = \sum_{i=1}^{2n} |\delta_i^l - \frac{1}{2n}\sum_{i=1}^{2n} Tr(v_i)|$$
$$= \begin{cases} |-\frac{7}{2}n+4| + 2|2n-\frac{7}{2}n+4| + (n-\frac{n}{2})|4n-4-\frac{7}{2}n+4| & \\ +(n-3)|3n-\frac{7}{2}n+4| + \frac{n}{2}|4n-2-\frac{7}{2}n+4| & 2 \mid n \\ |\frac{-7n+5}{2}| + |2n-\frac{7n-5}{2}| + (n-1)|\frac{-n+5}{2}| + n|4n-1-\frac{7n-5}{2}| & 2 \nmid n \end{cases}$$
$$= \begin{cases} 13n-24 & 3 \le n < 8, 2 \mid n \\ n^2+2n & n \ge 8, 2 \mid n \\ 10n-10 & 3 \le n \le 5, 2 \nmid n \\ n^2+3n & n > 5, 2 \nmid n \end{cases}.$$

Example 3.2. Consider the semi-dihedral group SD_{8n} of order $8n, n > 3$. By [12, Lemma 2.10], the commuting graph of SD_{8n} can be written as follows:

$$\mathcal{C}(SD_{8n}) = \begin{cases} K_2 + (2nK_2 \cup K_{4n-2}) & 2 \mid n \\ K_4 + (nK_4 \cup K_{4n-4}) & 2 \nmid n \end{cases}.$$

Apply Theorem 2.1 to deduce that

$$det(\mathcal{D}^L(\mathcal{C}(SD_{8n})) - \lambda I_n)$$
$$= \begin{cases} \lambda(\lambda-8n)^2(\lambda-(16n-4))^{2n}(\lambda-12n)^{4n-3}(\lambda-(16n-2))^{2n} & 2 \mid n \\ \lambda(\lambda-8n)^4(\lambda-(16n-8))^{3n}(\lambda-12n)^{4n-5}(\lambda-(16n-4))^{n} & 2 \nmid n \end{cases}.$$

Therefore,

$$\begin{aligned} LE_D(\mathcal{C}(SD_{8n})) &= \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}\sum_{i=1}^{8n} Tr(v_i)| \\ &= \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}\sum_{i=1}^{n} \delta_i^l| \\ &= \begin{cases} \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}(112n^2-32n)| & 2 \mid n \\ \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}(112n^2-56n)| & 2 \nmid n \end{cases} \\ &= \begin{cases} |-14n+2| + 2|-6n+2| + 2n|2n-2| + & \\ (4n-3)|-2n+2| + 2n|2n| & 2 \mid n \\ |-14n+72| + 4|-6n+7| + 3n|2n-1| + & \\ (4n-5)|-2n+7| + n|2n+3| & 2 \nmid n \end{cases} \\ &= \begin{cases} 16n^2+8n & 2 \mid n \\ 16n^2 & 2 \nmid n \end{cases}. \end{aligned}$$

Example 3.3. Consider the dicyclic group T_{4n} of order $4n$, $n \geq 2$. By [12, Lemma 2.7], $\mathcal{C}(T_{4n}) = K_2 + (nK_2 \cup K_{2n-2})$. Therefore,

$$\begin{aligned} det(\mathcal{D}^L(\mathcal{C}(T_{4n})) - \lambda I_n) &= \lambda(\lambda - 4n)^2(\lambda - 8n + 4)^n(\lambda - 6n)^{2n-3}(\lambda - 8n - 2)^n \\ &= \sum_{i=1}^{4n} |\delta_i^l - \frac{1}{4n}\sum_{i=1}^{4n} Tr(v_i)| \\ &= |-2n+4| + 2|-3n+4| + n|n| \\ &+ (2n-3)|-n+4| + n|n+2| \\ &= 4n^2 + 4n. \end{aligned}$$

Example 3.4. Consider the group V_{8n}. By [12, Lemma 2.10], the commuting graph of V_{8n} can be written as follows:

$$\mathcal{C}(V_{8n}) = \begin{cases} K_2 + (2nK_2 \cup K_{4n-2}) & 2 \nmid n \\ K_4 + (nK_4 \cup K_{4n-4}) & 2 \mid n \end{cases}.$$

Apply again Theorem 2.1 to deduce that

$$\begin{aligned} & det(\mathcal{D}^L(\mathcal{C}(V_{8n})) - \lambda I_n) \\ = & \begin{cases} \lambda(\lambda - 8n)^2(\lambda - (16n-4))^{2n}(\lambda - 12n)^{4n-3}(\lambda - (16n-2))^{2n} & 2 \nmid n \\ \lambda(\lambda - 8n)^4(\lambda - (16n-8))^{3n}(\lambda - 12n)^{4n-5}(\lambda - (16n-4))^{n} & 2 \mid n \end{cases}. \end{aligned}$$

Therefore,

$$\begin{aligned} LE_D(\mathcal{C}(V_{8n})) &= \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}\sum_{i=1}^{8n} Tr(v_i)| \\ &= \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}\sum_{i=1}^{n} \delta_i^l| \\ &= \begin{cases} \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}(112n^2 - 32n)| & 2 \nmid n \\ \sum_{i=1}^{8n} |\delta_i^l - \frac{1}{8n}(112n^2 - 56n)| & 2 \mid n \end{cases} \\ &= \begin{cases} |-14n+2| + 2|-6n+2| + 2n|2n-2| + \\ (4n-3)|-2n+2| + 2n|2n| & 2 \nmid n \\ |-14n+72| + 4|-6n+7| + 3n|2n-1| + \\ (4n-5)|-2n+7| + n|2n+3| & 2 \mid n \end{cases} \\ &= \begin{cases} 16n^2 + 8n & 2 \nmid n \\ 16n^2 & 2 \mid n \end{cases}. \end{aligned}$$

Example 3.5. Consider the group $U_{n,m}$. By [12, Theorem 2.3], the commuting graph of $U_{n,m}$ can be written as follows:

$$\mathcal{C}(U_{n,m}) = \begin{cases} K_{2n} + (\frac{m}{2}K_{2n} \cup K_{mn-2n}) & 2 \mid m \\ K_n + (mK_n \cup K_{mn-n}) & 2 \nmid m \end{cases}.$$

We now apply Theorem 2.1 to deduce that

$$det(\mathcal{D}^L(\mathcal{C}(U_{n,m}))-\lambda I_n) = \begin{cases} \lambda(\lambda-2mn)^{2n}(\lambda-(4mn-4n))^{\frac{m}{2}(2n-1)} & 2 \mid m \\ \times(\lambda-3mn)^{mn-2n-1}(\lambda-(4mn-2n))^{\frac{m}{2}} & \\ \lambda(\lambda-2mn)^{n}(\lambda-(4mn-2n))^{mn-m} & 2 \nmid m \\ \times(\lambda-3mn)^{mn-n-1}(\lambda-(4mn-n))^{m} & \end{cases}.$$

Therefore,

$$\begin{aligned} LE_D(\mathcal{C}(U_{n,m})) &= \sum_{i=1}^{2mn}|\delta_i^l - \frac{1}{2mn}\sum_{i=1}^{2mn}Tr(v_i)| \\ &= \sum_{i=1}^{2nm}|\delta_i^l - \frac{1}{2nm}\sum_{i=1}^{n}\delta_i^l| \\ &= \begin{cases} \sum_{i=1}^{2mn}|\delta_i^l - \frac{1}{2mn}(7m^2n^2-6mn^2-2mn)| & 2 \mid m \\ \sum_{i=1}^{2mn}|\delta_i^l - \frac{1}{2mn}(7m^2n^2-3mn^2-2mn)| & 2 \nmid m \end{cases} \\ &= \begin{cases} m^2n^2-2mn^2+\frac{1}{2}mn+3n+1 & m>6, 2 \mid m \\ 6mn^2-12n^2+7mn-10n-2 & 2<m\le 6, 2 \mid m \\ m^2n^2-mn^2+4mn & m\ge 4, 2\nmid m \\ 3mn^2-3n^2+7mn-5n-2 & 2\le m\le 3, 2\nmid m \end{cases}. \end{aligned}$$

We end this section by the following open question:

Question 3.6. Is it possible to find a closed formula for the distance Laplacian energy of the commuting graph of a CA-group?

4. Concluding Remarks

In this paper, the distance Laplacian spectrum of the commuting graph of CA-groups is computed. Our results were checked by computing the distance Laplacian spectrum of the commuting graph of some known CA-groups containing dihedral, semi-dihedral, dicyclic and some meta-cyclic groups.

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گراف جابجایی یک گروه متناهي G که آن را با $\mathcal{C}(G)$ نشان می‌دهیم گرافی ساده با مجموعه رئوس G است که در آن دو راس x و y مجاور هستند اگر و تنها اگر $xy = yx$. هدف این مقاله محاسبه طیف لاپلاسی فاصله و انرژی لاپلاسی فاصله گراف جابجایی CA-گروه‌های متناهی است.

کلمات کلیدی: ماتریس فاصله، گراف جابجایی، طیف لاپلاسی فاصله.