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$n-$ABSORBING $I-$PRIME HYPERIDEALS IN MULTIPLICATIVE HYPERRINGS

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Abstract. In this paper, we define the concept $I-$prime hyperideal in a multiplicative hyperring R. A proper hyperideal P of R is an $I-$prime hyperideal if for $a, b \in R$ with $ab \subseteq P - IP$ implies $a \in P$ or $b \in P$. We provide some characterizations of $I-$prime hyperideals. Also we conceptualize and study the notions $2-$absorbing $I-$prime and $n-$absorbing $I-$prime hyperideals into multiplicative hyperrings as generalizations of prime ideals. A proper hyperideal P of a hyperring R is an $n-$absorbing $I-$prime hyperideal if for $x_1, \cdots, x_{n+1} \in R$ such that $x_1 \cdots x_{n+1} \subseteq P - IP$, then $x_1 \cdots x_{i-1}x_{i+1} \cdots x_{n+1} \subseteq P$ for some $i \in \{1, \cdots, n+1\}$. We study some properties of such generalizations. We prove that if P is an $I-$prime hyperideal of a hyperring R, then each of $\frac{P}{J}$, $S^{-1}P$, $f(P)$, $f^{-1}(P)$, $\sqrt{P}$ and $P[x]$ are $I-$prime hyperideals under suitable conditions and suitable hyperideal I, where J is a hyperideal contains in P. Also, we characterize $I-$prime hyperideals in the decomposite hyperrings. Moreover, we show that the hyperring with finite number of maximal hyperideals in which every proper hyperideal is $n-$absorbing $I-$prime is a finite product of fields.

1. Introduction

Many concepts in modern algebra was generalized by generalizing their structures to hyperstructure. The French mathematician F. Marty in 1934 introduced the concept hyperstructure or multioperation by returning a set of values instead of a single value [11]. The

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hyperstructures theory was studied from many points of view and applied to several areas of mathematics especially in computer science and logic. In [11] the author presented the concept hypergroup and after that in 1937, the authors H. S. Wall [16] and M. Kranser [10] also gave their respective definitions of hypergroup as a generalization of groups.

The hyperrings were introduced by many authors. A type of hyperring where the multiplication is a hyperoperation while the addition is just an operation introduced by Rota in 1982 and called a multiplicative hyperring [13]. A well known example on multiplicative hyperring is that for a ring $(R, +, \cdot)$ and corresponding to every non-singleton subset $A \in P^*(R) = P(R)\backslash\{\phi\}$ where $P(R)$ is the power set of R, there exists a multiplicative hyperring with absorbing zero $(R_A, +, \circ)$ where $R_A = R$ and for any $x, y \in R_A$,

$$x \circ y = \{x \cdot a \cdot y : a \in A\}$$

(see [12, 15]). Another type of hyperring in which addition is a hyperoperation while the multiplication is an operation introduced by M. Krasner in 1983 and called Krasner hyperring [10]. The hyperrings in which the additions and multiplications are hyperoperations where introduced by De Salvo [8]. Procesi and Rota in [12] have conceptualized the notion of primeness of hyperideal in a multiplicative hyperring. A proper hyperideal P is called prime hyperideal if $ab \subseteq P$, then $a \in P$ or $b \in P$. The radical of a hyperideal P denoted by $\sqrt{P}$ is the intersection of all prime hyperideals that contains P. Some generalizations of prime hyperideals can be found in [3, 7, 14] .

In the recent years many generalizations of prime ideals were introduced. Here state some of them. The authors in [4] and [5] introduced the notions $2-$absorbing and $n-$absorbing ideals in commutative rings. A proper ideal P is called $2-$absorbing (or $n-$absorbing) ideal if whenever the product of three (or $n+1$) elements of R in P, the product of two (or n) of these elements is in P.

In [1] and [2], the author Akray introduced the notions $I-$prime ideal and $n-$absorbing $I-$ideal in classical rings as a generalization of prime ideals. For fixed proper ideal I of a commutative ring R with identity, a proper ideal P of R is an $I-$prime if for $a, b \in R$ with $a.b \in P - IP$, then $a \in P$ or $b \in P$. A proper ideal P of R is an $n-$absorbing $I-$ideal if for $x_1, \cdots, x_{n+1} \in R$ such that $x_1 \cdots x_{n+1} \in P - IP$, then $x_1 \cdots x_{i-1}x_{i+1} \cdots x_{n+1} \in P$ for some $i \in \{1, 2, \cdots, n+1\}$.

In this paper all hyperrings are commutative hyperring with identity. Here we want to define the $I-$prime hyperideal, $2-$absorbing $I-$hyperideal and $n-$absorbing $I-$hyperideal in multiplicative hyperrings. For fixed proper hyperideal I of a multiplicative hyperring R, a proper hyperideal P of R is an $I-$prime if $a, b \in R$ with $a.b \subseteq P - IP$, then $a \in P$ or $b \in P$. A proper hyperideal P of R is a $2-$absorbing $I-$prime hyperideal if for $x_1, x_2, x_3 \in R$ such that

$$x_1x_2x_3 \subseteq P - IP,$$

then $x_1x_2 \subseteq P$ or $x_1x_3 \subseteq P$ or $x_2x_3 \subseteq P$. A proper hyperideal P of R is an $n-$absorbing $I-$prime hyperideal if for $x_1, \cdots, x_{n+1} \in R$ such that $x_1 \cdots x_{n+1} \subseteq P - IP$, then $x_1 \cdots x_{i-1}x_{i+1} \cdots x_{n+1} \subseteq P$ for some $i \in \{1, 2, \cdots, n+1\}$.

In section two, we define $I-$prime hyperideal and we prove some equivalents of $I-$prime hyperideal (Theorem 2.18). Moreover, we establish $I-$prime hyperideals in finite product of hyperrings (Theorem 2.20). Section three devoted for $2-$absorbing $I-$prime and $n-$absorbing $I-$prime hyperideals and we prove Theorem 3.9 which state (Let $R = \prod_{i=1}^{n+1} R_i$ and P be a proper non-zero hyperideal of R. If P is an $(n+1)-$absorbing $I-$prime hyperideal of R, then

$$P = P_1 \times P_2 \times \cdots \times P_{n+1}$$

for some proper $n-$absobing I_i-prime hyperideals $P_1, \cdots, P_{n+1}$ of $R_1, \cdots, R_{n+1}$ respectively, where $I = \prod_{i=1}^{n+1} I_i$ and $I_i = R_i$, $\forall i = 1, 2, \cdots, n+1$). Also, we prove Theorem 3.11 that state (Let $\mid Max(R) \mid \geq n+1 \geq 2$. Then each proper hyperideal of R is an $n-$absorbing $I-$prime hyperideal if and only if each quotient of R is a product of $(n+1)-$fields). Finally, let P be an $n-$absorbing $I-$hyperideal of a hyperring R. Then there are at most n^{th} prime hyperideals of R that are minimal over P (Theorem 3.13).

2. $I-$PRIME HYPERIDEALS

We start this section by defining the concept of $I-$prime hyperideal and some example of it. A proper hyperideal P of R is an $I-$prime hyperideal if for $a, b \in R$ with $ab \subseteq P - IP$ implies $a \in P$ or $b \in P$.

In the following examples we show that the class of $I-$prime hyperideals contains properly the class of prime hyperideals.

Example 2.1. Consider the hyperring of integers $(\mathbb{Z}, +, \circ)$,

$$A = \{0, 1\} \subseteq \mathbb{Z}$$

and $n \circ m = \{nam : a \in A\} = \{0, nm\}$. So $4\mathbb{Z}$ is not prime hyperideal, since $2 \circ 2 = \{0, 4\} \subseteq 4\mathbb{Z}$ and $2 \notin 4\mathbb{Z}$. But $4\mathbb{Z}$ is $2\mathbb{Z}$-prime hyperideal, since $\forall a, b \in \mathbb{Z}, a \circ b = \{0, ab\} \nsubseteq 4\mathbb{Z} - (2\mathbb{Z} \circ 4\mathbb{Z}) = 4\mathbb{Z} - 8\mathbb{Z}$.

Example 2.2. Let $(\mathbb{Z}, +, \circ)$ be the hyperring of integers and $A = \{4, 8\} \subseteq \mathbb{Z}$ and $a \circ b = aAb = \{4ab, 8ab\}$. Then $1 \circ 1 = \{4, 8\} \subseteq 2\mathbb{Z}$ but $1 \notin 2\mathbb{Z}$ and hence $2\mathbb{Z}$ is not prime hyperideal. However $2\mathbb{Z}$ is not $8\mathbb{Z}-$prime hyperideal, since

$$2\mathbb{Z} - (8\mathbb{Z} \circ 2\mathbb{Z}) = 2\mathbb{Z} - (64\mathbb{Z} \cup 128\mathbb{Z}) = 2\mathbb{Z} - 64\mathbb{Z}$$

which contains $1 \circ 1$. Therefore, $2\mathbb{Z}$ is neither prime hyperideal nor $8\mathbb{Z}$-prime hyperideal of $(\mathbb{Z}, +, \circ)$.

The intersection of two $I-$prime hyperideals is not $I-$prime hyperideal let us explain our claim by this example.

Example 2.3. Consider the hyperring of integers $(\mathbb{Z}, +, \circ)$, where $a \circ b = \{2ab, 3ab\}$. Let $P = 2\mathbb{Z}$, $I = 3\mathbb{Z}$ and

$$\begin{aligned} P - IP &= 2\mathbb{Z} - (3\mathbb{Z}) \circ (2\mathbb{Z}) \\ &= 2\mathbb{Z} - 6A\mathbb{Z} \\ &= 2\mathbb{Z} - (12\mathbb{Z} \cup 18\mathbb{Z}). \end{aligned}$$

Thus P is $I-$prime hyperideal. Now, for $Q = 3\mathbb{Z}$ and $I = 3\mathbb{Z}$ we have

$$\begin{aligned} Q - IQ &= 3\mathbb{Z} - (3\mathbb{Z}) \circ (3\mathbb{Z}) \\ &= 3\mathbb{Z} - 9A\mathbb{Z} \\ &= 3\mathbb{Z} - (18\mathbb{Z} \cup 27\mathbb{Z}). \end{aligned}$$

So Q is $I-$prime hyperideal of $\mathbb{Z}$ while $P \cap Q = 6\mathbb{Z}$ is not $3\mathbb{Z}-$prime hyperideal, since

$$\begin{aligned} 6\mathbb{Z} - (3\mathbb{Z}) \circ (6\mathbb{Z}) &= 6\mathbb{Z} - (36\mathbb{Z} \cup 54\mathbb{Z})2 \circ 3 \\ &= \{12, 18\} \\ &\subseteq 6\mathbb{Z} - (36\mathbb{Z} \cup 54\mathbb{Z}), \end{aligned}$$

but neither $2 \in 6\mathbb{Z}$ nor $3 \in 6\mathbb{Z}$.

The following lemma is a generalization of Lemma 2.1 in [1].

Lemma 2.4. *Let P be a proper hyperideal of a hyperring $(R, +, \circ)$. Then P is an I-prime hyperideal if and only if P/IP is weakly prime hyperideal in R/IP.*

Proof. $(\Rightarrow)$ Let P be an I-prime hyperideal in $(R, +, \circ)$. Let $a, b \subseteq R$ with

$$\{0\} \neq (a+ IP)(b + IP) = a \circ b + IP \in P/IP.$$

Then $a \circ b \subseteq P - IP$ implies $a \subseteq P$ or $b \subseteq P$, hence $a + IP \subseteq P/IP$ or $b + IP \subseteq P/IP$. So P/IP is weakly prime hyperideal in R/IP.

($\Leftarrow$) Suppose that P/IP is weakly prime hyperideal in R/IP and take $r, s \subseteq R$ such that $r \circ s \subseteq P - IP$. Then

$$\{0\} \neq r \circ s + IP = (r + IP)(s + IP) \subseteq P/IP$$

so $r + IP \subseteq P/IP$ or $s + IP \subseteq P/IP$. Therefore $r \subseteq P$ or $s \subseteq P$. Thus P is an I-prime hyperideal in R. $\square$

Let $(R, +, \circ)$ be a hyperring and x be an indeterminate. Then $(R[x], +, \bullet)$ is a polynomial multiplicative hyperring where $ax^n \bullet bx^m = (a \circ b)x^{n+m}$ (see [6]).

Theorem 2.5. *If P is an $I-$prime hyperideal of $(R, +, \circ)$, then $P[x]$ is $I[x]-$prime hyperideal of $(R[x], +, \bullet)$.*

Proof. Let $a(x) \bullet b(x) \subseteq P[x] - I[x] \bullet P[x] = P[x] - (IP)[x]$. Without loss of generality, let $a(x) = cx^n$ and $b(x) = dx^m$, for $c, d \in R$. Thus $c \circ dx^{n+m} \subseteq P[x]$ so $c \circ d \subseteq P$ and $c \circ dx^{n+m} \nsubseteq IP[x]$ implies $c \circ d \nsubseteq IP$. P $I-$prime hyperideal gives us $c \in P$ or $d \in P$. Hence $a(x) = cx^n \in P[x]$ or $b(x) = dx^m \in P[x]$ and so $P[x]$ is an $I[x]-$prime. $\square$

Corollary 2.6. *Let P be an $I-$prime hyperideal of R. Then $P[x]$ is an $I-$prime hyperideal of $R[x]$.*

Theorem 2.7. *Let R be a hyperring and $f : R \longrightarrow R$ be a good epiomorphism and let P be an I-prime hyperideal of R with $Kerf \subseteq P$. Then $f(P)$ is an $I-$prime hyperideal.*

Proof. Firstly, we have to show that $f(P)$ is hyperideal of R. Let $\bar{r} \in R$ and $y \in f(P)$. Then $x = f^{-1}(y) \in P$ and there exists $r \in R$ such that $f(r) = \bar{r}$. So $\bar{r}.y = f(r).f(x) = f(r.x) \subseteq f(P)$. Now let us show that $f(P)$ is an $I-$prime hyperideal. To do this, we have for all $x, y \in R$ there exist $a, b \in R$ such that $x = f(a)$, $y = f(b)$. Then

$$x.y = f(a).f(b) = f(a.b) \subseteq f(P),$$

so $a.b \subseteq P + Kerf$. As P is an $I-$prime hyperideal, $a \in P$ or $b \in P$, that is $x = f(a) \in f(P)$ or $y = f(b) \in f(P)$. So $f(P)$ is an $I-$prime hyperideal of R. $\square$

Theorem 2.8. *Let $(R, +, \circ)$ be a hyperring and $f : R \longrightarrow R$ be a good homomorphism and let Q be an I-prime hyperideal of R. Then $f^{-1}(Q)$ is an $I-$prime hyperideal.*

Proof. Let $a \circ b \subseteq f^{-1}(Q)$. Then $f(a \circ b) = f(a) \circ f(b) \subseteq Q$ because f is a good homomorphism. As Q is $I-$prime hyperideal, $f(a) \in Q$ or $f(b) \in Q$. So, $a \in f^{-1}(Q)$ or $b \in f^{-1}(Q)$ and hence $f^{-1}(Q)$ is an $I-$prime hyperideal of R. $\square$

The following theorem generalizes Theorem 2.2 of [1].

Theorem 2.9. *(1) Let $I \subseteq J$ be two hyperideals of a multiplicative hyperring R. If P is an $I-$prime hyperideal of R, then it is a $J-$prime hyperideal.*

(2) Let R be a commutative multiplicative hyperring and P an I-prime hyperideal that is not prime hyperideal, then $P^2 \subseteq IP$. Thus, an I-prime hyperideal P with $P^2 \nsubseteq IP$ is a prime hyperideal.

Proof. (1) The proof comes from the fact that if $I \subseteq J$, then

$$P - JP \subseteq P - IP.$$

(2) Suppose that $P^2 \nsubseteq IP$, we show that P is prime hyperideal. Let $ab \subseteq P$ for $a, b \in R$. If $ab \nsubseteq IP$, then P I-prime gives $a \in P$ or $b \in P$. So assume that $ab \subseteq IP$. First, suppose that $aP \nsubseteq IP$; say $ax \nsubseteq IP$ for some $x \in P$. Then $a(x + b) \subseteq P - IP$. So $a \in P$ or $x + b \in P$ and hence $a \in P$ or $b \in P$. Hence we can assume that $aP \subseteq IP$ and in a similar way we can assume that $bP \subseteq IP$. Since $P^2 \nsubseteq IP$, there exist $y, z \in P$ with $yz \nsubseteq IP$. Then $(a + y)(b + z) \subseteq P - IP$. So P I-prime gives $a + y \in P$ or $b + z \in P$ and hence $a \in P$ or $b \in P$. Therefore P is a prime hyperideal of R see also [1] . □

Corollary 2.10. *Let P be an I-prime hyperideal of a hyperring R with $IP \subseteq P^3$. Then P is $\cap_{i=1}^{\infty} P^i$-prime hyperideal.*

Proof. If P is prime hyperideal, then P is $\cap_{i=1}^{\infty} P^i$-prime hyperideal. Assume that P is not prime hyperideal. By Theorem 2.5,

$$P^2 \subseteq IP \subseteq P^3.$$

Thus $IP = P^n$ for each $n \geq 2$. So $\cap_{i=1}^{\infty} P^i = P \cap P^2 = P^2$ and $\left(\cap_{i=1}^{\infty} P^i\right) P = P^2 P = P^3 = IP$. Being P is I-prime hyperideal implies P is $\cap_{i=1}^{\infty} P^i$-prime hyperideal. □

Remark 2.11. Let P be an I-prime hyperideal. Then $P \subseteq \sqrt{IP}$ or $\sqrt{IP} \subseteq P$. If $P \subsetneqq \sqrt{IP}$, then P is not prime hyperideal since otherwise $IP \subseteq P$ implies $\sqrt{IP} \subseteq \sqrt{P} = P$. While if $\sqrt{IP} \subsetneq P$, then P is a prime hyperideal. Now we give a way to construct I-prime ideals P when $\cap_{i=1}^{\infty} P^i \subseteq IP \subseteq P^3$.

Corollary 2.12. *Let P be an $I-$prime hyperideal of a hyperring R which is not prime hyperideal. Then $\sqrt{P} = \sqrt{IP}$.*

Proof. By Theorem 2.5, $P^2 \subseteq IP$ and hence $\sqrt{P} = \sqrt{P^2} \subseteq \sqrt{IP}$. The other containment always holds. □

Remark 2.13. Assume that P is an $I-$prime hyperideal, but not prime. Then by Theorem 2.5, if $IP \subseteq P^2$, then $P^2 = IP$. In particular, if P is weakly prime hyperideal (0-prime) but not prime hyperideal, then $P^2 = \{0\}$. Suppose that $IP \subseteq P^3$. Then $P^2 \subseteq IP \subseteq P^3$; So $P^2 = P^3$ and thus P^2 is an idempotent.

Lemma 2.14. *If P is an $I-$primary hyperideal of a hyperring R, then $\sqrt{P}$ is a $\sqrt{I}-$prime hyperideal of R.*

Proof. Let $ab \subseteq \sqrt{P} - \sqrt{I}\sqrt{P} = \sqrt{P} - \sqrt{IP}$ for $a, b \in R$. Then $(ab)^n = a^n b^n \subseteq P$ for some $n \in \mathbb{N}$ and $(ab)^m \nsubseteq IP$ for all $m \in \mathbb{N}$. So $a^n b^n \subseteq P - IP$ and as P is an $I-$primary hyperideal of R, $a^n \subseteq P$ or $b^n \subseteq \sqrt{P}$, that is $a \in \sqrt{P}$ or $b \in \sqrt{P}$ which means that $\sqrt{P}$ is a $\sqrt{I}-$prime hyperideal of R. □

The following theorem generalizes the result [1, Theorem 2.8].

Theorem 2.15. *(1) Let R and S be two commutative multiplicative hyperrings and P be $\{0\}-$prime hyperideal of R. Then $P \times S$ is I-prime hyperideal of $R \times S$ for each hyperideal I of $R \times S$ with*

$$\cap_{i=1}^{\infty}(P \times S)^i \subseteq I(P \times S) \subseteq P \times S.$$

(2) Let P be a finitely generated proper hyperideal of a commutative hyperring R. Assume P is an I-prime hyperideal with $IP \subseteq P^3$. Then either P is $\{0\}-$prime or $P^2 \neq \{0\}$ is idempotent and R decomposes as $T \times S$ where $S = P^2$ and $P = J \times S$ where J is a $\{0\}-$prime. Thus P is I-prime hyperideal for $\cap_{i=1}^{\infty} P^i \subseteq IP \subseteq P$.

Proof. (1) Let R and S be two commutative hyperrings and P be a $\{0\}-$prime hyperideal of R. Then $P \times S$ need not be a $\{0\}-$prime hyperideal of $R \times S$; In fact, $P \times S$ is $\{0\}-$prime if and only if $P \times S$ (or equivalently P) is prime hyperideal. However, $P \times S$ is an I-prime hyperideal for each I with $\cap_{i=1}^{\infty}(P \times S)^i \subseteq I(P \times S)$. If P is prime hyperideal, then $P \times S$ is a prime hyperideal and thus is I-prime for all I. Assume that P is not a prime hyperideal. Then $P^2 = \{0\}$ and $(P \times S)^2 = \{0\} \times S$. Hence $\cap_{i=1}^{\infty}(P \times S)^i = \cap_{i=1}^{\infty} P^i \times S = \{0\} \times S$. Thus

$$P \times S - \cap_{i=1}^{\infty}(P \times S)^i = P \times S - \{0\} \times S = (P - \{0\}) \times S.$$

Since P is $\{0\}-$prime hyperideal, $P \times S$ is $\cap_{i=1}^{\infty}(P \times S)^i$-prime hyperideal and as $\cap_{i=1}^{\infty}(P \times S)^i \subseteq I(P \times S)$, $P \times S$ is I-prime hyperideal.

(2) If P is a prime hyperideal, then P is $\{0\}-$prime. So we can assume that P is not prime hyperideal. Then $P^2 \subseteq IP$ and hence $P^2 \subseteq IP \subseteq P^3$. So $P^2 = P^3$. Hence P^2 is idempotent. Since P^2

is finitely generated, $P^2 =< e >$ for some idempotent $e \in R$. Suppose $P^2 = \{0\}$. Then $IP \subseteq P^3 = \{0\}$. So $IP = \{0\}$ and hence P is $\{0\}-$prime. So assume $P^2 \neq \{0\}$. Put $S = P^2 = <e>$ and $T =< 1 - e >$, so R decomposes as $T \times S$ where $S = P^2$. Let $J = P(1 - e)$, so $P = J \times S$ where

$$J^2 = (P(1-e))^2 = P^2(1-e)^2 =< e >< 1 - e >= \{0\}.$$

We show that J is $\{0\}-$prime hyperideal. Let $a \circ b \subseteq J - \{0\}$, so

$$\begin{aligned} (a,1)(b,1) &= (a \circ b, 1) \\ &\subseteq J \times S - (J \times S)^2 \\ &= J \times S - \{0\} \times S \\ &\subseteq P - IP. \end{aligned}$$

Since $IP \subseteq P^3$ implies $IP \subseteq P^3 = (J \times S)^3 = \{0\} \times S$. Hence $(a, 1) \in P$ or $(b, 1) \in P$ so $a \in J$ or $b \in J$. Therefore J is a $\{0\}-$prime hyperideal. □

Corollary 2.16. *Let $(R, +, \circ)$ be an indecomposable commutative hyperring and P a finitely generated I-prime hyperideal of $(R, +, \circ)$, where $IP \subseteq P^3$. Then P is a $\{0\}-$prime hyperideal.*

Corollary 2.17. *Let $(R, +, \circ)$ be a Noetherian integral hyperdomain. A proper hyperideal P of R is prime hyperideal if and only if P is P^2-prime hyperideal.*

The next theorem is a generalization of [1, Theorem 2.12].

Theorem 2.18. *Let P be a proper hyperideal of a hyperring R. Then the following assertions are equivalent:*

1. *P is I-prime hyperideal.*
2. *For $r \in R - P$, $(P : r) = P \cup (IP : r)$.*
3. *For $r \in R - P$, $(P : r) = P$ or $(P : r) = (IP : r)$.*
4. *For hyperideals J and K of R, $JK \subseteq P$ and $JK \nsubseteq IP$ imply $J \subseteq P$ or $K \subseteq P$.*

Proof. (1) ⇒ (2) Suppose $r \in R - P$. Let $s \in (P : r)$, so $rs \subseteq P$. If $rs \subseteq P - IP$, then $s \in P$. If $rs \subseteq IP$, then $s \in (IP : r)$. So $(P : r) \subseteq P \cup (IP : r)$. The other containment always holds.

(2) ⇒ (3) Note that if a hyperideal is a union of two hyperideals, then it is equal to one of them.

(3) ⇒ (4) Let J and K be two hyperideals of R with $JK \subseteq P$. Assume that $J \nsubseteq P$ and $K \nsubseteq P$. We claim that $JK \subseteq IP$. Suppose $r \in J$. First, let $r \notin P$. Then $rK \subseteq P$ gives $K \subseteq (P : r)$. Now $K \nsubseteq P$, so $(P : r) = (IP : r)$. Thus $rK \subseteq IP$. Next, let $r \in J \cap P$. Choose

$s \in J - P$. Then $r + s \in J - P$. By the first case $sK \subseteq IP$ and so $(r+s)K \subseteq IP$. Pick $t \in K$. Then $rt = (r+s)t - st \subseteq IP$ and $rK \subseteq IP$. Hence $JK \subseteq IP$.

(4) $\Rightarrow$ (1) Let $rs \in P - IP$. Then $(r)(s) \subseteq P$. But $(r)(s) \nsubseteq IP$. So $(r) \subseteq P$ or $(s) \subseteq P$ which means $r \in P$ or $s \in P$. □

Proposition 2.19. *Let P be an I-prime hyperideal of a hyperring R and $J \subseteq P$ be a hyperideal of R. Then P/J is $I-$prime hyperideal of R/J.*

Proof. Let $x, y \in R$ with $\bar{x} \circ \bar{y} \subseteq P/J - I(P/J) = P/J - (IP + J)/J$ where $\bar{x}$, $\bar{y}$ are the images of x, y in R/J. Thus $x \circ y \subseteq P - IP$. So $x \in P$ or $y \in P$. Therefore $\bar{x} \in P/J$ or $\bar{y} \in P/J$. So P/J is I-prime hyperideal. □

Let R_1 and R_2 be two hyperrings. It is known that the prime hyperideals of $R_1 \times R_2$ have the form $P \times R_2$ or $R_1 \times Q$, where P is a prime hyperideal of R_1 and Q is a prime hyperideal of R_2. We next generalize this result to I-prime hyperideals.

Theorem 2.20. *Let R_i be a hyperring and I_i a hyperideal of R_i for $i = 1, 2$. Let $I = I_1 \times I_2$. Then the I-prime hyperideals of $R_1 \times R_2$ have exactly one of the following three types:*

(1) $P_1 \times P_2$, where P_i is a proper hyperideal of R_i with $I_iP_i = P_i$.

(2) $P_1 \times R_2$ where P_1 is an I_1-prime hyperideal of R_1 and $I_2R_2 = R_2$.

(3) $R_1 \times P_2$, where P_2 is an I_2-prime hyperideal of R_2 and $I_1R_1 = R_1$.

Proof. We first prove that a hyperideal of $R_1 \times R_2$ having one of these three types is I-prime hyperideal. The first type is clear since

$$P_1 \times P_2 - I\left(P_1 \times P_2\right) = P_1 \times P_2 - \left(I_1P_1 \times I_2P_2\right) = \phi.$$

Suppose that P_1 is I_1-prime hyperideal and $I_2R_2 = R_2$. Let

$$\begin{aligned} (a,b)(x,y) &\subseteq P_1 \times R_2 - (I_1P_1 \times I_2R_2) \\ &= P_1 \times R_2 - (I_1P_1 \times R_2) \\ &= (P_1 - I_1P_1) \times R_2. \end{aligned}$$

Then $ax \subseteq P_1 - I_1P_1$ implies that $a \in P_1$ or $x \in P_1$, so $(a, b) \in P_1 \times R_2$ or $(x, y) \in P_1 \times R_2$. Hence $P_1 \times R_2$ is I-prime hyperideal. Similarly we can prove the last case. Next, let $P_1 \times P_2$ be I-prime and $ab \subseteq P_1 - I_1P_1$. Then

$$(a, 0)(b, 0) = (ab, 0) \in P_1 \times P_2 - I\left(P_1 \times P_2\right),$$

so $(a, 0) \in P_1 \times P_2$ or $(b, 0) \in P_1 \times P_2$, that is, $a \in P_1$ or $b \in P_1$. Hence P_1 is I_1-prime. Likewise, P_2 is I_2-prime.

Assume that $P_1 \times P_2 \neq I_1P_1 \times I_2P_2$, say $P_1 \neq I_1P_1$. Let $x \in P_1 - I_1P_1$ and $y \in P_2$. Then $(x,1)(1,y) = (x,y) \in P_1 \times P_2$. So $(x,1) \in P_1 \times P_2$ or $(1,y) \in P_1 \times P_2$. Thus $P_2 = R_2$ or $P_1 = R_1$. Assume that $P_2 = R_2$. Then $P_1 \times R_2$ is I-prime, where P_1 is I_1-prime.

□

3. $n-$ABSORBING $I-$PRIME HYPERIDEALS

We start this section by the definition of $n-$absorbing $I-$prime hyperideals.

Definition 3.1. A proper hyperideal P of a hyperring R is a $2-$absorbing $I-$prime hyperideal if for $x_1, x_2, x_3 \in R$ such that $x_1x_2x_3 \subseteq P - IP$, then $x_1x_2 \subseteq P$ or $x_1x_3 \subseteq P$ or $x_2x_3 \subseteq P$. A proper hyperideal P of R is an $n-$absorbing $I-$prime hyperideal if for $x_1, \cdots, x_{n+1} \in R$ such that $x_1 \cdots x_{n+1} \subseteq P - IP$, then

$$x_1 \cdots x_{i-1}x_{i+1} \cdots x_{n+1} \subseteq P$$

for some $i \in \{1, 2, \cdots, n+1\}$.

It is clear that the class of $n-$absorbing $I-$prime hyperideals contains properly the class of $n-$absorbing hyperideals. As we can see this in the following example.

Example 3.2. Let K be a hyperfield and $R = K[x_1, \cdots, x_{n+2}]$ be a polynomial multiplicative hyperring. Consider the hyperideals

$$P = < x_1 \cdots x_{n+1}, x_1^2 \cdots x_n, x_1^2x_{n+2} >$$

and $I = < x_1 \cdots x_n >$. So

$$\begin{aligned} P - IP &= < x_1 \cdots x_{n+1}, x_1^2 \cdots x_n, x_1^2x_{n+2} > \\ &\quad - < x_1 \cdots x_{n+1}, x_1^2 \cdots x_n, x_1^2 \cdots x_nx_{n+2} > . \end{aligned}$$

Hence P is an $n-$absorbing $I-$prime hyperideal but not $n-$absorbing hyperideal.

Lemma 3.3. *Let P be an $I-$prime hyperideal of R and K be a subset of R. For any $a \in R, aK \subseteq P$, $aK \nsubseteq IP$ and $a \notin P$ implies that $K \subseteq P$. (or $aK \subseteq P$ and $K \nsubseteq P$ imply that $a \in P$).*

Proof. Let $aK \subseteq P$ and $a \notin P$ for any $a \in R$. Then we have $aK = \cup ak_i \subseteq P$ for all $k_i \in K$. Hence $ak_i \subseteq P$ and $ak_i \nsubseteq IP$ for all $k_i \in K$. Since P is an $I-$prime hyperideal and $a \notin P$, $k_i \in P$, $\forall k_i \in K$. Thus $K \subseteq P$. □

Lemma 3.4. *Let P be an $I-$prime hyperideal of R and A, B be subsets of R. If $AB \subseteq P$ and $AB \nsubseteq IP$, then $A \subseteq P$ or $B \subseteq P$.*

Proof. Assume that $AB \subseteq P, AB \nsubseteq IP$ and $A \nsubseteq P$, $B \nsubseteq P$. Since $AB = \bigcup a_ib_i \subseteq P$, $a_ib_i \subseteq P$, for $a_i \in A, b_i \in B$. And as $A \nsubseteq P$ and $B \nsubseteq P$, we have $x \notin P$ and $y \notin P$ for some $x \in A, y \in B$. Then $xy \subseteq AB \subseteq P$ and $xy \nsubseteq IP$. From being P an $I-$prime hyperideal, we have $x \in P$ or $y \in P$ which is a contradiction. Thus $A \subseteq P$ or $B \subseteq P$. □

Every $I-$prime hyperideal is a $2-$absorbing $I-$prime hyperideal. Since for $(ab)c \subseteq P - IP$, we have $ab \subseteq P$ or $bc \subseteq P$. If $ab \nsubseteq P$ then by $I-$prime hyperideal of P, we have $c \in P$ and so $ac \in P$ or $bc \in P$. Hence P is a $2-$absorbing $I-$prime hyperideal of R.

Lemma 3.5. *Let P be a hyperideal of R and $P_1, P_2, \ldots, P_n$ be $2-$absorbing primary hyperideals of R such that $\sqrt{P_i} = P$ for all* i = $1, \ldots,$ n. *Then $\bigcap_{i=1}^n P_i$ is a 2-absorbing $I-$prime hyperideal and $\bigcap_{i=1}^n P_i = P$.*

Proof. Assume $P = \bigcap_{i=1}^n P_i$ and so $\sqrt{P} = \sqrt{\cap_{i=1}^n P_i} = \cap_{i=1}^n \sqrt{P_i} = P$. Let $xyz \subseteq P - IP$ with $xy \nsubseteq P$, for $x, y, z \in R$. Thus $xy \nsubseteq P_i$ for some $i = 1, 2, \cdots, n$. From being P_i a 2-absorbing primary hyperideal and $xyz \subseteq P - IP \subseteq P_i$, hence $xz \subseteq \sqrt{P_i} = P$ or $yz \subseteq \sqrt{P_i} = P$ which means that P is a 2-absorbing $I-$prime hyperideal of R. □

Theorem 3.6. *Let $h : R \to L$ be a bijective good homomorphism of hyperrings and P be a 2-absorbing $I-$prime hyperideal of L. Then $h^{-1}(P)$ is a 2-absorbing $h^{-1}(I)-$prime hyperideal of R.*

Proof. Suppose that $abc \subseteq h^{-1}(P), h^{-1}(I)h^{-1}(P) = h^{-1}(P) - h^{-1}(IP)$, for $a, b, c \in R$. So $h(abc) = h(a)h(b)h(a) \subseteq P$ and $h(abc) \nsubseteq IP$. From being P a 2-absorbing $I-$prime hyperideal, we have $h(a)h(b) \subseteq P$ or $h(a)h(c) \subseteq P$ or $h(b)h(c) \subseteq P$, that is $h(ab) \subseteq P$ or $h(ac) \subseteq P$ or $h(bc) \subseteq P$ which implies $ab \subseteq h^{-1}(P)$ or $ac \subseteq h^{-1}(P)$ or $bc \subseteq h^{-1}(P)$. So $h^{-1}(P)$ is a 2-absorbing $h^{-1}(I)-$prime hyperideal of R. □

Theorem 3.7. *Suppose that P is an $n-$absorbing $I-$prime hyperideal of R. Then $\sqrt{P}$ is an $n-$absorbing $\sqrt{I}-$prime hyperideal of R and $a^n \subseteq P$ for all $a \in \sqrt{P}$.*

Proof. Let $a \in \sqrt{P}$. Then $a^m \subseteq P$ for some $m \in \mathbb{N}$. If $m \leq n$, we are done. If $m > n$, by using the n-absorbing $I-$prime property on products a^m, we conclude that $a^n \subseteq P$. Now, consider

$$a_1 \cdots a_{n+1} \subseteq \sqrt{P} - \sqrt{I}\sqrt{P} = \sqrt{P} - \sqrt{IP}$$

for $a_1, \cdots, a_{n+1} \in R$. Thus $(a_1 \ldots a_{n+1})^n = a_1^n \cdots a_{n+1}^n \subseteq P$. If $a_1^n \cdots a_{n+1}^n \subseteq IP$, then $a_1 \cdots a_{n+1} \subseteq \sqrt{IP}$ which is a contradiction. Hence $a_1^n \cdots a_{n+1}^n \subseteq P - IP$ and P n-absorbing $I-$prime hyperideal gives us the desired. □

Lemma 3.8. *Let P_i be an n_i-absorbing $I-$prime hyperideal of a hyperring R for $i = 1, 2, \cdots, m$ and $IP_i = IP_j$, for $i \neq j$, Then $\cap_{i=1}^m P_i$ is an $n-$absorbing $I-$prime hyperideal where $n = \sum_{i=1}^m n_i$.*

Proof. Let $k > n$ and $x_1 \cdots x_k \subseteq \cap_{i=1}^m P_i - I \cap_{i=1}^m P_i$. Then by hypothesis for each $i = 1 \cdots m$, there exists a product of n_i of these $k-$elements in P_i. Let A_i be the collection of these elements and let $A = \cup_{i=1}^k A_i$. Thus A has at most $n-$elements. Now, as P_i is an $n-$absorbing $I-$prime hyperideal, the product of all elements of A must be in each P_i so $\cap P_i$ contains a product of at most $n-$elements and therefore it is an $n-$absorbing $I-$prime hyperideal of R. □

Theorem 3.9. *Let $R = \prod_{i=1}^{n+1} R_i$ and P be a proper non-zero hyperideal of R. If P is an $(n+1)-$absorbing $I-$prime hyperideal of R, then*

$$P = P_1 \times P_2 \times \cdots \times P_{n+1}$$

for some proper $n-$absobing I_i-prime hyperideals $P_1, \cdots, P_{n+1}$ of $R_1, \cdots, R_{n+1}$ respectively, where $I = \prod_{i=1}^{n+1} I_i$ and $I_i = R_i$, $\forall i = 1, 2, \cdots, n+1$.

Proof. Let $x_1, \cdots, x_{n+1} \in R$ with

$$x_1 \cdots x_{n+1} \subseteq P_1 - I_1 P_1$$

and suppose by contrary that P_1 is not an $n-$absorbing I_1-prime hyperideal of R_1. Set $a_i = (x_i, 1, 1, \cdots, 1)$ for $i = 1, 2, \cdots, n+1$ and $a_{n+2} = (1, 0, \cdots, 0)$. Then we have

$$a_1 a_2 \cdots a_{n+2} = (x_1 x_2 \cdots x_{n+1}, 0, 0, \cdots, 0) \subseteq P - IP$$

and

$$a_1 \cdots a_{i-1} a_{i+1} \cdots a_{n+2} = (x_1 x_2 \cdots x_{i-1} x_{i+1} \cdots x_{n+1}, 0, \cdots, 0) \nsubseteq P$$

for $i = 1, \cdots, n+1$, which contradicts with being P an $(n+1)-$absorbing $I-$prime hyperideal. Hence P_1 must be an $n-$absorbing I_1-prime hyperideal of R_1. By similar arguments, we can show that P_i is an $n-$absorbing I_i-prime hyperideal of R_i for $i = 1, \cdots, n+1$. □

Theorem 3.10. *Let $R = \prod_{i=1}^{n+1} R_i$, where R_i is a hyperring for $i \in \{1, \cdots, n+1\}$. If P is an $n-$absorbing $I-$prime hyperideal of R, then either $P = IP$ or*

$$P = P_1 \times P_2 \times \cdots \times P_{i-1} \times R_i \times P_{i+1} \cdots \times P_{n+1}$$

for some $i \in \{1, \cdots, n+1\}$ and if $P_j \neq R_i$ for $j \neq i$, then P_j is an $n-$absorbing hyperideal in R_i.

Proof. Let $P = \prod_{i=1}^{n+1} P_i$ be an $n-$absorbing $I-$prime hyperideal of R. Then there exists $(x_1, \cdots, x_{n+1}) \subseteq P - IP$, and so

$$\begin{aligned}(x_1, 1, \cdots, 1)(1, x_2, 1 \cdots, 1) \cdots (1, 1, \cdots, 1, x_{n+1}) &= (x_1, x_2, \cdots, x_{n+1}) \\ &\subseteq P - IP.\end{aligned}$$

As P is an $n-$absorbing $I-$prime hyperideal, we have

$$(x_1, x_2, \cdots, x_{i-1}, 1, x_{i+1}, \cdots, x_{n+1}) \subseteq P$$

for some $i \in \{1, 2, \cdots, n+1\}$. Thus $(0, 0, \cdots, 0, 1, 0, \cdots, 0) \in P$ and hence $P = P_1 \times P_2 \times \cdots \times P_{i-1} \times R_i \times P_{i+1} \cdots \times P_{n+1}$. If $P_j \neq R_i$ for $j \neq i$, then we have to prove P_j is an $n-$absorbing hyperideal of R_i. Let $i < j$ and take $x_1 x_2 \cdots x_{n+1} \subseteq P_j$. Then

$$\begin{aligned}&(0, 0, \cdots, 0, 1, 0, \cdots, 0, x_1 x_2 \cdots x_{n+1}, 0 \cdots, 0) \\ &= (0, 0, \cdots, 1, 0, \cdots, 0, x_1, 0 \cdots, 0)(0, 0, \cdots, 1, 0, \cdots, 0, x_2, 0 \cdots, 0) \\ &\quad \cdots (0, 0, \cdots, 1, 0, \cdots, 0, x_{n+1}, 0 \cdots, 0) \\ &\subseteq P - IP.\end{aligned}$$

Since P is an $n-$absorbing $I-$prime hyperideal,

$$(0, 0, \cdots, 0, 1, 0, \cdots, 0, x_1 x_2 \cdots x_{k-1} x_{k+1} \cdots x_{n+1}, 0, \cdots, 0) \in P$$

for some $k \in \{1, 2, \cdots, n+1\}$. Thus $x_1 x_2 \cdots x_{k-1} x_{k+1} \cdots x_{n+1} \in P_j$ and hence P_j is an $n-$absorbing hyperideal of R_i. We can do similar arguments for the case $i > j$. □

In the following result, we characterize hyperrings in which every proper hyperideal of R is an $n-$absorbing $I-$prime hyperideal.

Theorem 3.11. *Let $\mid Max(R) \mid \geq n+1 \geq 2$. Then each proper hyperideal of R is an $n-$absorbing $I-$prime hyperideal if and only if each quotient of R is a product of $(n+1)-$fields.*

Proof. ($\Rightarrow$) Let P be a proper hyperideal of R. Then

$$\tfrac{R}{IP} \cong F_1 \times \cdots \times F_{n+1}$$

and $\frac{P}{IP} \cong P_1 \times \cdots \times P_{n+1}$, where P_i is a hyperideal of F_i, $i = 1, \cdots, n+1$. If $P = IP$ then there is nothing to prove, otherwise we have $P_j = 0$, for at least one $j \in \{1, \cdots, n+1\}$ since $\frac{P}{IP}$ is proper. So, $\frac{P}{IP}$ is an $n-$absorbing $0-$prime hyperideal of $\frac{R}{IR}$ which means P is an $n-$absorbing $I-$prime hyperideal of R.

($\Leftarrow$) Let $m_1, \cdots, m_{n+1}$ be distinct maximal hyperideals of R. Then $m = \prod_{i=1}^{n+1} m_i$ is an $n-$absorbing $I-$hyperideal of R. we claim that m is not an $n-$absorbing hyperideal. First, if $m_i \subseteq \cup_{i\neq j} m_j$, then there exist m_j with $m_i \subseteq m_j$ by Prime Avoidance Lemma and this contradicts the maximality of m_i. Hence $m_i \nsubseteq \cup_{i\neq j} m_j$ and so, there exists $x_i \in m_i - \cup_{i\neq j}^{n+1} m_j$ so that $x_1 \cdots x_{n+1} \subseteq m$. If there exists $j \in \{1, \cdots, n+1\}$ with

$$a = x_1 x_2 \cdots x_{j-1} x_{j+1} \cdots x_{n+1} \subseteq m \subseteq m_j,$$

then $x_i \in m_j$ for some $i \neq j$ which is a contradiction. Hence m is not an $n-$absorbing hyperideal and so $m^{n+1} = Im$. Then by Chinese Remainder Theorem we have $\frac{R}{Im} \simeq \frac{R}{m_1^{n+1}} \times \frac{R}{m_2^{n+1}} \times ... \times \frac{R}{m_{n+1}^{n+1}}$. Put $F_i = \frac{R}{m_1^{n+1}}$. If F_i is not a field, then it has a nonzero proper hyperideal H and so $0 \times 0 \times \cdots \times 0 \times H \times 0 \times \cdots \times 0$ is an $n-$absorbing $0-$hyperideal of $\frac{R}{Im}$. Thus, by Lemma 3.10 we have $H = F_i$ or $H = 0$ which is impossible. Hence F_i is a field. □

Corollary 3.12. *Suppose $\mid Max(R) \mid \geq n+1 \geq 2$. Then each proper hyperideal of R is an $n-$absorbing $0-$hyperideal if and only if $R \cong F_1 \times \cdots \times F_{n+1}$, where $F_1, \cdots, F_{n+1}$ are fields.*

Theorem 3.13. *Let P be an $n-$absorbing $I-$prime hyperideal of a hyperring R. Then there are at most n^{th} prime hyperideals of R that are minimal over P.*

Proof. Let $C = \{q_i : q_i$ is a prime hyperideal minimal over $P\}$ and let C has at least n elements. Assume $q_1, \cdots, q_n \in C$ are distinct elements and $x_i \in q_i - \cup_{j\neq i} q_j$ for $i = 1, \cdots, n$. By [9, Theorem 2.1], there is a $y_i \notin q_j$ such that $y_i x_i^{t_i} \subseteq P$ for $i = 1, \cdots, n$ and for some positive integers $t_1, \cdots, t_n$. Since $x_i \notin \cap_{j=1}^n q_j$ and P an $n-$absorbing $I-$prime hyperideal, we have $y_i x_i^{n-1} \in P$. As $x_i \notin \cap_{j=1}^n q_j$ and

$$y_i x_i^{n-1} \subseteq P \subseteq \cap_{j=1}^n q_j,$$

we get $y_i \in q_i - \cup_{j\neq i} q_j$, and so $y_i \notin \cap_{j=1}^n q_j$ for $i = 1, \cdots, n$. Since $y_i x_i^{n-1} \subseteq P$, $\sum_{j=1}^n y_j \prod_{i=1}^n x_i^{n-1} \subseteq P$ and clearly $\sum_{j=1}^n y_j \notin q_i$, for $i = 1, \cdots, n$, and being P an $n-$absorbing $I-$prime hyperideal, we have $y_i x_i^{n-1} \in P$. As $x_i \notin \cap_{j=1}^n q_j$ and $y_i x_i^{n-1} \subseteq P \subseteq \cap_{i=1}^n q_i$, we get $y_i \in q_i - \cup_{j\neq i} q_j$ and so $y_i \notin \cap_{i=1}^n q_i$ for $i = 1, \cdots n$. Since $y_i x_i^{n-1} \subseteq P$, $\Sigma_{j=1}^n y_j \prod_{i=1}^n x_i^{n+1} \subseteq P$ and clearly $\Sigma_{j=1}^n y_j \notin q_i$, for $i = 1, \cdots n$ and being P an $n-$absorbing $I-$prime hyperideal, we have

$$\prod_{i=1}^n x_i^{n-1} \subseteq P.$$

Now, suppose $q_{n+1} \in C$ such that $q_{n+1} \neq q_i$, for $i = 1, \cdots, n$ and consequently $z_i \in q_{n+1}$ for $i = 1, \cdots, n$ which is a contradiction. Therefore C has at least n elements. □

In a multiplicative hyperring $(R, +, \circ)$ a non empty subset L of R is called a multiplicative set whenever $a, b \in A \Rightarrow a \circ b \cap A \neq \phi$.

We can contract the localization of a multiplicative hyperring R as follows: Let S be a multiplicative closed subset of R, that is, S is closed under the hypermultiplication and contains the identity. Let $S^{-1}R$ be the set $(R \times S/\sim)$ of equivalence classes where

$$(r_1, s_1) \sim (r_2, s_2) \iff \exists s \in S \text{ such that } ss_1r_2 = ss_2r_1.$$

Let r/s be the equivalence class of $(r, s) \in R \times S$ under the equivalence relation $\sim$. The operation addition and the hyperoperation multiplication are defined by

$$\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{s_1r_2 + s_2r_1}{s_1s_2} = \{\frac{a+b}{c} : a \in s_1r_2, b \in s_2r_1, c \in s_1s_2\}$$

$$\frac{r_1}{s_1} \cdot \frac{r_2}{s_2} = \frac{r_1r_2}{s_1s_2} = \{\frac{a}{b}, a \in r_1r_2, b \in s_1s_2\}.$$

Note that the localization map $f : R \to S^{-1}R$, $f(r) = \frac{r}{1}$ is a homomorphism of hyperrings. It is easy to see that the localization of a hyperideal is a hyperideal.

Proposition 3.14. *Let P be an $I-$prime hyperideal of R with $S \cap P = \emptyset$. Then $S^{-1}P$ is an $S^{-1}I-$prime hyperideal of $S^{-1}R$.*

Proof. $\frac{r_1}{s_1}, \frac{r_2}{s_2} \in S^{-1}R$ with

$$\tfrac{r_1}{s_1}\tfrac{r_2}{s_2} = \tfrac{r_1r_2}{s_1s_2} \subseteq S^{-1}P - S^{-1}IS^{-1}P = S^{-1}P - S^{-1}(IP).$$

For each $n \in r_1r_2, s \in s_1s_2$, we have $\frac{n}{s} \in \frac{r_1r_2}{s_1s_2}$ and $\frac{n}{s} = \frac{a}{t}$, where $a \in P$, $t \in S$. So there exists $q \in S$ such that $qtn = qsa$. Hence $qtn \subseteq P - IP$ and so $qr_1r_2 \subseteq P - IP$. As P is an $I-$prime hyperideal, we have $qr_1 \subseteq P$ or $r_2 \in P$. Thus $\frac{r_1}{s_1} = \frac{qr_1}{qs_1} \in P$ or $\frac{r_2}{s_2} \in S^{-1}P$. Therefore $S^{-1}P$ is an $S^{-1}I-$prime hyperideal of $S^{-1}R$. □

4. Conclusion

In this article we transfer the notions $I-$prime ideals and $n-$absorbing $I-$ideals in multiplicative hyperrings and named them as $I-$prime hyperideal and $n-$absorbing $I-$prime hyperideal. We study some properties of such two concepts and we see that they have analogous properties of prime ideals. During the study, we found out similar concepts that one can think about like $2 - I-$primal hyperideals, $2 - I-$primal hypersubmodules and $2 - I-$prime hypersubmodules.

Questions Readers can think about the following subjects:

(1) $2-I-$primal hyperideals
(2) $2-I-$primal hypersubmodules
(3) $2-I-$prime hypersubmodules

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n-ABSORBING I-PRIME HYPERIDEALS IN MULTIPLICATIVE HYPERRINGS

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در این مقاله، ابتدا هایپر ایده‌آل I-اول در هایپر حلقه‌ی ضربی R را تعریف می‌کنیم. ایده‌آل سره‌ی P از R را یک هایپر ایده‌آل I-اول می‌نامیم هرگاه $ab \subseteq P - IP$ نتیجه دهد $a \in P$ یا $b \in P$. سپس مشخصه‌سازی‌هایی از هایپر ایده‌آل‌های I-اول بیان می‌کنیم. همچنین مفاهیم ایده‌آل‌های I-اول ۲-جذب کننده و I-اول n-جذب کننده در هایپر حلقه‌های ضربی به عنوان تعمیمی از ایده‌آل‌های اول را معرفی و مورد بررسی قرار می‌دهیم. یک هایپر ایده‌آل سره‌ی P از هایپر حلقه‌ی ضربی R را هایپر ایده‌آل I-اول n-جذب کننده می‌نامیم هرگاه برای هر $x_1, \dots, x_{n+1} \in R$، اگر

$$x_1 \cdots x_{n+1} \subseteq P - IP,$$

آن‌گاه $x_1 \cdots x_{i-1}x_{i+1} \cdots x_{n+1} \subseteq P$ برای برخی $i \in \{1, \dots, n+1\}$. نشان می‌دهیم که اگر P یک هایپر ایده‌آل I-اول از هایپر حلقه‌ی R باشد، آن‌گاه تحت شرایط خاصی روی I، P/J، $S^{-1}P$، $f(P)$، $f^{-1}(P)$، $\sqrt{P}$ و $P[x]$ نیز هایپر ایده‌آل I-اول هستند که J نیز یک هایپر ایده‌آل مشمول در P است. همچنین در هایپر حلقه‌های تجزیه‌پذیر، یک مشخصه‌سازی از هایپر ایده‌آل‌های I-اول بیان می‌کنیم. به‌علاوه، نشان می‌دهیم اگر یک هایپر حلقه دارای تعداد متناهی هایپر ایده‌آل ماکسیمال باشد و هر هایپر ایده‌آل سره I-اول n-جذب کننده باشد، آن‌گاه R حاصل‌ضرب تعداد متناهی میدان است.

کلمات کلیدی: هایپر حلقه، هایپر حلقه‌ی ضربی، هایپر ایده‌آل اول، هایپر ایده‌آل I-اول.