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## Specimens Size Effect on Mechanical and Fracture Properties of Rocks: a Review

Alireza Dolatshahi and Hamed Molladavoodi\*

Department of Mining Engineering, Amirkabir University of Technology, Tehran, Iran

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### Abstract

The structure's response to the region's prevailing loading conditions guides the engineers in estimating the resilience of the structural materials and their reinforcement. One of the main concerns in designing rock structures is paying attention to the size effect phenomenon. The size effect influences the nominal strength, brittleness, load capacity, stress intensity factor, the characteristics of the fracture process zone at the crack tip, and the way and path of crack propagation. Therefore, studying the size effect law will make a guideline for correct decision-making, design, and implementation of efficient support systems. As a comprehensive review, this work investigates specimen size effect on the rock's mechanical and fracture properties. With a comprehensive look at this issue, it explains the essential points that help the engineers design rock structures. During the investigations carried out in this work, it is shown that the specimen size affects the fracture and mechanical properties of the rock. The severity of this phenomenon depends on various factors such as the brittleness index, the shape of the notch or crack length, and the size of the particles that create the rock. In concrete, it depends on the additive boosting materials in the concrete.

## 1. Introduction

Investigating the resistance to growth and the path of crack propagation in rock materials in many issues such as oil and gas extraction industries, storage of carbon dioxide gas (CO<sub>2</sub>), and preventing its wastage and leakage during drilling and maintenance, mining, geothermal production energy, tunnel construction, slope stability, dams, bridges, and well construction; exploitation of underground water is of extraordinary significance [1]. Attention to safety issues is proportional to the importance and sensitivity of the project. The engineers must control and predict crack growth, check the path of crack extension, and take measures against it to achieve an immune structure. For this reason, it is essential to expand awareness of the failure mechanism and its impact from various factors.

A rock and rock-like materials structure can be subjected to specific loading conditions that dictate

the intensity of the prevailing type of loading, and subsequent processes, such as designing an efficient support system, strengthening the support system, and even stopping a project. The type of loading conditions prevailing in each project is one of the most critical factors that neglect it causing many irreparable financial and human losses. For example, we can mention the failure in the inner concrete lining of the tunnel project in China under tensile loading and shear failure in the industrial wastewater transfer tunnel in North America, which, in addition to high costs and wasted time, caused the spread of widespread pollution in an area. Accordingly, the structure's response to the loading conditions is significant. Based on that and other conditions, the engineers can make intelligent plans to stabilize a structure [2, 3].

Furthermore, the structure size effect is another factor that has an enormous role in the stability of

✉ Corresponding author: [davoodi@aut.ac.ir](mailto:davoodi@aut.ac.ir) (H. Molladavoodi)

rock structures and rock-like materials in projects. The role of this factor can be seen in the sensitivity of projects and the requirement to maintain larger structures compared to similar structures but in terms of small dimensions.

Rock strength parameters are usually measured using standards in which laboratory specimens have specific geometry, shape, and size limitations proposed by ISRM (International Society for Rock Mechanics) or ASTM (American Society for Testing and Methods). According to the proposed procedures for determining materials' mechanical and fracture parameters, the material characteristic is assumed to be almost the same for different specimen sizes with or without cracks. The mechanical parameters for the specimen in different dimensions are assumed to be the same. However, various experimental studies by the researchers have shown that this proposition is only sometimes true. Likewise, the specimen's size can significantly affect the mechanical parameters of objects containing or without cracks. Therefore, in addition to determining the values of strength parameters measured by standard laboratory specimens, to predict the resistance of large structures to small structures, it is necessary to investigate the relationship between the resistance parameters and the specimen size [4, 5].

The size effect has permanently been the compass of rock mechanics as a bridge between laboratory tests and engineering sites. For this reason, knowing this phenomenon comprehensively is very effective in designing a stable rock structure. The effect of size can be observed in tunneling, and the effect of tunnel dimensions or the thickness of the tunnel's concrete inner lining is one of the size effect examples, which needs to be paid attention for durability in loading conditions.

The main purpose of this article is to describe the dependence of mechanical properties and rock fracture on specimen size and the influencing factors to accurately show the specimen size effect because paying attention to the size effect on the fracture mechanics of materials is considered as a key parameter in designing. This article reviews the studies conducted on the specimen size effect on the fracture and mechanical properties of brittle and semi-brittle materials such as rocks. The investigation of this phenomenon is comprehensive and coherent. This article contains three main sections, which are the description of the size-dependent fracture parameters for an adequate

understanding of the subject, the investigation of the size effect on the fracture and mechanical properties of the rocks with the most essential and efficient investigation containing achievements of various researchers, and finally, discussion about challenging issues about size effect.

## 2. Size-dependent Fracture Properties

The fracture mechanics science is based on investigating the objects' crack expansion, resistance against crack growth, and the factors affecting the expansion of pre-existing or germinating cracks in objects. This science was previously generally accepted in the failure mechanism of metallic structures, especially in aerospace, marine, and nuclear engineering. It appeared in the field of rocks and concrete structures later. One of the reasons for adopting the fracture mechanics approach for brittle and semi-brittle materials such as rocks and artificial rock structures is the particular focus of this science on the phenomenon of size effect. Accordingly, researchers presented a series of the most common and straightforward fracture criteria in this science and developed them over time. With the progress of science and industry and gaining experience from many projects, it became clear that some of the old criteria of rock mechanics cannot justify some problems based on rock fractures. The justification of the existence of a series of natural intricacies as an inherent characteristic of quasi-brittle materials like rocks and concrete such as joints, cracks, microcracks, and layer boundary discontinuity that are caused by various factors such as physical and chemical weathering, defects in the construction and production of structures, inherent defects of artificial rocks such as concrete and cement mortar, the seismic history of the region and the loading conditions, proved that this factor is one of the essential factors of the structure breaking before reaching the (theoretical) exemplary strength of a structure [6].

The first initial steps in the classical linear elastic fracture mechanics science and paying attention to the problem of cracking and its growth as well as the fracture resistance of materials were proposed by Inglis' article in 1913, the main goal of which is to acquire the stress field at the tip of an ellipse and the inclination of that ellipse to cracking was carried out by reducing one of the main diameters of the ellipse compared to the other diameters (Figure 1) [7].

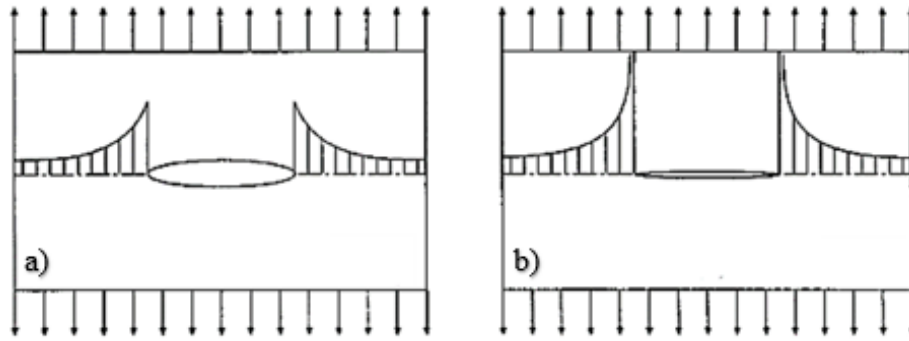


Figure 1. a) Stress field at the tip of the oval and .b) Stress field at the tip of the crack [7].

The inefficiency of linear elastic fracture mechanics for rocks and rock-like materials is that a stress singularity is observed in the crack tip region (Figure 1). The question arises whether there are materials with an infinite strength at the crack tip. The answer to this question must be simple because no existing material has such properties. For this reason, other criteria by Griffith in 1920, based on the energy release rate, the crack tip opening criterion, and another criterion as stress intensity coefficients were replaced by the stress field criterion by Irwin in 1952 to 1954. This causes the front of the crack tip to exhibit inelastic and non-linear behavior for a certain length. The non-linear region at the crack tip can absorb much

energy and cause rapid and stable crack growth. Therefore, it is essential to study many studies in the plastic zone at the front of the crack tip in quasi-brittle materials [8].

**2.1. Fracture process zone**

While studying crack growth in quasi-brittle materials such as rocks and cement-based materials, it was found that a particular region consisting of multiple micro-cracks at the crack tip in rocks has an inelastic behavior. This zone at the crack tip is FPZ (fracture process zone), as shown in Figure 2 [9].

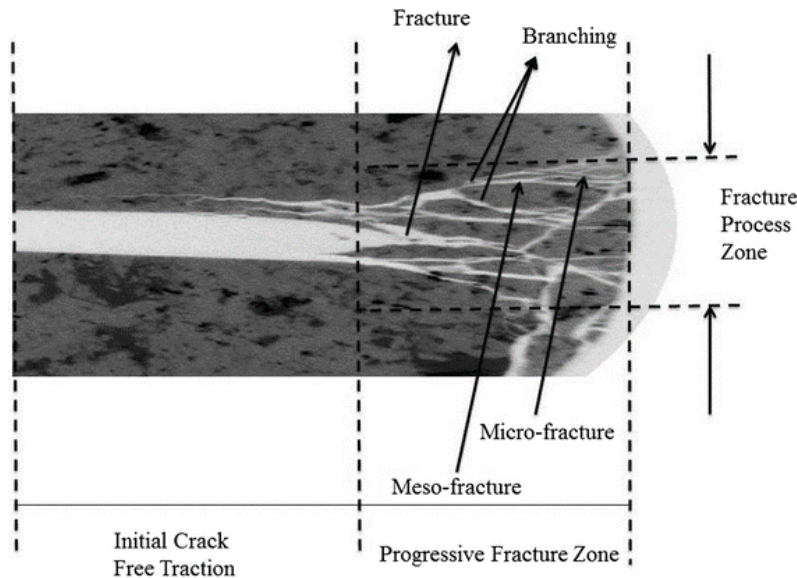


Figure 2. Fracture process zone [9].

By creating micro-cracks at the crack tip, interweaving a part of the crack, and creating larger

cracks, the fracture surfaces become more extensive and therefore cause the final failure of the rock.

In linear elastic fracture mechanics, the plasticity region at the crack tip is ignored to simplify equations and solve problems with idealism. However, this factor makes the use of the linear elastic fracture mechanics approach not valid for rocks and rock-like materials. Studies show that by increasing the dimensions of the specimen, if the FPZ area at the crack tip is minimal compared to the dimensions of the specimen and can be neglected, the solution in the linear method is reasonable for such problems. However, in other cases, it should be noted that the FPZ present at the crack tip affects the results of the tests and should be taken into account [9, 10].

## 2.2. Fracture toughness

The two main criteria of the stress intensity coefficient at the crack tip and the energy release rate are expressed in rock fracture mechanics problems, which are used in linear elastic and non-

linear fracture mechanics. Examining these parameters depends on the loading condition of the rock [8].

A rock or rock-like structure can be subjected to different loading conditions. The structure's response to the various loading conditions is of particular significance and is based on the loading conditions. The types of loading conditions on the rock based on crack propagation are [11]:

- Tensile or opening mode (mode I) in which the crack surfaces are separated from each other in the direction perpendicular to the crack surface.
- Shear or sliding mode (mode II) in which the crack faces are displaced in the crack plane but separated perpendicular to the crack front.
- Tearing mode (mode III) in which the crack faces are displaced parallel to the crack front.
- Most of the time, the type of loading is in the form of a mixed mode which can be analyzed based on the combination of loading modes from the first to the third mode (Figure 3).

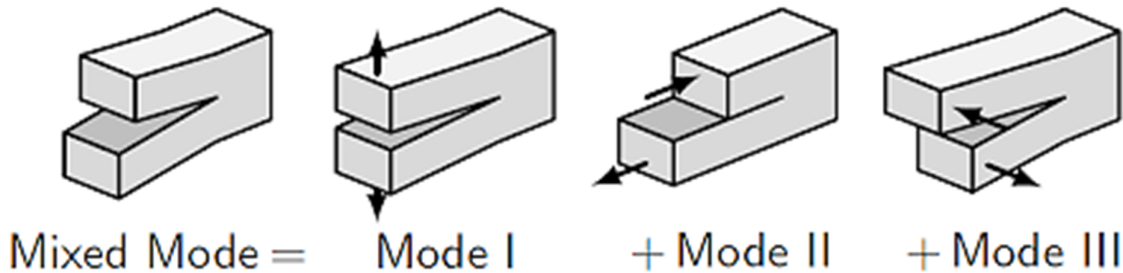


Figure 3. Different modes of crack propagations [12].

Based on the loading conditions on the semi-brittle materials, the stress intensity coefficients  $K_I$ ,  $K_{II}$ , and  $K_{III}$  (respectively, the stress intensity coefficient in mode I, mode II, and mode III) can be calculated in different ways. However, the critical value of these coefficients is known as rock fracture toughness or rock resistance against crack growth [7, 10].

In addition to the critical stress intensity coefficients of different states, paying attention to the effective stress intensity for the simultaneous occurrence of two loading modes (I +II) is important because the dominant loading on the structures is of a mixed mode. With the aid of different methods, such as analytical, experimental, and numerical methods, the fracture mechanics parameters of a structure can be found. The experimental methods are the most critical and reliable, and numerical methods are the easiest and fastest procedures to achieve rock fracture

toughness without preparing specimens. Analytical methods and the other two methods are usually used for validation [13].

Based on the standards proposed by the ISRM (international society for rock mechanics), the investigation of the resistance characteristics of rocks can be divided into three general categories based on how they are loaded in the laboratory. Tests were divided based on the loading condition including bending, direct tension, and compressive loading and on the specimen geometry. According to each proposed method type, the conditions for making and preparing specimens for laboratory study are different [13].

The most common proposed experimental tests based on the type of loading for Mode I are as follows [14-16]:

- Laboratory procedures based on direct tensile loading: short rod test (SR)

- Laboratory procedures based on compressive loading: cracked straight through Brazilian disc test (CSTBD), cracked chevron notched Brazilian disc test (CCNBD), and flattened Brazilian disc test (FBD).
- Laboratory procedures based on bending loading: semi-circular bending test (SCB), straight edge cracked round bar bending test (SECRBB), and chevron bending test (CB).

The most common proposed experimental tests based on the type of loading for Mode II are as follows [14-16]:

- Laboratory procedures based on compressive loading: cracked straight through Brazilian disc test (CSTBD), cracked chevron notched Brazilian disc test (CCNBD).
- Laboratory procedures based on shear loading: box shear cutting test (BSC), and punch trough shearing (p-TS) test.
- Laboratory procedures based on bending loading: semi-circular bending test (SCB).

The most common proposed experimental tests based on the type of loading for Mode III are as follows [14]:

- Laboratory procedures based on bending loading: edge notched bending disc (ENDB).

In addition to the above categories, the proposed ISRM tests can be divided into two groups of tests containing disk specimens and cylindrical samples based on the geometry specimen; the essential parameters for each different loading condition can be found. It should be noted that finding the fracture mechanics characteristics of rock is more comprehensive than those proposed methods by ISRM. Instead, these suggested series may be upgraded based on the study conditions of any material. For example, there are other methods to determine the rock's fracture toughness in mode I, mode II, and mode III, which include Brazilian disk (BD), modified ring (MR), and diagonal compression test (DC), which were presented and developed based on different researchers for specific specimens of materials [15, 16]. The efficiency of each test to calculate the fracture toughness of rock in various types of loading is given in Table 1.

**Table 1. Efficiency of any laboratory test method for different fracture modes**

Test	Mixed-mode	Mode III	Mode II	Mode I
SR	---	---	---	***
CSTBD	***	---	***	***
DC	---	---	---	***
CCNBD	***	---	***	***
MR	---	---	---	***
BD	---	---	---	***
FBD	---	---	---	***
SNSCB	***	---	***	***
CNSCB	***	---	***	***
ENBD	***	***	---	***

The strength parameters of rocks, such as nominal strength, resistance to crack growth or fracture toughness, and the unique characteristics of the FPZ at the crack tip, such as its length and width and the energy release rate, are considered. The following section discusses the role of the size effect phenomenon on the mechanical and fracture properties of rocks [7, 12].

### 3. Size Effect

The history of paying attention to the size effect states that the size effect was first raised with a question in 1500 AD by Leonard DaVinci about the resistance of a string. This question asked why

the string's resistance decreases as the string's length increases. Multiple theories were raised and proposed around this question after that. Nearly half a century later, Mariotte justified the cause of the size effect with statistical reasoning for the first time in 1688. This argument was based on the fact that a long string is likely to have more defects than a shorter string, which causes the string's resistance to drop compared to a shorter string. later, Griffis, by studying glass fiber, found out that its resistance decreases as the diameter of glass fiber increases in 1983. Based on his studies, Griffis stated that cracks and discontinuities cause a sudden decrease in strength in isotropic materials. The results of Griffith's studies strengthened Mariotte's statistical

reasoning based on the statistical size effect (probability of defects in larger samples than in smaller samples) [4, 17].

The fracture mechanism of quasi-brittle materials such as rocks and rock-like materials showed that crack propagation highly depends on the damaged area at the crack tip. The intensity of stress at the crack tip, the fracture toughness of the rock, and many factors including the direction and manner of crack propagation, can be influenced by the plastic area at the crack tip. It should be noted that the size effect affects the dimensions of the damaged area on the crack, the amount of deformation, nominal strength, the behavior of the cracks, and the structure's load capacity. Therefore, the justification of the size effect should always be investigated during studies based on the fracture mechanics for quasi-brittle materials.

Structures with different dimensions but geometrically similar have different fracture toughness and load capacity in different loading conditions. It is impossible to justify this subject in the strength of materials science. Therefore, the fracture mechanics science of rock is fundamental because one of the most prominent features of fracture mechanics is its linear and non-linear elastic approach to problem-solving and its attention and sensitivity to the size effect factor [17].

By studying various materials, it is determined that their strength decreases with increments in the dimensions of a structure. Various methods such as statistical, experimental, and analytical were implemented to explain and show the size effect. The most essential statistical theory is the Weibull theory. Weibull (1956) proposed his statistical size effect theory and showed that the nominal strength of a specimen, which may have more elements with minor lower critical strength due to its larger dimensions, shows a decreasing trend [8, 17]. The Weibull theory can be explained in the following Equation 1.

$$\bar{\sigma}_{Nu} = \bar{\sigma}_{0Nu} \left( \frac{D_0}{D} \right)^{n_d/m} \quad (1)$$

where:

- 
- $\bar{\sigma}_{Nu}$ : Mean nominal strength.
  - $\bar{\sigma}_{0Nu}$ : Basic and theoretical mean nominal strength.
  - D: Structure size constant.
  - $D_0$ : Basic and theoretical structure size constant.
  - m: Weibull's statistical constant.
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$n_d$ : Constant which depends on structure 1D, 2D, and 3D dimension.

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Weibull's theory was only responsible for glass and other brittle materials that break when the first micro-cracks sprouting. Rocks and cement-based materials such as cement mortar, concrete, and brick are excluded from this rule. Because many factors are not considered in the Weibull's theory for rock and semi-rock materials such as failure to pay attention to the FPZ at the crack tip, the heterogeneous distribution of cracks in the material, or the problem of stable crack growth before the object reaches its breaking strength, caused the use of Weibull's theory to be rejected for semi-brittle materials [17].

Statistical justification is insufficient for the size effect and its impact on the strength parameters of cement-based materials. Additional factors such as the effect of boundary layers, diffusion phenomenon, and heat of hydration have a weak effect in many studies. For this reason, the size effect in fracture mechanics is in another way as Bazant's theory, or experimental such as multi-fractal theory for concrete and granular samples were raised [17].

One of the experimental theories of the size effect on precast concrete samples without cracks is the multi-fractal theory [18], where the relationship between the tensile strength and the dimensions of the specimen is as follows.

$$\sigma_t = f_T \left[ a \cdot \frac{D_{max}}{D} + 1 \right]^{-0.5} \quad (2)$$

where:

- 
- $\sigma_t$ : Size-dependent tensile strength.
  - a: Structure constant
  - D: Structure size constant.
  - $f_T$ : the tensile strength of materials based on ISRM recommendation and not size-dependent parameter.
  - $D_{max}$ : Largest grain size of concrete
- 

To take into account the effect of specimen size on linear elastic and non-linear fracture mechanics, Bazant considered the energy released at the time of failure as a function of the size of the specimen and the size of the FPZ at the crack tip. Based on the dimensional analysis, he expressed the nominal strength of the specimen for similar structures with different sizes as an infinite series according to the following Equation [8, 19].

$$\sigma_{Nu} = Bf'_T \left[ \frac{D}{D_0} + 1 + L_1 \left( \frac{D}{D_0} \right)^{-1} + L_2 \left( \frac{D}{D_0} \right)^{-2} + \dots \right]^{-0.5} \tag{3}$$

where:

- $\sigma_{Nu}$ : Nominal strength.
- D: Structure size constant.
- $D_0$ : Basic and theoretical structure size constant.
- B,  $L_1$ ,  $L_2$ , ...: Structure constant
- $f'_t$ : Tensile strength of materials which is not dependent on specimen size.

Equation 3 can be changed to the following equation for the condition  $\frac{D}{D_0} \geq \frac{1}{20}$ :

$$\sigma_{Nu} = Bf'_T \left[ \frac{D}{D_0} + 1 \right]^{-0.5} = \frac{Bf'_T}{\sqrt{\frac{D}{D_0} + 1}} \tag{4}$$

The mentioned theories are only one of several theories developed on the effect of specimen size on strength properties. Among these theories, Bazant's theory is the most well-known as the foundation of many theories related to the law of size effect [19]. Therefore, a more detailed description of the size effect on each mechanical and rock fracture property has been discussed in the following.

### 3.1. Size effect on nominal strength of rocks

As stated in the Bazant's theory, the nominal strength of a structure is a function of its dimensions and its measured tensile strength, so a structure's strength corresponding to two

dimensions is proportional. They do not have the same nominal strength.

The strength does not depend on the size effect in the strength of material criteria. It considers the resistance strength of an object to be constant regardless of its dimensions in the strength criterion, that is not the case in linear and non-linear elastic fracture mechanics. In fracture mechanics science, the nominal strength of a beam whose dimensions are larger than the other beam has less strength [19].

With the increase in the dimensions of the structure in rocks and rock-like materials due to the shrinking of the FPZ at the crack tip compared to the dimensions of the structure, the use of linear elastic fracture mechanics methods and criteria is responsible. In the conditions of structures with small dimensions, the nominal strength of the structure is very close to the nominal strength obtained from the material strength criterion because the fracture area at the tip is close to the dimensions of the structure. Hence the behavior of the material remains like a non-dependent size effect material. Therefore, the use of the material strength criterion for most lab is suitable (Figure 4) [8, 19].

Correspondingly, in addition to changes in the nominal resistance strength of the sample with changes in size, the ductility of the sample changes. As the dimensions of a sample become smaller, the tendency for more plasticity in the sample increases and vice versa (Figure 5) [21].

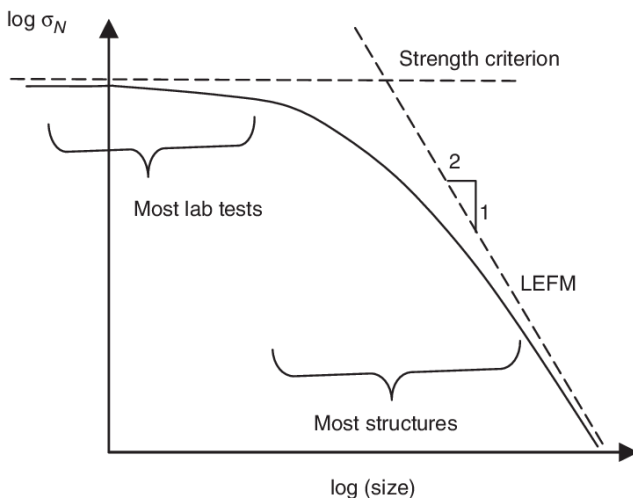


Figure 4. Nominal strength changes with size effect [20].

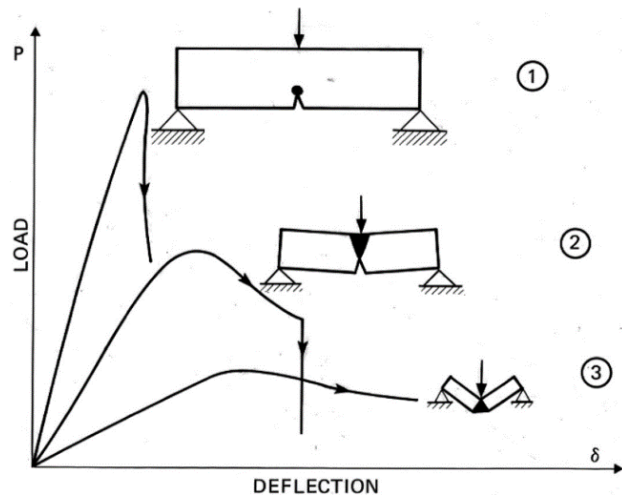


Figure 5. Size effect on load-deflection diagrams [21].

According to the study conducted by different researchers, it is concluded that the studies on the size effect on rock and pseudo-rock materials are divided into two general categories based on 1) rock and rock-like materials without cracks on a macroscopic scale and 2) rock and rock-like materials contain pre-existing cracks. In the first category, studies focus on the influence of the size effect on strength, load capacity, strain, the behavior of cracks about each other, and the process of their changes.

On the other hand, in the second category, studies focus on the effect of the size effect on the nominal strength of the samples and the combination of mechanical properties of rock fracture and semi-brittle materials such as rock fracture toughness, rock fracture energy, and the impact of FPZ at the crack tip which an analysis of more advanced issues such as growth, path, and angle of crack propagation and initiation. In this article, both categories of studies are discussed [22].

Considerable studies have been achieved on the effect of specimen size on mechanical properties, which can oblige engineers to design rock structures in critical and sensitive projects. For example, Nguyen *et al.* [23], investigated the size effect on the flexural stress of the specimen and the average number of multiple cracks and their spacing on the specimen under case loading on concrete reinforced with hybrid fibers. In this study, two soft and flexible hybrid fibers have been used in the concrete mixing design. The results of their study showed that increasing the dimensions of the specimen decreases the flexural strength of the concrete beam. As the size of the specimen increases, the spacing between the cracks and the average number of cracks increases, which is the reason for the increase in toughness and energy release rate during crack propagation in the specimen.

Bahaldini [24], by studying the sandstone of Sydney, in different diameters, investigated the size effect on the indirect (Brazilian) tensile strength of this rock. In this study, specimens were scheduled in diameters 19, 25, 38, 50, 66, 96, 118, and 145 mm, with a diameter to a thickness equal to 0.5. For each diameter, the Brazilian tensile strength test was repeated five times, and finally, their average value was used to interpret the results. His studies indicate that the Brazilian tensile strength reduces with the specimens' diameter increase. Kong *et al.* [25], with a laboratory study on cylindrical samples of sandstone and concrete

with diameters of 20 to 150 mm, also showed that the compressive and tensile strength decreases with the increase of Young's modulus and diameter. Furthermore, during the direct shear strength test on cubic specimens with dimensions of 20 to 150 mm, the shear strength reduced with the specimen size enlargement. Usol'Tseva and Tosi [26], with a laboratory study on a cylindrical specimen with a diameter of 10, 30, and 60 mm and a ratio of diameter to length of 0.5 hornfels, siltstone, gypsum, and sandstone, showed the compressive and tensile strength of sandstone, hornfels, gypsum, and siltstone reduced with increasing specimen diameter. More studies on the size effect on the mechanical properties of the rock are given in Table 2, and their results and achievements are pithily conveyed.

### 3.2. The size effect on FPZ

It is vital to pay attention to the size effect on the characteristics of the FPZ at the crack tip because this zone is the main reason for the dependence on the size of the fracture toughness. Thus a study to obtain the governing equations surrounding the FPZ under specimen size changes is necessary. In this regard, multiple primary governing equations to express the influence of the size effect phenomenon on the FPZ at the crack tip were proposed by Dugdale, 1960 and Barnblatt, 1980, for ductile materials. Hillerborg *et al.* 1976, used the Barnblatt, 1980 model to develop their model for quasi-brittle materials. Bazant, 1980 Schmidt, 1980, and Whitman *et al.*, 1990 proposed other models for the relationships governing size changes on the FPZ [40, 41].

To justify the size effect on FPZ, two beam-shaped structures with different sizes though similar geometry can be assumed, as shown in Figure 5. The specimens were subjected to three-point bending loading. The FPZ dimension increases when the load reaches its maximum. If it is assumed that the FPZ is constant in Figure 5-a and it should be constant in the giant beam as well, in this case, based on the fracture mechanics, which stated that the nominal strength decreases with the increase in the dimensions of the structure, it is no longer valid. In this case, if it is considered that the dimensions (width and length) of FPZ are the same in two similar structures with different dimensions, it means that two different stress regimes have been applied at the crack tip in the larger specimen [42].



**Table 2. More reviews of the studies conducted on the effect of the size effect on mechanical properties of rocks**

Ref.	Researchers (+year)	Mechanical properties	Materials	Shortly discussion
[27]	Zi <i>et al.</i> (2014)	Flexural strength, and Brazilian tensile strength	Concrete	Three different sizes of a circular disc with a diameter of 30, 48, and 75 mm and beams were subjected to bending loading, and the test results show an increase in the amount of action at the moment of failure, a decrease in biaxial tensile strength, biaxial and flexural strength with increasing diameter.
[28]	Lin <i>et al.</i> (2015)	Brazilian tensile strength	---	Numerical modeling in FLAC-3D software of the effect of size on samples with different diameter-to-thickness ratios was investigated. It was found that the value of this parameter is significant, and Brazilian tensile strength is a size-dependent parameter.
[29]	Kaklis <i>et al.</i> (2015)	Brazilian tensile strength, uniaxial compressive strength, young's modulus, and Poisson's ratio	Alfas stone	The Brazilian tensile strength and uniaxial compressive strength of samples with three diameters of 54, 75, and 100 mm was investigated. The results showed that the tensile strength, and uniaxial compressive strength of Alfas stone reduce with the increase in diameter. However, the Poisson's ratio and young's modulus had not changed a lot with different size.
[30]	Al-Rkaby <i>et al.</i> (2015)	Uniaxial compressive strength, and young's modulus	Limestone	The effect of size (ratio L/H of the specimen, where H is the height of the specimen) on uniaxial compressive strength and elastic modulus has been investigated. The results of this study show that when the L/H ratio is equal to 1, the uniaxial compressive strength and Young's modulus reach their lowest levels. If the L/H ratio becomes greater and less than 1, Young's modulus increases.
[31]	Khosravi <i>et al.</i> (2017)	Brazilian tensile strength	Gabbro, Basalt, Micro-Gabbro	Their study investigated the size effect by preparing samples with a length-to-diameter ratio of 0.2 to 1.5 to examine tensile strength. The results exhibited that the surface roughness and tensile strength decreased with the increase in the length ratio to the diameter of micro-gabbro and gabbro specimens. However, no changes were observed in basalt specimens.
[32]	Li (2018)	Brazilian tensile strength	Marl	The Brazilian tensile strength of specimens with three diameters of 40, 60, and 80 mm was restricted, and the results showed that increasing the diameter does not change the Brazilian tensile strength.
[33]	Fládr <i>et al.</i> (2018)	Flexural strength	Fibers reinforced concrete	Their study investigated size effects on 40, 100, 150, and 200 mm cubic specimens under compressive and bending loading conditions. The test results showed that the compressive and flexural strength decreased with the increase in the dimensions of the specimen.
[34]	Yu <i>et al.</i> (2018)	Uniaxial compressive strength	Fine, medium, and coarse grain granite	This study, with the assistance of image processing to investigate the effect of grain size in a small-scale and laboratory study, it was shown that the uniaxial compressive strength of granite reduces with the increase of grain size.
[35]	Zhai <i>et al.</i> (2020)	Tensile strength, and young's modulus	Limestone	The size effect on the compressive strength and Young's modulus Iceland limestone with a constant ratio of length to the diameter of the specimen equivalent to 2 and different diameters from 26 to 285 mm was investigated. The results revealed that in all the specimens, the tensile strength shows an increasing trend and then a decreasing trend with the increase of the sample diameter. In comparison, the changes in Young's modulus do not show any particular trend with respect to the increase in the specimen diameter.
[36]	Han <i>et al.</i> (2021)	Triaxial compressive strength	Granite and Basalt	The size effect on Young's modulus and triaxial compressive strength of granite and basalt stones were investigated. The results revealed a decrease in triaxial compressive strength for increasing the dimensions of the rectangular cube specimen. However, the changes in Young's modulus have been incremental and insignificant.
[37]	Zhao <i>et al.</i> (2022)	Young's modulus, uniaxial, and triaxial compressive strength	Shale	The size effect of the anisotropic shale specimen on both experimental and numerical levels was done with the PFC (particle flow code) software on Young's modulus, uniaxial, and triaxial compressive strength. The results showed that the size effect affects Young's modulus only in the case where the direction of loading on the sample is parallel to the planes of anisotropy.
[38]	Cao <i>et al.</i> (2022)	Flexural strength	Reinforced concrete	Two types of specimens with dimensions of 150x2300x250 mm as a small specimen and 300x4600x500 mm as a large specimen and armed with high strength and normal fibers were used to investigate the effect of size. results showed that the flexural strength decreased with the increase in the dimensions of the specimen.
[26]	Usol'Tseva <i>et al.</i> (2022)	Tensile strength, uniaxial compressive strength, young's modulus, and decreased modulus	Copper ore (rich copper, ordinary copper, and porphyry copper), and Siltstone	Tensile strength and uniaxial compressive strength, changes of two parameters, Young's modulus and decreased modulus of rich copper ore, ordinary copper ore, porphyry copper ore, and siltstone, were investigated. The results indicated that with the increase in the diameter of the specimens, the modulus of the descent of all the specimens decreased, and the tensile and uniaxial compressive strength of specimens increased.
[39]	Asadi and Fakhimi (2023)	Dynamic and static uniaxial compressive strength	Granular sandstone	The results indicated that with the increase in the diameter of the specimens, dynamic, and static uniaxial compressive strength of specimens decreased.

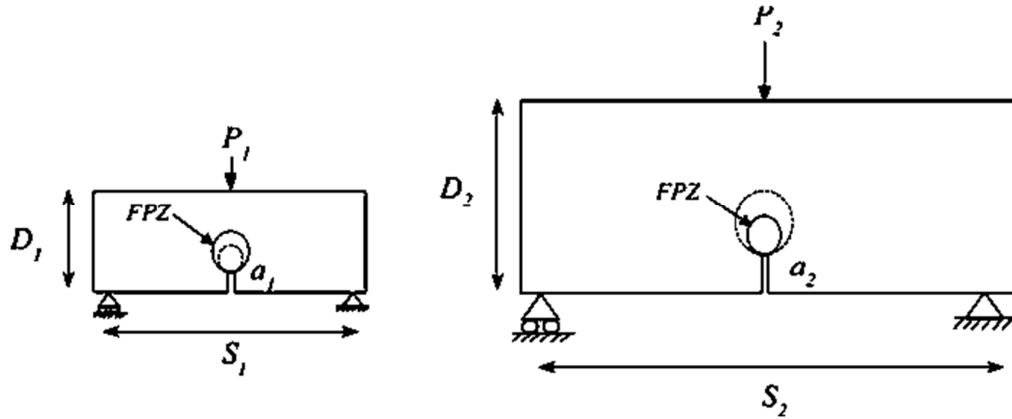


Figure 5. FPZ in two similar beams with different size [42]

Experimental studies show that three basic modes occur, assuming that the FPZ region at the crack tip has a width of ( $w$ ) and a length of ( $l$ ) with size ( $D$ ) changes. Some studies show that in the rectangle rock with  $D_w$  and  $D_l$ , representing the width and length of rectangle form of rock specimen, three scenarios occur. In the first

scenario, If  $D < D_w$ ,  $l_w$  increases for the FPZ region. In the second scenario, If  $D_w < D < D_l$ , in this case,  $w$  of the FPZ area remains constant, but the changes of  $l$  of the FPZ area can occur incrementally. In the third scenario,  $D > D_l$ , in which case the number of changes in the FPZ area remains constant, Figure 6 [42].

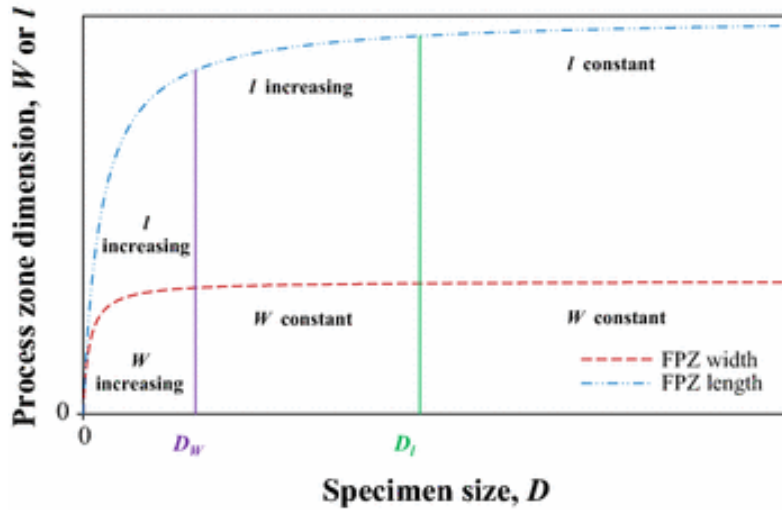


Figure 6. Size effect on FPZ dimension [42]

### 3.3. Size effect on fracture toughness

Determining the fracture toughness parameter of rock is essential while preparing samples, for it is time-consuming and difficult. This parameter is affected by laboratory equipment, sample shape, temperature, and lateral pressure. The sample size is one of the most important factors influencing the fracture toughness of rock. As stated earlier, the damaged area at the crack tip changes due to the size effect phenomenon. Its changes cause variations in the direction, manner, and stress intensity at the crack tip. Therefore, it is necessary to study its effect in the laboratory to guide the

design of large structures in civil engineering. Studies on the size effect phenomenon on rock fracture toughness are critical. The first steps of the study on the fracture toughness of materials including rocks and metals, emphasized the principle that the fracture toughness of rocks depends on the type of material. Subsequently, it was shown by Bazant with the creation of the size effect law that specimen size is effective on the strength and fracture toughness of rock. The changes in the surface of nominal strength and fracture toughness of rocks relative to specimen size are shown in Figure 7.

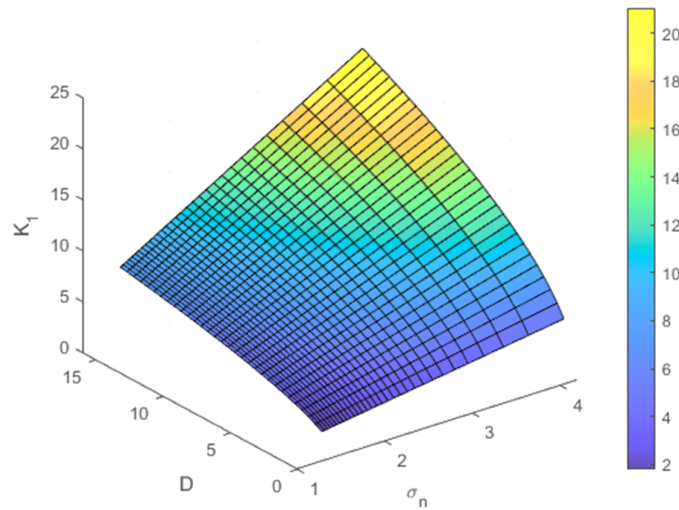


Figure 7. Size effect of nominal strength and fracture toughness mode I.

Based on linear elastic fracture mechanics, when the applied load on the specimen reaches its maximum value, the stress intensity coefficient of the specimen reaches its critical level. Afterward, the crack spreads in the sample. The value of rock fracture toughness can be found approximately based on the proposed linear elastic fracture mechanics equations. However, it should be noted that due to a plastic zone at the crack's tip, the toughness obtained by the tests differs from the rock's inherent toughness [43].

An expansive range of studies of the size effect on specimens containing pre-existed cracks, and the main emphasis of the studies are to investigate the modifications in rock fracture toughness, energy release rate, and characteristics of the FPZ at the crack tip in different modes. For example, Zhang *et al.* study was done on limestone with SCB disk radius values of 25 mm, 37.5 mm, 50 mm, and 75 mm, and the crack length ratio to the disk sample's radius was 0.4, 0.5, and 0.6. The results of Zhang *et al.* research show that by increasing the radius of the semi-circular specimen increases the tensile mode fracture toughness of limestone. Also, with the fixed specimen size, the increase in the crack length ratio to the specimen's radius causes a reduction in fracture toughness mode I [44].

Ayatollahi and Akbaridoust [45] prepared the SCB samples of marble with the radius of 25, 50, 95, and 190 mm, and constant thickness of 26 mm, and the ratio of crack length to the constant radius of 0.5 and an angle of the crack relative to the direction of load application on the specimen under two zero angles (for Mode-I) and 41 degrees (for Mode-II). The load at the moment of failure, the

length of the FPZ area, the fracture toughness Mode I and Mode II of marble increases experimentally and analytically with the aid of the developed maximum tangential stress (MTS) criterion, numerically with the help of Abaqus software with the increase of the specimen dimensions.

The rock fracture mechanism under Mode III conditions is less common than in the previous Mode I or II, and its occurrence is less frequent. However, the fracture mechanism of multiple structures at splendid depths is Mode-III. For example, Bidadi *et al.* [46] conducted a laboratory study on changes in the thickness of rock structures, which is an essential issue in the design of rock structures. Therefore, in their study, several Mode-I and Mode-III fracture toughness tests have been performed on a Brazilian sample containing an edge crack under ENDB three-point bending loading made of a type of marble to investigate the effect of sample thickness on the respective values of  $K_{Ic}$  and  $K_{IIIc}$ . In this study, the diameter of the discs was fixed, and only their thickness was changed to 15, 25, 50, and 80 mm. The results obtained from the experimental evaluation of Bidadi *et al.* showed that Mode-I and Mode-III toughness of marble increases with the increase in thickness. As the thickness increases, the amount of applied load for sample failure increases. However, always in all samples, the increase of Mode-I compared to Mode-III is more.

Aliha *et al.* [47] by examining two laboratory methods for calculating the toughness of the Brazilian disc sample method containing a significant crack and the semi-Brazilian sample

method containing an edge crack under three-point bending on a limestone sample to investigate the effect of specimen radius size equal to 25, 50, 75, and 150 mm under the mixed-mode loading condition (I+II) showed that the crack propagation and the crack path move from the tip of the crack towards the point of the maximum applied load. In

specimens with a larger radius, the crack propagation path deviates less from the direction of the applied load axis on the specimen. More studies on the size effect on the fracture properties of the rock are given in Table 3, and their results and achievements are pithily conveyed.

**Table 3. More reviews of the studies conducted on the size effect on fracture properties of rocks**

Ref.	Researchers (+year)	Fracture parameter	Mode(s)	Type of specimen	Material	Results
[48]	Khoramishad <i>et al.</i> (2014)	Fracture toughness, and the FPZ's length	I	CB	Limestone	Increasing with the increase in the dimensions of the specimen.
[49]	Ayatollahi <i>et al.</i> (2014)	Fracture toughness, and the FPZ's length	I	CSTBD	Nayriz Marble	Increasing with the increase in the dimensions of the specimen.
[50]	Aliha <i>et al.</i> (2015)	Fracture toughness	I, and III	SCB	Asphalt concrete	Increasing with the increase in the dimensions of the specimen.
[51]	Akbardoost (2016)	Dynamic fracture toughness	I	MR	Marble	Increasing with the increase in the dimensions of the specimen.
[52]	Ayatollahi <i>et al.</i> (2016)	Fracture toughness, and the FPZ's length	Mixed-mode (I+II)	SCB	Graphite	Increasing with the increase in the dimensions of the specimen.
[53]	Akbardoost and Rastin (2016)	Fracture toughness	I, II, and mixed-mode (I+II)	CSTBD	Concrete	Increasing with the increase in the dimensions of the specimen.
[54]	Jeong <i>et al.</i> (2017)	Fracture toughness	I	SCB	Granite	Increasing with the increase in the dimensions of the specimen.
[55]	Ghasemi-Ghalebahman <i>et al.</i> (2017)	Fracture toughness	Mixed-mode (I+II)	SCB	Nano-composite	Increasing with the increase in the dimensions of the specimen.
[56]	Moazzami <i>et al.</i> (2020)	Fracture toughness, and the FPZ's length	I, II, and mixed-mode (I+II)	CB, and SCB	Marble, and sandstone	Increasing with the increase in the dimensions of the specimen.
[57]	Ayatollahi <i>et al.</i> (2020)	Fracture toughness	II	CB	Semi-brittle epoksi	Increasing with the increase in the dimensions of the specimen.
[58]	Zhang <i>et al.</i> (2021)	Fracture energy release rate, and fracture toughness	I	CSTBD	Sandstone	Increasing with the increase in the dimensions of the specimen.
[59]	Muñoz-Ibáñez <i>et al.</i> (2021)	Fracture toughness	I	SCB	Sandstone, and granite	Increasing with the increase in the dimensions of the specimen.
[60]	Nejati <i>et al.</i> (2021)	Fracture toughness	II	SCB	Anisotropic granite	Increasing with the increase in the dimensions of the specimen.
[61]	Zhang <i>et al.</i> (2022)	Fracture toughness	I	CB	Reinforced concrete	Increasing with the increase in the dimensions of the specimen.
[62]	Wang <i>et al.</i> (2022)	Fracture toughness	I	CSTBD	Reinforced concrete	Increasing with the increase in the dimensions of the specimen.
[63]	Xie <i>et al.</i> (2022)	Fracture toughness, and the FPZ's length	I	CSTBD	Shale	Increasing with the increase in the dimensions of the specimen.
[64]	Pijaudier-Cabot <i>et al.</i> (2022)	Fracture energy release rate	I	CB	Shale, and limestone	Increasing with the increase in the dimensions of the specimen.
[65]	Li <i>et al.</i> (2023)	Fracture toughness, and the FPZ's length	I	CB	Anisotropic Shale	Increasing with the increase in the dimensions of the specimen.
[66]	Davis <i>et al.</i> (2023)	Fracture toughness, and the FPZ's length	I	CB	Quasi-brittle sandstone	Increasing with the increase in the dimensions of the specimen.
[67]	Karimi <i>et al.</i> (2023)	Fracture toughness	I, and II	ENDB	Concrete	Increasing with the increase in the dimensions of the specimen.
[68]	Daneshfar <i>et al.</i> (2023)	Fracture energy release rate, and Fracture toughness	I	CB	Reinforced concrete	Increasing with the increase in the dimensions of the specimen.

### 3.4. MTS criterion and size effect phenomena

The linear elastic approach of fracture mechanics needs to be revised to justify the changes in fracture toughness and FPZ length compared to the changes in specimen size. On the other hand, the distance

parameter from each mentioned point to the crack tip is effective. Many criteria were proposed based on the Williams (in 1957) expansion, and one of the most important criteria is the MTS (maximum tensile stress) criterion. In tensile loading conditions, considering the Williams expansion

(Equation 5), it is evident that with the decrease of  $r$ , all terms of the expansion except the first term can be neglected. It represents the increase of tangential stress at the crack tip. Therefore, this

criterion is called the maximum tangential stress criterion. The changes in fracture toughness depend on the type of loading and geometry [66].

$$\sigma_{\theta\theta} = \sum_{n=1}^{\infty} \frac{n}{2} A_n r^{\frac{n}{2}-1} \left[ \left(\frac{n}{2} + 1\right) \cos\left(\frac{n}{2} - 1\right)\theta - \left(\frac{n}{2} + (-1)^n\right) \cos\left(\frac{n}{2} + 1\right)\theta \right] \quad (5)$$

where:

- $\sigma_{\theta\theta}$ : Tangential stress
- $A_n$ : William's constant (fracture toughness)
- $r, \theta$ : Polar coordinates of a point from the crack tip

For tensile loading conditions, it is necessary to set  $\theta$  equal to zero, then the William's (1957) expansion becomes as shown in Equation 6;

$$\sigma_{\theta\theta|\theta=0} = \frac{A_1}{\sqrt{r}} + 3A_3\sqrt{r} + 5A_5r^{\frac{3}{2}} + \dots \quad (6)$$

The criterion of maximum tangential stress states that only the first term of Equation 6 should be equal to the inherent tensile strength of the rock when failure occurs in the material sample. Therefore, the criterion of maximum tangential stress will be as (Equation 7):

$$f_t = \frac{A_1}{\sqrt{r}} \quad (7)$$

$A_1$  parameter depends on the stress intensity coefficient in the crack opening mode and this parameter can be calculated as below (Equation 8):

$$A_1 = \frac{K_{Ic}}{\sqrt{2\pi}} \quad (8)$$

To estimate the maximum tangential stress, the critical value of  $K_{Ic}$  (the fracture toughness) is used and the value of  $r$  can be approximated based on the critical value of the distance representing the length of the FPZ region. If the Equation 8 is substituted in Equation 7, the maximum tangential stress criterion will be (Equation 9):

$$f_t = \frac{K_{Ic}}{\sqrt{2\pi}} r_c^{-0.5} \quad (9)$$

where:

$f_t$ : Tensile strength of materials based on ISRM recommendation and not size-dependent parameter.

MTS criterion can be also applied for various other situations such as mode II or III. However, it

shows that the fracture toughness of rock increases with the increase of FPZ radius. The essential issue in the MTS criterion is that it assumes constant tensile strength against the increase in the dimensions of the specimen. Based on this, it shows that with the increase in the dimensions of specimens, the length of the FPZ area and toughness increases, while the Bazant theory states that with the increase in the dimensions of the specimen, the tensile strength is also decreased.

### 3.5. MMTS criterion and size effect phenomena

A new approach called the modified MTS criterion was developed by Ayatollahi and Akbaridoost [66]. This developed approach is the maximum tensile stress criterion based on the expansion of Williams et al.1957, which was for brittle materials such as glass and semi-brittle materials. For each study, there is a difference in the number of parameters involved in the provided criterion relationship according to the loading conditions and the structure's geometry. The difference between MMTS and the MTS criterion is that it is more effective for semi-brittle materials such as rock and concrete. At the tip of the crack, the FPZ region contains micro-cracks that cause the softening of that region and justify the ineffectiveness of linear elastic fracture mechanics (LEFM).

The MMTS criterion states that the first two terms of the Williams expansion must be equal to the tensile strength of the rock in order for the fracture to occur in the material specimen (Equation 10);

$$f_t = \frac{A_1}{\sqrt{r}} + 3A_3\sqrt{r} \quad (10)$$

The MMTS criterion in the critical state, in which fracture and crack propagation is obvious, is as equation 11;

$$f_t = \frac{A_{1c}}{\sqrt{r_c}} + 3A_{3c}\sqrt{r_c} \quad (11)$$

$A_{1C}$ , and  $A_{3C}$  base on nominal strength are as equations 12, 13:

$$A_{1C} = \frac{K_{IC}}{\sqrt{2\pi}} = \sigma_N \cdot \sqrt{W} \cdot A_1^* \quad (12)$$

$$A_{3C} = \frac{\sigma_N}{\sqrt{W}} \cdot A_3^* \quad (13)$$

where:

w: Structure's dimension

$A_1^*$ : Crack length rate-dependent parameter to w

$A_3^*$ : Parameter dependent on crack length rate and loading rate

If Equations 12 to 13 are inserted in Equation 11 the MMTS criterion will be as Equation 14:

$$f_t = \frac{K_{IC}}{\sqrt{2\pi r_c}} \left(1 + 3 \frac{A_3^* r_c}{A_1^* w}\right) \quad (14)$$

$K_{IC}$  based on equation 14 and nominal strength ( $\sigma_N$ ) based on Equations 14 and 12 are as below:

$$K_{IC} = f_t \sqrt{2\pi r_c} \left(1 + 3 \frac{A_3^* r_c}{A_1^* w}\right)^{-1} \quad (15)$$

$$\sigma_N = \frac{f_t}{A_1^* \sqrt{\frac{W}{r_c}} \left(1 + 3 \frac{A_3^* r_c}{A_1^* w}\right)} \quad (16)$$

The accuracy of this result was shown by an experimental study on rock fracture toughness under tensile loading on rectangular cube specimens of limestone and concrete containing cracks under different three-point bending tests, by Ayatollahi and Akbari [66]. As the dimensions

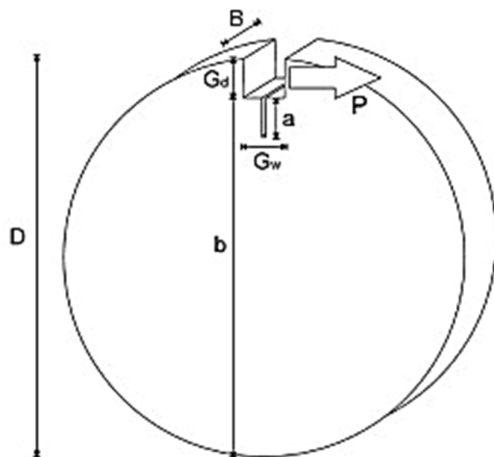


Figure 9. Schematic representation of pCT specimen [57].

of limestone and concrete specimens increase, the fracture toughness of rock in the tensile loading mode and the length of the fracture process zone at the crack tip increases, and the nominal strength decreases.

#### 4. Extra Key Factors in Size Effect

With the aid of various studies on the size effect on the mechanical fracture properties of rocks, it was determined that several factors affect the intensity of the specimen size effect, which are:

- The size effect in both microscopic and macroscopic scales cause a change in the nominal strength. In semi-brittle and cement-based materials, such as cement mortar and concrete, the change in grain size causes changes in the nominal strength of the specimen. By reducing the grain size of the materials used, the nominal resistance increases because of filling all the pores with minuscule particles and reducing the effect of the boundary layer. The likelihood of a crack unfurling between the grains is more heightened in the specimen with coarse grains than in fine grains size or impact rocks (Figure 8) [67-69].
- The intensity of the specimen size's effect on the mechanical properties of rocks depends on the number of metallic minerals (ore) in the rock. As metallic minerals increase, their strength increases with the specimen size [33, 70-72].
- In some laboratory methods, increasing the dimensions of the specimen does not affect the fracture toughness. For example, side effects are not palpable in pCT (a method for finding fracture toughness mode I in direct tensile loading) specimens (Figure 9) compared to the SCB method [59].

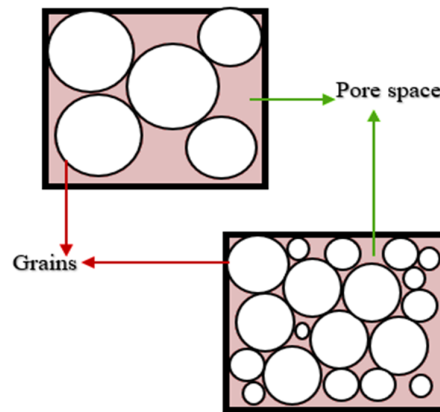


Figure 8. Grain size effect on pore space

- Torabi *et al.* [73], in a numerical and experimental study on the size effect on graphite fracture toughness, investigated the increase in the radius of SCB specimens and the geometry of cracks and notches. In this study, graphite SCB specimens with different crack and notch shapes, like Figure 9, with two angles of 45 and 90 degrees at radii of 5, 10, 20, and 35 mm under loading conditions Mode I were found. In addition to the experimental test, the calculation of toughness based on the criterion of maximum tangential stress at the crack tip was also investigated. Test showed that graphite's fracture toughness increased with the specimen radius increase in all specimens with all types of notches. However, FPZ's length only in crack and U-notch increased with dimensions increasing [73-75].

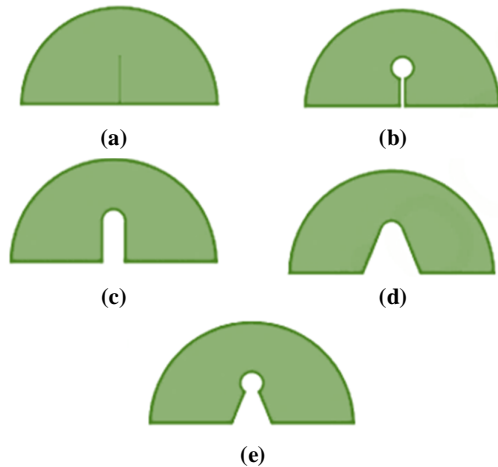


Figure 10. a) Crack, b) U-notch, c) key-hole notch, d) RV notch, and e) RO notch

Based on the review of various studies aforementioned, it was determined that the additives in concrete materials and the mineralogical characteristics of the rocks have an essential effect on the severity or mildness of the specimen size effect. Moreover, reinforcing the concrete causes the specimen size effect to decrease. Golewski and Sadowski [76], showed that concrete containing 20% fly ash increases the fracture toughness of concrete compared to normal concrete against tensile loading.

Furthermore, Golewski and Sadowski [77, 78], in another study showed that the resistance against

crack growth of concrete under mode I, mode II, and mode III loading increases with the addition of 20% fly ash. If this percentage increases, the resistance parameters of concrete decrease. This study shows that it is always necessary to determine the optimal percentage of materials in the design of concrete for better performance in various conditions. This work can play an essential role in the internal coating of concrete.

Modifying the binder composition with three pozzolanic active materials increased the analyzed mechanical parameters for each of the combinations compared to the results obtained for the control concrete. In addition, as the content of fly ash rises throughout each quaternary concrete series, the material becomes more ductile and shows less brittle failure. Therefore, with high fracture toughness and lower brittleness, this boosted concrete can be used in reinforced concrete structures subjected to dynamic or cyclic loads [79].

### 5. Discussion

In order to verify the results obtained from the comprehensive study of the size effect on mechanical properties and rock fracture, simple two-dimensional modeling has been done in ABAQUS software, the characteristics of this model are based on Table 4. In this modeling, two rectangular concrete plates containing a central crack with dimensions of 3 m x 6 m, and 2 m (crack length) and 6 m x 12 m, and 4 m (crack length) are subjected to the same tensile loading with the same loading speed of 0.5 MPa/s and this loading is up to the maximum amount of ten times the tensile strength of concrete, which is almost equal to the compressive strength of concrete, continues. The constitutive model of concrete has been investigated only from the elastic aspect, although the plastic parameters have an effect. For ease of modeling, the elastic constitutive model has been considered. The modeling results show that the strength and damage parameter in the structure with larger dimensions is less than in the smaller structure. As the damage parameter decreases, the tensile fracture toughness of the rock increases (Figure 11) [80].

Table 4. Concrete plate properties [81].

Young's Modulus	Poison's ratio	Tensile strength	Compressive strength
21 GPa	0.23	4.58 MPa	47.1 MPa

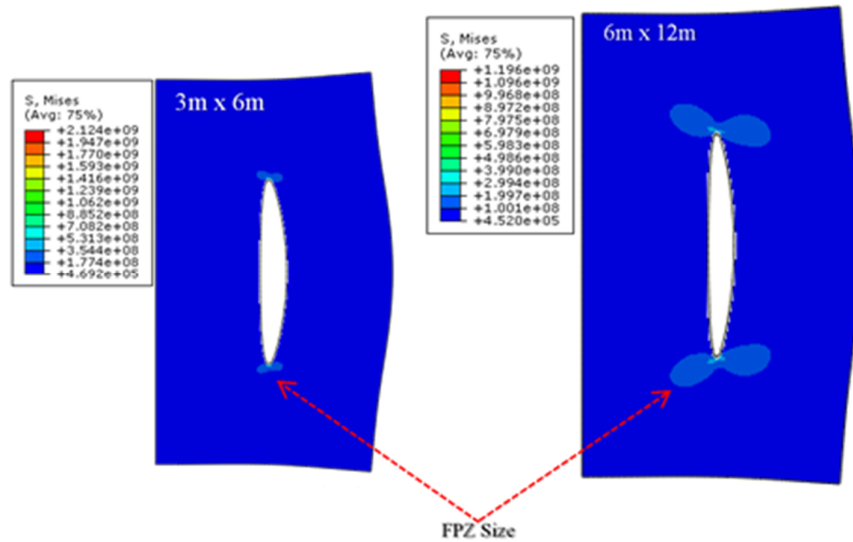


Figure 11. Numerical model results.

Although it was found by investigating various studies that the sample size causes a change in the number of mechanical properties and rock fracture, the main phenomenon on the mechanical properties is mineralogical composition and material type. As long as the materials have metal elements, they can be insensitive to the specimen size. From this point of view, an essential parameter of the material should be proposed to indicate the brittleness or ductility of the material. From this point of view, it is one of the applications of the brittleness index in justifying the size effect phenomenon [82, 83].

Numerous studies have been conducted in the field of examining the strength of samples containing several cracks. Nevertheless, studies have yet to be done on the effect of sample size on the fracture behavior of samples containing multiple cracks. Therefore, it is suggested to conduct more studies in this field [87-88].

## 6. Conclusions

The size effect is among the most influential factors in engineering rock structures. In the intelligent design of a rock structure, depending on the dominant type of loading, resilience and resistance to rock crack growth should be estimated due to their significant importance. The engineers in the design of rock structures should always check the effect of size and make the right decisions based on analytical, experimental, and numerical studies. This article showed that the intensity of the influence of the specimen size effect on the mechanical properties is as much as the size of the particles that make up the sample on

the micro-scale and mineralogical properties. Furthermore, the intensity of the influence of the specimen size effect on the fracture properties of rock depends on various factors such as the geometry of the specimen and the geometry of cracks or notch. With a comprehensive study conducted in this work. The following results can be obtained:

- In semi-brittle materials such as rocks and concrete, the length and width of the FPZ at the crack tip and the determination of stress intensity coefficient, nominal strength, load capacity, and deformation are sensitive to changes in specimen size.
- Increasing the dimensions of the specimen causes a decrease in the compressive and tensile strength. It increases the load at the moment of failure, the distance between the cracks in the beams under bending, and the width of the crack opening. The changes in the modulus of elasticity are variable compared to the changes in the size of the sample, so the changes are small and incremental in many rocks.
- In the loading conditions of Mode I, Mode II, Mode III, and Mixed-Mode, with the increase of specimen dimensions, the fracture toughness and the length of the FPZ zone increase in most quasi-brittle materials such as rocks and concrete. This is while in fragile materials such as glass, increasing the dimensions of the specimen does not cause any sensitivity in the change of fracture toughness.
- Under different loading conditions, other factors such as loading rate, mineralogical characteristics, type of geometry of the manufactured sample, and anisotropy have an



effect on the fracture toughness and length of the FPZ region, and the intensity of the size effect phenomenon is different.

- In concrete specimens adding pozzolanic materials can reduce effects of specimens and boosting mechanical parameters of concrete.

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## اثر اندازه نمونه بر خواص مکانیکی و شکست سنگ‌ها: یک مطالعه مروری

علیرضا دولت‌شاهی و حامد ملاداودی\*

دانشکده مهندسی معدن، دانشگاه صنعتی امیرکبیر، تهران، ایران

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\* نویسنده مسئول مکاتبات: davoodi@aut.ac.ir

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### چکیده:

پاسخ سازه به شرایط بارگذاری غالب منطقه، مهندسین را در برآورد انعطاف‌پذیری و شکنندگی مصالح سازه‌ای و تقویت آن‌ها راهنمایی می‌کند. یکی از دغدغه‌های اصلی در طراحی سازه‌های سنگی توجه به پدیده اثر اندازه است. پدیده اثر اندازه بر مقاومت اسمی، شکل‌پذیری، ظرفیت بار، ضریب شدت تنش، ویژگی‌های ناحیه شکست در نوک ترک، و نحوه و مسیر انتشار ترک تأثیرگذار است. بنابراین، مطالعه قانون اثر اندازه، راهنمای تصمیم‌گیری، طراحی و اجرای صحیح سیستم‌های نگهداری کارآمد در حین اجرا و پس از آن برای سازه خواهد بود. به عنوان یک بررسی جامع، این کار اثر اندازه نمونه بر خواص مکانیکی و شکست سنگ را بررسی کرده است. با نگاهی جامع به این موضوع، نکات ضروری را توضیح می‌دهد که به مهندسین در طراحی سازه‌های سنگی کمک می‌کند. در خلال بررسی‌های انجام شده در این کار، نشان داده شده است که اندازه نمونه بر خواص مکانیکی شکست سنگ تأثیر می‌گذارد. شدت این پدیده به عوامل مختلفی از جمله شاخص شکنندگی سنگ، شکل شکاف یا طول ترک و اندازه ذرات سازنده سنگ بستگی دارد. همچنین در بتن، به مواد تقویت کننده افزودنی در بتن بستگی دارد.

**کلمات کلیدی:** اثر اندازه، خواص مکانیکی، ناحیه شکست سنگ، چقرمگی شکست.

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