Journal of Algebraic Systems, Vol. 13(No. 1): (2025), pp 105-134.
https://doi.org/10.22044/JAS.2023.12745.1688

APPLICATIONS OF ROUGH SOFT TO EXTENSIONS SEMIHYPERGROUPS INDUCED BY OPERATORS AND CORRESPONDING DECISION-MAKING METHODS

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Abstract. In this paper, we apply a rough soft set to a special algebraic hyperstructure, and give the concept of a rough soft semihypergroup. We propose the notion of lower and upper approximations concerning a special semihypergroup and obtain some properties. Moreover, we consider a connection between the lower(upper) approximation of a special semihypergroup and the lower(upper) approximation of the associated Γ-hypergroupoid. In the last section of this research, we discuss the decision-making algorithm of rough soft semihypergroups. Afterward, we obtain a relation between the decision-making algorithm of rough soft semihypergroups and their associated rough soft Γ-hypergroupoids for a special semihypergroup.

1. Introduction

A semigroup is an algebraic structure consisting of a non-empty set equipped with an associative binary operation in the set. Semigroup plays an essential role in some areas of mathematics, including coding theory, combinatorics, and mathematical analysis. In 1986, Sen and Saha [37] defined an idea Γ-semigroup as a generalization of a semigroup. Some classical ideas from semigroup have developed in Γ-semigroups such as communicatively, regularity and ideal [40, 19, 4].

The concept of algebraic hyperstructure theory was introduced in 1934 by Marty [25]. The hyperstructure theory has many applications in hypergraphs and graphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, codes, artificial intelligence, and probabilities [5, 6]. The concept of Γ-semihypergroup as a generalization of semigroup, semihypergroup, and Γ-semigroup was introduced by Davvaz et al. [7, 16, 17, 18]. Moreover, Anvariyeh et al. introduced Pawlak approximations in Γ-semihypergroups [3]. After that, Davvaz et al. presented the concept of Γ-semihyperring and investigated some properties of Γ-semihyperring. [9, 10, 11] At the same time,

Published online: 1 April 2024
MSC(2010): Primary: 65F05; Secondary: 46L05, 11Y50.
Keywords: Γ-semihypergroup; Regular relation; Pawlak space; Rough soft set; Decision-making.
Received: 18 February 2023, Accepted: 24 June 2023.
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Dehkordi and Heidari introduced the T-functor, Θ-relation, and Fundamental group for general Γ-hypergroups [31]. The notion of a Γ-hyperideal of a Γ-semihypergroup is introduced by Anvariyeh [2].

The rough set theory was introduced by Pawlak [33] and he used this concept as a method for data mining. Many researchers and practitioners in various fields of science and technology work on this concept and there are many applications in various areas such as knowledge discovery, economics, finance, engineering, and even arts and culture [21, 27, 26, 34, 35, 36, 38, 39, 41, 42]. Davvaz et al. generalized rough sets for modules and quotient hypermodules base on fuzzy sets [8, 20]. There are many problems in real life like engineering, medical science, economics, environments, etc. have various uncertainties. Some kinds of theories are given such as the theory of fuzzy sets [42] and the theory of rough sets [33]. The concept of soft set theory was introduced by Molodsov [28] as a mathematical tool for handling uncertainties. He applied this theory in several directions [28, 29, 30]. Park et al. studied soft sets in algebra[32]. Also, Feng et al. in 2008 studied soft semirings [13]. The application of soft set in the decision-making problem was considered by Maji et al. [24, 23]. Also, Ma et al. [22] introduced and considered decision-making algorithm of rough soft sets to Krasner (m, n)-hyperrings. The concept of soft groups was introduced and studied by Aktas et al. [1]. Zhan et al. presented parameter reduction of soft sets and corresponding decision-making algorithms [44]. Feng et al. [15] studied an interesting connection between two mathematical approaches to vagueness: Rough sets and soft sets, called soft approximation spaces and soft rough sets. Furthermore, Zhan et al. introduced rough soft n-semigroup, covering-based soft rough set, covering-based soft fuzzy rough set, and fuzzy soft β-covering based fuzzy rough set and presented decision-making algorithms of them [43, 50, 46, 47, 49]. These algorithms are useful for solving multiple criteria decision-making (MCDM) problems. In recent days, Zhan et al. have studied some applications of multiple attribute decision-making (MADM) algorithms [12, 45, 48, 51].

This paper is structured as follows. In the second section, we define a new set and we introduce the semihypergroup associated with this new hyperstructure obtained from a union of two Γ-semihypergroups which don't meet each other and we study relations between them. In the third section, we define rough semihypergroups, soft semihypergroups, and prove that there is a relation between them and associated rough and soft Γ-hypergroupoids. Also, we define rough soft semihypergroups by using a regular relation and showing that there are relations between rough soft semihypergroups and their associated rough soft Γ-hypergroupoids. Finally, in the last section, we present the decision-making algorithm of rough soft semihypergroups, then we find a relation between the decision-making algorithm of this rough soft semihypergroup and its associated rough soft Γ-hypergroupoid.

2. $G_1[G_2]$ AND ASSOCIATED SEMIHYPERGROUP

In this section, we introduce the semihypergroup associated with $G_1[G_2]$, where G_1 and G_2 are Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$ and study relations between them. First, we recall some notions and results about hyperstructures that we shall use in the following paragraphs.

Let G be a non-empty set and $\mathcal{P}^*(G)$ be the set of all non-empty subsets of G. A map $\circ : G \times G \longrightarrow \mathcal{P}^*(G)$ is called hyperoperation on G and the couple $(G, \circ)$ is called hypergroupoid. When $(x, y) \in G^2$ then its image under $\circ$ is denoted by $x \circ y$. Let A and B be non-empty subsets of hypergroupoid G. Then, $A \circ B$ is given by $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$. Also, $x \circ A$ is used for $\{x\} \circ A$. A hypergroupoid $(G, \circ)$ is called semihypergroup if hyperoperation $\circ$ is associative $(x \circ y = y \circ x, \forall x, y \in G)$ and a semihypergroup is hypergroup if for all $x \in G$, $G = x \circ G = G \circ x$.

Definition 2.1. Let H and Γ be non-empty sets and for every α of Γ there are hyperoperations $\otimes_\alpha : H \times H \longrightarrow \mathcal{P}^*(H)$. Then, we say that H is a Γ-*semihypergroup*, when

$$(x \otimes_\alpha y) \otimes_\beta z = x \otimes_\alpha (y \otimes_\beta z),$$

for every α and β of Γ and $x, y, z \in H$. A Γ-semihypergroup H is called Γ-*hypergroup*, when

$$x \otimes_\alpha H = H \otimes_\alpha x = H,$$

for every α of Γ and $x \in H$

If for every $\alpha \in \Gamma$, there exists $e_\alpha \in H$, such that

$$x \otimes_\alpha e_\alpha = e_\alpha \otimes_\alpha x = x,$$

for every $x \in H$, then H is called Γ-semihypergroup with a *unit*. Let A be a non-empty subset of Γ-semihypergroup H. Therefore A is called a left (right) Γ-hyperideal if for all $h \in H$, $h \otimes_\Gamma A \subseteq A (A \otimes_\Gamma h \subseteq A)$. Also, A is a Γ-hyperideal of H if it is both a right and a left Γ-hyperideal.

Example 2.2. Let $(G, \circ)$ be a group and $\Gamma = \{\alpha, \beta\}$. Then, we define $x \otimes_\alpha y = G$ and $x \otimes_\beta y = x \circ y$. Thus, G is a Γ-semihypergroup.

Example 2.3. Let $A = \{A_g\}_{g\in G}$ be a collection of disjoint and non-empty sets such that $(G, \circ)$ be a semigroup and Γ be a non-empty subset of G. Then, $T = \bigcup_{g\in G} A_g$ is a Γ-semihypergroup by the following hyperoperation:

$$x_1 \otimes_\alpha x_2 = A_g,$$

where $x_1 \in A_{g_1}, x_2 \in A_{g_2}, g = g_1\alpha g_2$, $\alpha \in \Gamma$ and $x_1, x_2 \in G$. We denote this Γ-semihypergroup by T_{AG}.

Example 2.4. Let $G = \{a, b\}$ and $\Gamma = \{\alpha, \beta\}$. Then, G is a Γ-semihypergroup such that hyperoperations are defined in Tables 1 and 2. Also, G is a

TABLE 1. Hyperoperation $\otimes_\alpha$ on $G = \{a, b\}$

$\otimes_\alpha$	a	b
a	a	b
b	b	a

TABLE 2. Hyperoperation $\otimes_\beta$ on $G = \{a, b\}$

$\otimes_\beta$	a	b
a	b	a
b	a	b

Γ-semihypergroup with unit and $e_\alpha = a$ and $e_\beta = b$, because for every $z \in G$, $a \otimes_\alpha z = z \otimes_\alpha a = z$ and $b \otimes_\beta z = z \otimes_\beta b = z$.

Let G be a Γ-semihypergroup and the relation ρ defined on

$$G \times \Gamma = \{(x, \alpha) : x \in H, \alpha \in \Gamma\},$$

as follows:

$$(x, \alpha)\rho(y, \beta) \iff \forall z \in G,\ x \otimes_\alpha z = y \otimes_\beta z.$$

Thus, ρ is an equivalence relation and the set

$$\widehat{G} = \{[(x, \alpha)]_\rho : x \in H, \alpha \in \Gamma\}$$

is a semihypergroup by following hyperoperation:

$$[(x, \alpha)]_\rho \circ [(y, \beta)]_\rho = \{[(z, \beta)]_\rho : z \in x \otimes_\alpha y\}.$$

Proposition 2.5. *Let $(G_1, \oplus_\alpha)$ and $(G_2, \otimes_\alpha)$ be Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$. Then, we define a hyperoperation on a new system*

$G_1[G_2] = G_1 \cup G_2$ *as follows:*

$$x \odot_\alpha y = \begin{cases} x \oplus_\alpha y & x, y \in G_1, \\ x & x \in G_2, y \in G_1, \\ y & x \in G_1, y \in G_2, \\ x \otimes_\alpha y & x, y \in G_2, \end{cases}$$

for all $x, y \in G_1[G_2]$ *and* $\alpha \in \Gamma$. *We define the relation* ρ *on* $G_1[G_2] \times \Gamma$ *as follows:*

$$(x, \alpha)\rho(y, \beta) \Longleftrightarrow \forall z \in G_1[G_2],\ x \underset{\alpha}{\odot} z = y \underset{\beta}{\odot} z.$$

Hence ρ *is an equivalence relation and*

$$\widehat{G_1[G_2]} = \{[(x, \alpha)]_\rho : x \in G_1[G_2],\ \alpha \in \Gamma\} = \widehat{G_1} \cup \widehat{G_2}$$

is a semihypergroup by the following hyperoperation. Furthermore, $\widehat{G_1[G_2]}$ *is called an extension of semihypergroup by semihypergroup.*

$$[(x_1, \alpha_1)]_\rho \circledcirc [(x_2, \alpha_2)]_\rho = \begin{cases} [(x_1, \alpha_1)]_\rho \circ [(x_2, \alpha_2)]_\rho & [(x_1, \alpha_1)]_\rho, [(x_2, \alpha_2)]_\rho \in \widehat{G_1}, \\ [(x_2, \alpha_2)]_\rho & [(x_2, \alpha_2)]_\rho \in \widehat{G_2}, [(x_1, \alpha_1)]_\rho \in \widehat{G_1}, \\ [(x_1, \alpha_1)]_\rho & [(x_1, \alpha_1)]_\rho \in \widehat{G_2}, [(x_2, \alpha_2)]_\rho \in \widehat{G_1}, \\ [(x_1, \alpha_1)]_\rho \circ [(x_2, \alpha_2)]_\rho & [(x_1, \alpha_1)]_\rho, [(x_2, \alpha_2)]_\rho \in \widehat{G_2}. \end{cases}$$

Proof. The proof is straightforward. □

Example 2.6. Let $G_1 = \{a, b\}$ be a Γ-semihypergroup with unit such that $\Gamma = \{\alpha, \beta\}$, $e_\alpha = a, e_\beta = b$ and the hyperoperations are defined in Tables 3 and 4 and $G_2 = \{c, d, e\}$ is a Γ-semihypergroup with unit, where hyperoperations $\otimes_\alpha$ and $\otimes_\beta$ are given in Tables 5 and 6 and $e_\alpha = c, e_\beta = d$. Then, the hyperoperations $\odot_\alpha$ and $\odot_\beta$ are defifnded on $G_1[G_2]$ in Tables 7 and 8. But

TABLE 3. Hyperoperation α on $G_1 = \{a, b\}$

$\oplus_\alpha$	a	b
a	a	b
b	b	a

TABLE 4. Hyperoperation β on $G_1 = \{a, b\}$

$\oplus_\beta$	a	b
a	b	a
b	a	b

TABLE 5. Hyperoperation α on $G_2 = \{c, d, e\}$

$\otimes_\alpha$	c	d	e
c	c	{d,e}	{d,e}
d	{d,e}	c	c
e	{d,e}	c	c

TABLE 6. Hyperoperation β on $G_2 = \{c, d, e\}$

$\otimes_\beta$	c	d	e
c	{d,e}	c	c
d	c	{d,e}	{d,e}
e	c	{d,e}	{d,e}

TABLE 7. Hyperoperation $\odot_\alpha$ on $G_1[G_2]$

$\odot_\alpha$	a	b	c	d	e
a	a	b	c	d	e
b	b	a	c	d	e
c	c	c	c	{d,e}	{d,e}
d	d	d	{d,e}	c	c
e	e	{d,e}	c	c	c

TABLE 8. Hyperoperation $\odot_\beta$ on $G_1[G_2]$

$\odot_\beta$	a	b	c	d	e
a	b	a	c	d	e
b	a	b	c	d	e
c	c	c	{d,e}	c	c
d	d	d	c	{d,e}	{d,e}
e	e	e	c	{d,e}	{d,e}

TABLE 9. Hyperoperation $\oplus_\alpha$ on $G_1 = \{a, b, c\}$

$\oplus_\alpha$	a	b	c
a	a	{b,c}	{b,c}
b	{b,c}	a	a
c	{b,c}	a	a

TABLE 10. Hyperoperation $\oplus_\beta$ on $G_1 = \{a, b, c\}$

$\oplus_\beta$	a	b	c
a	{b,c}	a	a
b	a	{b,c}	{b,c}
c	a	{b,c}	{b,c}

TABLE 11. Hyperoperation $\otimes_\alpha$ on $G_2 = \{d, e, f\}$

$\otimes_\alpha$	d	e	f
d	d	{e,f}	{e,f}
e	{e,f}	d	d
f	{e,f}	d	d

TABLE 12. Hyperoperation $\otimes_\beta$ on $G_2 = \{d, e, f\}$

$\otimes_\beta$	d	e	f
d	{e,f}	d	d
e	d	{e,f}	{e,f}
f	d	{e,f}	{e,f}

$G_1[G_2]$ is not Γ-semihypergroup, since

$$(c \underset{\alpha}{\odot} b) \underset{\beta}{\odot} e = c \underset{\beta}{\odot} e = c,\ c \underset{\alpha}{\odot} (b \underset{\beta}{\odot} e) = c \underset{\alpha}{\odot} e = \{d, e\},$$

$$(c \underset{\alpha}{\odot} b) \underset{\beta}{\odot} e \neq c \underset{\alpha}{\odot} (b \underset{\beta}{\odot} e).$$

$\widehat{G_1}$ and $\widehat{G_2}$ are semihypergroups, Also, $(\widehat{G_1}[\widehat{G_2}], \circledcirc)$ is a semihypergroup.

Example 2.7. If $G_1 = \{a, b, c\}$ and $G_2 = \{d, e, f\}$ be Γ-semi- hypergroups with unit, where $\Gamma = \{\alpha, \beta\}$ and the hyperoperation are defined in Tables 9, 10, 11 and 12. Then, by Proposition 2.5, $\widehat{G}_1[\widehat{G}_2]$ is a semihypergroup.

Example 2.8. Let $A = \{A_g\}_{g\in G}$ and $B = \{B_k\}_{k\in K}$ be collections of disjoint and non-empty sets such that $(G, \circ)$ and $(K, \circ)$ are semigroups, $G \cap K = \emptyset$ and $\Gamma, \Gamma^{'}$ be non-empty subsets of G and K, respectively. Then, $H_1 = \bigcup\limits_{g\in G} A_g$ and $H_2 = \bigcup\limits_{k\in K} B_k$ are Γ-semihypergroup and $\Gamma^{'}$-semihypergroup, respectively, by the following hyperoperations:

$$x_1, x_2 \in H_1, \alpha \in \Gamma, x_1 \otimes_\alpha x_2 = A_g : x_1 \in A_{g_1}, x_2 \in A_{g_2}, g = g_1 \circ \alpha \circ g_2.$$

$$x_1, x_2 \in H_2, \alpha^{'} \in \Gamma^{'}, x_1 \underset{\alpha^{'}}{\otimes} x_2 = B_k : x_1 \in B_{k_1}, x_2 \in B_{k_2}, k = k_1 \circ \alpha^{'} \circ k_2.$$

Note that H_1 and H_2 do not have units. From $G \cap K = \emptyset$, we have $\Gamma \cap \Gamma^{'} = \emptyset$, therefore $\Gamma[\Gamma^{'}] = \Gamma \cup \Gamma^{'}$. Also, we conclude that $A_g \cap B_k = \emptyset$. This implies that $H_1 \cap H_2 = \emptyset$. So, $\widehat{H_1} \cap \widehat{H_2} = \emptyset$. Then, $\widehat{H_1[H_2]}$ is a semihypergroup but $H_1[H_2]$ is not $\Gamma[\Gamma^{'}]$-semihypergroup.

Proposition 2.9. *Let* G_1 *and* G_2 *be* Γ*-semihypergroups with unit such that* $G_1 \cap G_2 = \emptyset$*. Then,* $\widehat{G_1[G_2]} = \widehat{G_1}[\widehat{G_2}]$.

Proof. Suppose that $[(x,\alpha)]_\rho \in \widehat{G_1[G_2]}$. Hence, there exist $y \in G_1[G_2]$ and $\beta \in \Gamma$ such that $[(x,\alpha)]_\rho = [(y,\beta)]_\rho$. Thus $y \in G_1$ or $y \in G_2$. Then, $[(y,\beta)]_\rho \in \widehat{G_1}$ or $[(y,\beta)]_\rho \in \widehat{G_2}$. So, $[(x,\alpha)]_\rho = [(y,\beta)]_\rho \in \widehat{G_1} \cup \widehat{G_2}$ and $\widehat{G_1} \cup \widehat{G_2} = \widehat{G_1}[\widehat{G_2}]$, Also, $\widehat{G_1} \cap \widehat{G_2} = \emptyset$. Since, $G_1 \cap G_2 = \emptyset$. We obtain $\widehat{G_1[G_2]} \subseteq \widehat{G_1}[\widehat{G_2}]$.

Now, let $[(x,\alpha)]_\rho \in \widehat{G_1}[\widehat{G_2}]$. Thus,

$$[(x,\alpha)]_\rho \in \widehat{G_1} \text{ or } [(x,\alpha)]_\rho \in \widehat{G_2}.$$

Then, there exist $t_1 \in G_1, \beta_1 \in \Gamma$ or $t_2 \in G_2, \beta_2 \in \Gamma$ such that $[(x,\alpha)]_\rho = [(t_1,\beta_1)]_\rho$ or $[(x,\alpha)]_\rho = [(t_2,\beta_2)]_\rho$. We conclude that for every $z \in G_1, x \otimes_\alpha z_1 = t_1 \underset{\beta_1}{\otimes} z_1$ and for every $z_2 \in G_2, x \otimes_\alpha z_2 = t_2 \underset{\beta_2}{\otimes} z_2$. We set $z_1 = e_\alpha, z_2 = e_\alpha$. Then, $x \in t_1 \underset{\beta_1}{\otimes} e_\alpha \subseteq G_1$ and $x \in t_2 \underset{\beta_2}{\otimes} e_\alpha \subseteq G_2$. So, $x \in G_1$ or $x \in G_2$. We obtain $x \in G_1[G_2]$ and $\alpha \in \Gamma$. This means that $[(x,\alpha)]_\rho \in \widehat{G_1[G_2]}$. Therefore, $\widehat{G_1}[\widehat{G_2}] \subseteq \widehat{G_1[G_2]}$. □

Definition 2.10. Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$, $A \subseteq G_1[G_2]$ and $B \subseteq \widehat{G}_1[\widehat{G}_2]$. We define

$$\widehat{A} = \{[(x,\alpha)]_\rho : x \in A, \alpha \in \Gamma\},$$

$$B^{'} = \{x \in G_1[G_2] : \exists \alpha \in \Gamma : [(x,\alpha)]_\rho \in B\}.$$

Proposition 2.11. *Let* G_1 *and* G_2 *be* Γ*-semihypergroups with unit such that* $G_1 \cap G_2 = \emptyset$*,* $A, B \in \mathcal{P}^*(G_1[G_2])$*, and* $C \in \mathcal{P}^*(\widehat{G}_1[\widehat{G}_2])$*. Then, the following statements hold:*

1)$C \subseteq \widehat{(C')}$.

And if A *and* B *are* Γ*-hyperideals of* $G_1[G_2]$*, then*

2)$\widehat{A \cap B} = \widehat{A} \cap \widehat{B}$,

3)$\widehat{A \cup B} = \widehat{A} \cup \widehat{B}$,

4)$(\widehat{A})^{'} = A$,

5)$\widehat{A \underset{\Gamma}{\otimes} B} = \widehat{A} \circ \widehat{B}$.

Proof. The proof is straightforward. □

In the Proposition 2.11 (1), in general C is not equal to $\widehat{(C')}$.

Example 2.12. In Example 2.6, let $C = \{[(a,\alpha)]_\rho, [(c,\beta)]_\rho\} \subseteq \widehat{G_1}[\widehat{G_2}]$. Then, $C^{'} = \{a, c\}$. Therefore,

$$\widehat{(C')} = \{[(a,\alpha)]_\rho, [(c,\alpha)]_\rho, [(a,\beta)]_\rho, [(c,\beta)]_\rho\}.$$

We obtain $C \subseteq \widehat{(C')}$ but $C \neq \widehat{(C')}$, because $\widehat{(C')} \nsubseteq C$.

Proposition 2.13. *Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$ and $A, B \in \mathcal{P}^*(\widehat{G}_1[\widehat{G}_2])$ such that $A \cap B \neq \emptyset$, and $C \in \mathcal{P}^*(G_1[G_2])$ be a Γ-hyperideal of $G_1[G_2]$. Then, the following statements hold:*
1)$(\widehat{C})^{'} = C$;
2)$(A \cap B)^{'} \subseteq A^{'} \cap B^{'}$;
3)$(A \cup B)^{'} = A^{'} \cup B^{'}$;
4)$A \subseteq \widehat{(A')}$.

Proof. The proof is straightforward. □

Example 2.14. In Example 2.6, we have

$$\begin{aligned}\widehat{G_1}[\widehat{G_2}] =& \widehat{G_1} \cup \widehat{G_2}\\ =& \{[(a,\alpha)]_\rho, [(a,\beta)]_\rho, [(b,\alpha)]_\rho, [(b,\beta)]_\rho,\\ & [(c,\alpha)]_\rho, [(c,\beta)]_\rho, [(d,\alpha)]_\rho, [(d,\beta)]_\rho, [(e,\alpha)]_\rho, [(e,\beta)]_\rho\}.\end{aligned}$$

Let $A = \{[(a,\alpha)]_\rho, [(b,\beta)]_\rho\}$ and $B = \{[(a,\alpha)]_\rho, [(b,\alpha)]_\rho\}$. Therefore, $A^{'} = \{a, b\}$ and $B^{'} = \{a, b\}$. So, $A^{'} \cap B^{'} = \{a, b\}$. Moreover, we have

$$A \cap B = \{[(a,\alpha)]_\rho\}.$$

We obtain $(A \cap B)^{'} = \{a\}$. We conclude that $A^{'} \cap B^{'} \nsubseteq (A \cap B)^{'}$.
Moreover, $A^{'} = \{a, b\}$. We have

$$\widehat{(A')} = \{[(a,\alpha)]_\rho, [(a,\beta)]_\rho, [(b,\alpha)]_\rho, [(b,\beta)]_\rho\}.$$

So, $A \subseteq \widehat{(A')}$ but $\widehat{(A')} \nsubseteq A$.

Proposition 2.15. *Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$ A, B are nonempty subsets of $G_1[G_2]$ such that B is a right Γ-hyperideal of $G_1[G_2]$. Then,*

$$A \subseteq B \Longleftrightarrow \widehat{A} \subseteq \widehat{B}.$$

Proof. Assume that $[(x,\alpha)]_\rho \in \widehat{A}$, then there exist $y \in A$ and $\beta \in \Gamma$ such that $[(x,\alpha)]_\rho = [(y,\beta)]_\rho$. Also, we have $y \in A \subseteq B$. So, $y \in B$ and $[(y,\beta)]_\rho \in \widehat{B}$. We conclude that $[(x,\alpha)]_\rho \in \widehat{B}$ and this means that $\widehat{A} \subseteq \widehat{B}$.

Conversely, let $x \in A$. Then, for every $\alpha \in \Gamma$, $[(x,\alpha)]_\rho \in \widehat{A} \subseteq \widehat{B}$. This implies that $[(x,\alpha)]_\rho \in \widehat{B}$. So, there exist $y \in B$ and $\beta \in \Gamma$ such that $[(x,\alpha)]_\rho = [(y,\beta)]_\rho$. Hence, for every z of $G_1[G_2]$, $x \otimes_\alpha z = y \underset{\beta}{\otimes} z$ and for every $z \in G_1$ or $z \in G_2$, $x \otimes_\alpha z = y \underset{\beta}{\otimes} z$. If $z \in G_1$, we set $z = e_\alpha$ (the unit element corresponding α of G_1), then

$$x = x \otimes_\alpha e_\alpha \in y \underset{\beta}{\otimes} e_\alpha \subseteq B \underset{\Gamma}{\otimes} G_1 \subseteq B \underset{\Gamma}{\otimes} G_1[G_2] \subseteq B.$$

If $z \in G_2$, in the same way, we obtain $x \in B$. We conclude that $A \subseteq B$. $\square$

Proposition 2.16. *Let G_1 and G_2 are Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$ and A, B be nonempty subsets of $\widehat{G_1}[\widehat{G_2}]$. Then,*

$$A \subseteq B \Longrightarrow A^{'} \subseteq B^{'}.$$

Proof. The proof is straightforward. $\square$

Definition 2.17. Let G_1, G_2, H_1, and H_2 be Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$ and $H_1 \cap H_2 = \emptyset$. Then, a map

$$\Phi : \widehat{G_1}[\widehat{G_2}] \longrightarrow \widehat{H_1}[\widehat{H_2}]$$

is called a *homomorphism*, if for all $[(x_1,\alpha)]_\rho, [(x_2,\beta)]_\rho \in \widehat{G_1}[\widehat{G_2}]$,

$$\Phi([(x_1,\alpha)]_\rho \circ [(x_2,\beta)]_\rho) = \Phi([(x_1,\alpha)]_\rho) \circ \Phi([(x_2,\beta)]_\rho).$$

Definition 2.18. Let G_1, G_2, H_1, and H_2 be Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$ and $H_1 \cap H_2 = \emptyset$. Then, we define a hyperoperation $\underset{(\alpha_1,\alpha_2)}{\otimes}$ on $G_1[G_2] \times H_1[H_2]$ as follows:

$$\underset{(\alpha_1,\alpha_2)}{\otimes} : (G_1[G_2] \times H_1[H_2]) \times (G_1[G_2] \times H_1[H_2]) \longrightarrow \mathcal{P}^*(G_1[G_2] \times H_1[H_2],)$$

$$(x_1,y_1) \underset{(\alpha_1,\alpha_2)}{\otimes} (x_2,y_2) = \{(t_1,t_2) : t_1 \in x_1 \underset{\alpha_1}{\otimes} x_2,\ t_2 \in y_1 \underset{\alpha_2}{\otimes} y_2\},$$

where

$$\underset{\alpha_1}{\otimes} : G_1[G_2] \times G_1[G_2] \longrightarrow \mathcal{P}^*(G_1[G_2]),$$

and

$$\underset{\alpha_2}{\otimes} : H_1[H_2] \times H_1[H_2] \longrightarrow \mathcal{P}^*(H_1[H_2]).$$

Also, we define

$$\begin{aligned} \widehat{G_1[G_2] \times H_1[H_2]} = \{&[((x_1,x_2),(\alpha_1,\alpha_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}}. \\ &: (x_1,x_2) \in G_1[G_2] \times H_1[H_2], \alpha_1,\alpha_2 \in \Gamma\}. \end{aligned}$$

Theorem 2.19. *Let $G_1, G_2, H_1,$ and H_2 be Γ-semimihypergroups s with unit such that $G_1 \cap G_2 = \emptyset$ and $H_1 \cap H_2 = \emptyset$. Then,*

$$(\widehat{G_1[G_2] \times H_1[H_2]}) \cong \widehat{G_1}[\widehat{G_2}] \times \widehat{H_1}[\widehat{H_2}].$$

Proof. Suppose that $\rho_{G_1[G_2]\times H_1[H_2]}$, $\rho_{G_1[G_2]}$, and $\rho_{H_1[H_2]}$ are equivalence relations defined on $G_1[G_2] \times H_1[H_2], G_1[G_2]$, and $H_1[H_2]$, respectively. Let

$$\Psi : (\widehat{G_1[G_2] \times H_1[H_2]}) \longrightarrow \widehat{G_1}[\widehat{G_2}] \times \widehat{H_1}[\widehat{H_2}]$$

be defined as follows:

$$\begin{aligned}&\Psi([((x_1,x_2),(\alpha_1,\alpha_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}})\\ &= ([(x_1,\alpha_1)]_{\rho_{G_1[G_2]}}, [(x_2,\alpha_2)]_{\rho_{H_1[H_2]}}).\end{aligned}$$

Hence,

$$[((x_1,x_2),(\alpha_1,\alpha_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}} = [((y_1,y_2),(\beta_1,\beta_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}},$$

if and only if for every $(z_1, z_2) \in G_1[G_2] \times H_1[H_2]$,

$$(x_1,x_2) \underset{(\alpha_1,\alpha_2)}{\otimes} (z_1,z_2) = (y_1,y_2) \underset{(\beta_1,\beta_2)}{\otimes} (z_1,z_2),$$

if and only if for every $z_1 \in G_1[G_2]$ and $z_2 \in H_1[H_2]$,

$$x_1 \underset{\alpha_1}{\otimes} z_1 = y_1 \underset{\beta_1}{\otimes} z_1, \qquad x_2 \underset{\alpha_2}{\otimes} z_2 = y_2 \underset{\beta_2}{\otimes} z_2,$$

if and only if

$$[(x_1,\alpha_1)]_{\rho_{G_1[G_2]}} = [(y_1,\beta_1)]_{\rho_{G_1[G_2]}}, [(x_2,\alpha_2)]_{\rho_{H_1[H_2]}} = [(y_2,\beta_2)]_{\rho_{H_1[H_2]}}.$$

Hence, Ψ is well-defined and one-to-one.
If $([(x_1,\alpha_1)]_{\rho_{G_1[G_2]}}, [(x_2,\alpha_2)]_{\rho_{H_1[H_2]}}) \in \widehat{G_1}[\widehat{G_2}] \times \widehat{H_1}[\widehat{H_2}]$, then

$$[(x_1,\alpha_1)]_{\rho_{G_1[G_2]}} \in \widehat{G_1}[\widehat{G_2}], \qquad [(x_2,\alpha_2)]_{\rho_{H_1[H_2]}} \in \widehat{H_1}[\widehat{H_2}].$$

Hence,

$$[(x_1,\alpha_1)]_{\rho_{G_1[G_2]}} \in \widehat{G_1} \text{ or } [(x_1,\alpha_1)]_{\rho_{G_1[G_2]}} \in \widehat{G_2}$$

and we have $[(x_2,\alpha_2)]_{\rho_{H_1[H_2]}} \in \widehat{H_1}$ or $[(x_2,\alpha_2)]_{\rho_{H_1[H_2]}} \in \widehat{H_2}$. Also, $x_1 \in G_1[G_2]$ and $x_2 \in H_1[H_2]$, because $G_1, G_2, H_1,$ and H_2 are Γ-semihypergroup with the unit. This implies that $(x_1, x_2) \in G_1[G_2] \times H_1[H_2]$ and $[(x_1,x_2),(\alpha_1,\alpha_2)]_{\rho_{G_1[G_2]\times H_1[H_2]}} \in \widehat{G_1[G_2] \times H_1[H_2]}$. So, Ψ is onto. Now, we show that Ψ is homomorphism of semihypergroups:

$$\Psi([((x_1,x_2),(\alpha_1,\alpha_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}} \circ [((y_1,y_2),(\beta_1,\beta_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}})$$

$$= \Psi([((x_1, x_2) \underset{(\alpha_1,\alpha_2}{\otimes} (y_1, y_2), (\beta_1, \beta_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}})$$
$$= \Psi([((x_1 \underset{\alpha_1}{y_1}, x_2 \underset{\alpha_2}{\otimes} y_2), (\beta_1, \beta_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}})$$
$$= ([(x_1 \underset{\alpha_1}{\otimes} y_1, \beta_1)]_{\rho_{G_1[G_2]}}, [(x_2 \underset{\alpha_2}{\otimes} y_2, \beta_2)]_{\rho_{H_1[H_2]}})$$
$$= ([(x_1, \alpha_1)]_{\rho_{G_1[G_2]}} \circ [(y_1, \beta_1)]_{\rho_{G_1[G_2]}}, [(x_2, \alpha_2)]_{\rho_{H_1[H_2]}} \circ [(y_2, \beta_2)]_{\rho_{H_1[H_2]}})$$
$$= ([(x_1, \alpha_1)]_{\rho_{G_1[G_2]}}, [(x_2, \alpha_2)]_{\rho_{H_1[H_2]}}) \circ ([(y_1, \beta_1)]_{\rho_{G_1[G_2]}}, [(y_2, \beta_2)]_{\rho_{H_1[H_2]}})$$
$$= \Psi([((x_1, x_2), (\alpha_1, \alpha_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}}) \circ \Psi([((y_1, y_2), (\beta_1, \beta_2))]_{\rho_{G_1[G_2]\times H_1[H_2]}}).$$

This completes the proof. □

3. Relations between rough soft semihypergroups and their associated rough soft sets

In this section first, we define rough semihypergroups and soft semihypergroups and prove a relation between them and associated rough and soft sets. After that we define rough soft semihypergroups by defining a regular relation and we show that there are relations between rough soft semihypergroups and their associated rough soft sets.

Definition 3.1. Suppose that G_1 and G_2 are Γ-semihypergroups where $G_1 \cap G_2 = \emptyset$ and Θ is an equivalence relation on $G_1[G_2]$. Then, we define the relation $\widehat{\Theta}$ on $\widehat{G_1}[\widehat{G_2}]$ as follows:

$$[(x, \alpha)]_\rho \widehat{\Theta} [(y, \beta)]_\rho \Longleftrightarrow (x \otimes_\alpha z) \overline{\Theta} (y \underset{\beta}{\otimes} z) \quad \text{for all } z \in G_1[G_2].$$

Definition 3.2. Let G_1 and G_2 be Γ-semihypergroups, let $G_1 \cap G_2 = \emptyset$ and let Θ be an equivalence relation on $G_1[G_2]$. Then, the upper and lower approximations of $A \subseteq G_1[G_2]$ defined by

$$\overline{Apr}_\Theta(A) = \{x \in G_1[G_2] : [x]_\Theta \cap A \neq \emptyset\},$$
$$\underline{Apr}_\Theta(A) = \{x \in G_1[G_2] : [x]_\Theta \subseteq A\}.$$

Example 3.3. Let $G_1 = \{a, b, c\}$ be a Γ-semihypergroup such that $\Gamma = \{\alpha, \beta\}$. The hyperoperations α and β are defined in Tables 13 and 14. and $G_2 = \{(1\ 2\ 3), (1\ 3\ 2), (1)\}$ (as a subset of the symmetric group of degree 3) be a Γ-semihypergroup such that $\Gamma = \{\cdot\}$, where " $\cdot$ " is the multiplication of S_3 . Then, we define an equivalence relation Θ on

$$G_1[G_2] = \{a, b, c, (1\ 2\ 3), (1\ 3\ 2), (1)\}$$

TABLE 13. Hyperoperation α on $G_1 = \{a, b, c\}$

α	a	b	c
a	a	{b,c}	{b,c}
b	{b,c}	a	a
c	{b,c}	a	a

TABLE 14. Hyperoperation β on $G_1 = \{a, b, c\}$

β	a	b	c
a	{b,c}	a	a
b	a	{b,c}	{b,c}
c	a	{b,c}	{b,c}

as follows:

$$x\Theta y \Longleftrightarrow \text{if } x, y \in G_1, \exists \gamma \in \Gamma : x \in y \underset{\gamma}{\otimes} y,$$
$$\text{if } x, y \in G_2,\ x = y.$$

Thus, we obtain the equivalence classes of $G_1[G_2]$ w.r.t Θ as follows:

$$[a]_\Theta = [b]_\Theta = [c]_\Theta = G_1,$$
$$[(1\ 2\ 3)]_\Theta = \{(1\ 2\ 3)\}, [(1\ 3\ 2)]_\Theta = \{(1\ 3\ 2)\}, [(1)]_\Theta = \{(1)\}.$$

Moreover,

$$\widehat{G_1}[\widehat{G_2}] = \{[(a, \alpha)]_\rho, [(a, \beta)]_\rho, [(b, \alpha)]_\rho, [(b, \beta)]_\rho, [(c, \alpha)]_\rho, [(c, \beta)]_\rho, ((1\ 2\ 3), \cdot)]_\rho, [((1\ 3\ 2), \cdot)]_\rho, [((1), \cdot)]_\rho\},$$

is a semihypergroup. By the Definition 3.1, the equivalence classes of $\widehat{\Theta}$ are as follows:

$$[([(a, \alpha)]_\rho)]_{\widehat{\Theta}} = [([(b, \alpha)]_\rho)]_{\widehat{\Theta}} = [([(c, \alpha)]_\rho)]_{\widehat{\Theta}} = [([(a, \beta)]_\rho)]_{\widehat{\Theta}}$$
$$= [([(b, \beta)]_\rho)]_{\widehat{\Theta}} = [([(c, \beta)]_\rho)]_{\widehat{\Theta}} = \widehat{G_1}.$$
$$[([((1\ 2\ 3), \cdot)]_\rho)]_{\widehat{\Theta}} = \{[((1\ 2\ 3), \cdot)]_\rho\},$$
$$[([((1\ 3\ 2), \cdot)]_\rho)]_{\widehat{\Theta}} = \{[((1\ 3\ 2), \cdot)]_\rho\},$$
$$[([((1), \cdot)]_\rho)]_{\widehat{\Theta}} = \{[((1), \cdot)]_\rho\}.$$

Example 3.4. Let G_1 and G_2 be canonical hypergroups such that $G_1 \cap G_2 = \emptyset$ and $\{X_g\}_{g\in G_1[G_2]}$ be collection of disjoint nonempty sets and $\Gamma_1 \subseteq G_1$, $\Gamma_2 \subseteq G_2$, $H_1 = \bigcup_{g\in G_1} X_g$, and $H_2 = \bigcup_{g\in G_2} X_g$. Then, H_1 and H_2 are

Γ_1-semihypergroup and Γ_2-semihypergroup, respectively by

$$x_1 \odot_\alpha x_2 = X_g : x_1 \in X_{g_1}, x_2 \in X_{g_2},\ g \in g_1 \otimes_{\alpha_1} g_2, \alpha_1 \in \Gamma_1,$$

and

$$x_1 \odot_{\alpha_2} x_2 = X_g : x_1 \in X_{g_1}, x_2 \in X_{g_2},\ g \in g_1 \otimes_{\alpha_2} g_2,\ \alpha_2 \in \Gamma_2.$$

We define an equivalence relation Θ on $H_1[H_2]$ as follows:

$$x_1 \Theta x_2 \Longleftrightarrow \exists 1 \leq i \leq 2,\ g_i \in G_i : \{x_1, x_2\} \subseteq X_{g_i}.$$

For every $A \subseteq H_1[H_2]$, we have

$$\begin{aligned}
\overline{Apr}_\Theta(A) &= \{x \in H_1[H_2] : [x]_\Theta \cap A \neq \emptyset\} \\
&= \{x \in H_1[H_2] : \exists t \in [x]_\Theta \cap A\} \\
&= \{x \in H_1[H_2] : t \in [x]_\Theta, t \in A\} \\
&= \{x \in H_1[H_2] : \exists 1 \leq i \leq 2, g_i \in G_i : \{t, x\} \subseteq X_{g_i}, t \in A\} \\
&= \{x \in H_1[H_2] : \exists 1 \leq i \leq 2, g_i \in G_i : t \in X_{g_i} \cap A\} \\
&= \{x \in H_1[H_2] : \exists 1 \leq i \leq 2, g_i \in G_i : X_{g_i} \cap A \neq \emptyset\},
\end{aligned}$$

$$\begin{aligned}
\underline{Apr}_\Theta(A) &= \{x \in H_1[H_2] : [x]_\Theta \subseteq A\} \\
&= \{x \in H_1[H_2] : \forall t \in [x]_\Theta \Rightarrow t \in A\} \\
&= \{x \in H_1[H_2] : \exists 1 \leq i \leq 2, g_i \in G_i : \{t, x\} \subseteq X_{g_i}, t \in A\} \\
&= \{x \in H_1[H_2] : \exists 1 \leq i \leq 2, g_i \in G_i : t \in X_{g_i} \Rightarrow t \in A\} \\
&= \{x \in H_1[H_2] : \exists 1 \leq i \leq 2, g_i \in G_i : X_{g_i} \subseteq A\}.
\end{aligned}$$

Definition 3.5. Let Θ be an equivalence relation on semihypergroup $\widehat{G_1}[\widehat{G_2}]$. Then, we define the relation Θ' on $G_1[G_2]$ as follows:

$$x \Theta' y \Longleftrightarrow [(x, \Gamma)]_\rho \overline{\Theta} [(y, \Gamma)]_\rho,$$

where $x, y \in G_1[G_2]$.

Example 3.6. Let H_1, H_2 be Γ_1-semihypergroup and Γ_2-semihypergroup, respectively, as defined in Example 3.4. Suppose that $\{X_g\}_{g \in G_1[G_2]}$ is a collection of disjoint hyperideals of $H_1[H_2]$. Hence, $(\widehat{H_1}[\widehat{H_2}], \circledcirc)$ is a semihypergroup. We define an equivalence relation Θ on $\widehat{H_1}[\widehat{H_2}]$ as follows:

$$[(x, \alpha)]_\rho \Theta [(y, \beta)]_\rho \Longleftrightarrow \exists 1 \leq i \leq 2,\ g_i \in G_i : \{[(x, \alpha)]_\rho, [(y, \beta)]_\rho\} \subseteq \widehat{X_{g_i}}.$$

We obtain

$$x \Theta' y \Longleftrightarrow \exists 1 \leq i \leq 2,\ g_i \in G_i : \{x, y\} \subseteq X_{g_i}.$$

Because

$$\begin{aligned}
x\Theta^{'}y &\Leftrightarrow \exists\alpha\in\Gamma:[(x,\alpha)]_\rho\Theta[(y,\alpha)]_\rho\\
&\Leftrightarrow \exists\alpha\in\Gamma,\ 1\le i\le 2, g_i\in G_i:\{[(x,\alpha)]_\rho,\ [(y,\alpha)]_\rho\}\subseteq\widehat{X_{g_i}}\\
&\Leftrightarrow \exists\alpha,\alpha^{'},\ \beta^{'}\in\Gamma,\ 1\le i\le 2, g_i\in G_i,\ x^{'},y^{'}\in X_{g_i}:\\
&\quad [(x,\alpha)]_\rho=[(x^{'},\alpha^{'})]_\rho,[(y,\alpha)]_\rho=[(y^{'},\beta^{'})]_\rho\\
&\Leftrightarrow \exists\alpha,\alpha^{'},\ \beta^{'}\in\Gamma,\ 1\le i\le 2, g_i\in G_i, x^{'},y^{'}\in X_{g_i}, z\in H_1[H_2]:\\
&\quad x\otimes_\alpha z=x^{'}\underset{\alpha^{'}}{\otimes}z,\ y\otimes_\alpha z=y^{'}\underset{\beta^{'}}{\otimes}z\\
&\Leftrightarrow \exists\alpha,\ \alpha^{'},\ \beta^{'}\in\Gamma,\ 1\le i\le 2,\ g_i\in G_i,\ x^{'},\ y^{'}\in X_{g_i}:\\
&\quad x\otimes_\alpha e_\alpha=x^{'}\underset{\alpha^{'}}{\otimes}e_\alpha, y\otimes_\alpha e_\alpha=y^{'}\underset{\beta^{'}}{\otimes}e_\alpha\\
&\Leftrightarrow \exists 1\le i\le 2,\ g_i\in G_i: x\in x^{'}\underset{\alpha^{'}}{\otimes}e_\alpha\subseteq X_{g_i}\underset{\Gamma}{\otimes}H_1[H_2]\subseteq X_{g_i},\\
&\quad y\in y^{'}\underset{\beta^{'}}{\otimes}e_\alpha\subseteq X_{g_i}\underset{\Gamma}{\otimes}H_1[H_2]\subseteq X_{g_i}\\
&\Leftrightarrow \exists 1\le i\le 2,\ g_i\in G_i:\{x,y\}\subseteq X_{g_i}.
\end{aligned}$$

Proposition 3.7. *Let Θ be an regular equivalence relation on $G_1[G_2]$. Then, for every $[(y,\beta)]_\rho\in\widehat{G}_1[\widehat{G_2}]$,*

$$[([(y,\beta)]_\rho)]_{\widehat{\Theta}}\subseteq\widehat{[y]_\Theta}.$$

Proof. The proof is straightforward. □

Proposition 3.8. *Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1\cap G_2=\emptyset$ and Θ be a regular equivalence relation on $G_1[G_2]$, and A be a right Γ-hyperideal of $G_1[G_2]$. Then,*

$$\widehat{\underline{Apr}_\Theta(A)}=\underline{Apr}_{\widehat{\Theta}}(\widehat{A}).$$

Proof. The proof is straightforward. □

Theorem 3.9. *Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1\cap G_2=\emptyset$ and Θ be a regular equivalence relation on $\widehat{G_1}[\widehat{G_2}]$ and A be a right hyperideal of $\widehat{G_1}[\widehat{G_2}]$. Then,*

$$(\underline{Apr}_\Theta(A))^{'}\subseteq\underline{Apr}_{\Theta^{'}}A^{'}.$$

Proof. Suppose that $x \in (\underline{Apr}_{\Theta}A)^{'}$. There exists $\alpha \in \Gamma$ such that $[(x,\alpha)]_{\rho} \in \underline{Apr}_{\Theta}A$. Hence, $[([(x,\alpha)]_{\rho})]_{\Theta} \subseteq A$. By Proposition 2.16, we conclude that $[([(x,\alpha)]_{\rho})]^{'}_{\Theta} \subseteq A^{'}$. Let $y \in [x]_{\Theta^{'}}$. Then, $y\Theta^{'}x$ and $[(y,\Gamma)]_{\rho}\overline{\Theta}[(x,\Gamma)]_{\rho}$. For $\alpha \in \Gamma$, there exists $\beta \in \Gamma$ such that $[(y,\beta)]_{\rho}\Theta[(x,\alpha)]_{\rho}$ and implies that $[(y,\beta)]_{\rho} \in [([(x,\alpha)]_{\rho})]_{\Theta}$. This implies that $y \in [([(x,\alpha)]_{\rho})]^{'}_{\Theta}$. We obtain $[x]_{\Theta^{'}} \subseteq [([(x,\alpha)]_{\rho})]^{'}_{\Theta}$ and we have $[([(x,\alpha)]_{\rho})]^{'}_{\Theta} \subseteq A^{'}$. Hence, $[x]_{\Theta^{'}} \subseteq A^{'}$ and $x \in \underline{Apr}_{\Theta^{'}}(A^{'})$. Therefore, $(\underline{Apr}_{\Theta}(A))^{'} \subseteq \underline{Apr}_{\Theta^{'}}A^{'}$. □

Proposition 3.10. *Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$, and Θ be a regular equivalence relation on $G_1[G_2]$, and A be a Γ-hyperideal of $G_1[G_2]$. Then,*

$$\widehat{\overline{Apr}_{\Theta}(A)} = \overline{Apr}_{\widehat{\Theta}}(\widehat{A}).$$

Proof. The proof is straightforward. □

Theorem 3.11. *Let G_1 and G_2 be Γ-semihypergroups with unit such that $G_1 \cap G_2 = \emptyset$ and Θ be a regular equivalence relation on $\widehat{G_1}[\widehat{G_2}]$, and A be a Γ-hyperideal of $\widehat{G_1}[\widehat{G_2}]$. Then,*

$$(\overline{Apr}_{\Theta}A)^{'} = \overline{Apr}_{\Theta^{'}}A^{'}.$$

Proof. Let $x \in (\overline{Apr}_{\Theta}A)^{'}$. There exists $\alpha \in \Gamma$ where

$$[(x,\alpha)]_{\rho} \in \overline{Apr}_{\Theta}(A).$$

Then, $[([(x,\alpha)]_{\rho})]_{\Theta} \cap A \neq \emptyset$ and there exists $[(y,\beta)]_{\rho} \in [([(x,\alpha)]_{\rho})]_{\Theta} \cap A$. So, $[(y,\beta)]_{\rho}\Theta[(x,\alpha)]_{\rho}$ and $y \in A^{'}$. This implies that $y\Theta^{'}x$. We obtain $y \in [x]_{\Theta^{'}} \cap A^{'}$ which means that $[x]_{\Theta^{'}} \cap A^{'} \neq \emptyset$. Thus, $x \in \overline{Apr}_{\Theta^{'}}A^{'}$. We conclude that

$$(\overline{Apr}_{\Theta}A)^{'} \subseteq \overline{Apr}_{\Theta^{'}}A^{'}.$$

Conversely, let $x \in \overline{Apr}_{\Theta^{'}}A^{'}$. Then, $[x]_{\Theta^{'}} \cap A^{'} \neq \emptyset$. Hence, there exists $y \in [x]_{\Theta^{'}} \cap A^{'}$. Thus, $y \in [x]_{\Theta^{'}}$ and $y \in A^{'}$. There exists $\alpha \in \Gamma$ such that $[(y,\alpha)]_{\rho} \in A$ and we have $[(y,\Gamma)]_{\rho}\overline{\Theta}[(x,\Gamma)]_{\rho}$. For $\alpha \in \Gamma$, there exists $\beta \in \Gamma$ such that $[(y,\alpha)]_{\rho} \in [([(x,\beta)]_{\rho})]_{\Theta}$. We conclude that $[([(x,\beta)]_{\rho})]_{\Theta} \cap A \neq \emptyset$. This means that $[(x,\beta)]_{\rho} \in \overline{Apr}_{\Theta}A$. We obtain $x \in (\overline{Apr}_{\Theta}A)^{'}$ and we have $\overline{Apr}_{\Theta^{'}}A^{'} \subseteq (\overline{Apr}_{\Theta}A)^{'}$. □

Proposition 3.12. *Let G_1 and G_2 be Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$ and Θ be a regular relation on $G_1[G_2]$. Then,*

$$[G_1[G_2] : \Theta] = \{[x]_{\Theta} : x \in G_1[G_2]\}$$

is a Γ*-hypergroupoid by the following hyperoperation:*

$$[x_1]_\Theta \underset{\alpha}{\oplus} [x_2]_\Theta = \{[z]_\Theta : z \in x_1 \otimes_\alpha x_2\}.$$

Proof. The proof is straightforward. □

There is a connection between a regular relation Θ on $\widehat{G_1}[\widehat{G_2}]$ and the regular relation $\Theta^{'}$ on associated Γ-hypergroupoid $G_1[G_2]$ as follows:

Proposition 3.13. *Let* Θ *be a regular equivalence relation on* $G_1[G_2]$*. Then* $[([x_1]_\Theta, \alpha)]_\rho = [([x_2]_\Theta, \beta)]_\rho$ *if and only if* $[(x_1, \alpha)]_\rho \widehat{\Theta} [(x_2, \beta)]_\rho$*.*

Proof. The proof is straightforward. □

Proposition 3.14. *Let* Θ *be a regular relation on semihypergroup* $\widehat{G_1}[\widehat{G_2}]$*. Then,*

$$[([(x,\alpha)]_\rho)]_\Theta = [([(y,\alpha)]_\rho)]_\Theta \Longleftrightarrow [([x]_{\Theta'}, \alpha)]_\rho = [([y]_{\Theta'}, \alpha)]_\rho.$$

Proof. Suppose that $[(x,\alpha)]_\rho, [(x,\beta)]_\rho \in \widehat{G_1}[\widehat{G_2}]$. Hence,

$$\begin{aligned} [([(x,\alpha)]_\rho)]_\Theta = [([(y,\beta)]_\rho)]_\Theta &\Longleftrightarrow [(x,\alpha)]_\rho \Theta [(y,\beta)]_\rho \\ &\Longleftrightarrow x\Theta^{'} y \\ &\Longleftrightarrow [x]_{\Theta'} = [y]_{\Theta'} \\ &\Longleftrightarrow [([x]_{\Theta'}, \alpha)]_\rho = [([y]_{\Theta'}, \beta)]_\rho. \end{aligned}$$

This completes the proof. □

Proposition 3.15. *Let* G_1 *and* G_2 *be* Γ*-semihypergroups such that* $G_1 \cap G_2 = \emptyset$ *and* Θ *be a regular relation on* $G_1[G_2]$*. Then, the relation* $\widehat{\Theta}$ *is a regular relation on* $\widehat{G_1}[\widehat{G_2}]$*.*

Proof. The proof is straightforward.

□

Proposition 3.16. *Let* Θ *be a regular equivalence relation on semihypergroup* $\widehat{G_1}[\widehat{G_2}]$*. Then,* $\Theta^{'}$ *is a regular equivalence relation on* $G_1[G_2]$*.*

Proof. Suppose that $x\Theta^{'}y$ and $z \in G_1[G_2]$. Then, $[(x,\Gamma)]_\rho \overline{\Theta} [(y,\Gamma)]_\rho$. Let $t_1 \in x \otimes_\alpha z$. Then, $[(t_1,\alpha)]_\rho \in [(x,\alpha)]_\rho \circ [(z,\alpha)]_\rho$. From the regularity of Θ, we have

$$[(x,\alpha)]_\rho \circ [(z,\alpha)]_\rho \overline{\Theta} [(y,\alpha)]_\rho \circ [(z,\alpha)]_\rho.$$

There exists $[(t_2,\alpha)]_\rho \in [(y,\alpha)]_\rho \circ [(z,\alpha)]_\rho$ such that $[(t_1,\alpha)]_\rho \Theta [(t_2,\alpha)]_\rho$. We conclude that $t_1\Theta^{'}t_2$. So, we obtain $(x \otimes_\alpha z)\overline{\Theta^{'}}(y \otimes_\alpha z)$. □

Theorem 3.17. *Let G_1 and G_2 be Γ-semihypergroups, let $G_1 \cap G_2 = \emptyset$ and Θ be a regular relation on $G_1[G_2]$. Then, we have*

$$[\widehat{G_1[G_2] : \Theta}] = [\widehat{G_1}[\widehat{G_2}] : \widehat{\Theta}].$$

Definition 3.18. Let G_1 and G_2 be Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$, Θ_1 and Θ_2 be a regular relations on $G_1[G_2]$, and $\Theta_1 \subseteq \Theta_2$. Then, the relation Θ_1/Θ_2 defined on $[G_1[G_2] : \Theta_2]$ define as follows:

$$([x]_{\Theta_2}, [y]_{\Theta_2}) \in \Theta_1/\Theta_2 \Longleftrightarrow (x, y) \in \Theta_1.$$

Proposition 3.19. *Let G_1 and G_2 be Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$, Θ_1 and Θ_2 be a regular relations on $G_1[G_2]$, and $\Theta_1 \subseteq \Theta_2$. Then,*

1) $\widehat{\Theta_1} \subseteq \widehat{\Theta_2}$,
2) *The relation Θ_1/Θ_2 is regular,*
3) $\widehat{\Theta_1/\Theta_2} = \widehat{\Theta_1}/\widehat{\Theta_2}$,
4) $[[G_1[G_2] \widehat{: \Theta_2]} : \Theta_1/\Theta_2] \simeq [G_1\widehat{[G_2] : \Theta_1}]$.

Proof. (1) Suppose that $[(x,\alpha)]_\rho \widehat{\Theta_1}[(y,\beta)]_\rho$. So, $x \otimes_\alpha z\overline{\Theta_1}y \underset{\beta}{\otimes} z$, for all $z \in G_1[G_2]$. We conclude that $x \otimes_\alpha z\overline{\Theta_2}y \underset{\beta}{\otimes} z$, because $\Theta_1 \subseteq \Theta_2$. This implies that $[(x,\alpha)]_\rho \widehat{\Theta_2}[(y,\beta)]_\rho$. Therefore, $\widehat{\Theta_1} \subseteq \widehat{\Theta_2}$.

(2) Let $[x]_{\Theta_2}\Theta_1/\Theta_2[y]_{\Theta_2}$ and $[z]_{\Theta_2} \in [G_1[G_2] : \Theta_2]$. Then, $x\Theta_1 y$. We obtain $x \otimes_\alpha z\overline{\Theta_1}y \otimes_\alpha z$, because Θ_1 is regular. We conclude that

$$[x \otimes_\alpha z]_{\Theta_2}\overline{\Theta_1/\Theta_2}[(y \otimes_\alpha z)]_{\Theta_2}.$$

This implies that

$$[x]_{\Theta_2} \underset{\alpha}{\oplus} [z]_{\Theta_2}\overline{\Theta_1/\Theta_2}[y]_{\Theta_2} \underset{\alpha}{\oplus} [z]_{\Theta_2}.$$

So, Θ_1/Θ_2 is regular.

(3) We prove the equation as follows:

$$\begin{aligned}
[([(x,\alpha)]_\rho)]_{\widehat{\Theta_2}}\widehat{\Theta_1}/\widehat{\Theta_2}[([(y,\beta)]_\rho)]_{\widehat{\Theta_2}} &\Leftrightarrow [(x,\alpha)]_\rho\widehat{\Theta_1}[(y,\beta)]_\rho \\
&\Leftrightarrow x \otimes_\alpha z\overline{\Theta_1}y \underset{\beta}{\otimes} z \text{ for all} z \in G_1[G_2] \\
&\Leftrightarrow [x \otimes_\alpha z]_{\Theta_2}\overline{\Theta_1/\Theta_2}[y \underset{\beta}{\otimes} z]_{\Theta_2} \\
&\Leftrightarrow [([(x,\alpha)]_\rho)]_{\widehat{\Theta_2}}\widehat{\Theta_1/\Theta_2}[([(y,\beta)]_\rho)]_{\widehat{\Theta_2}}.
\end{aligned}$$

(4) We define the relation

$$\Psi : [[G_1[G_2]\widehat{: \Theta_2]} : \Theta_1/\Theta_2] \longrightarrow [G_1\widehat{[G_2] : }\Theta_1]$$

such that

$$\Psi([([([x]_{\Theta_2})]_{\Theta_1/\Theta_2}, \alpha)]_\rho) = [([x]_{\Theta_1}, \alpha)]_\rho.$$

Now, we show that Ψ is well-defined, one to one, and onto. Suppose that $[([x]_{\Theta_1}, \alpha)]_\rho = [([y]_{\Theta_1}, \beta)]_\rho$. Then, $[x]_{\Theta_1} \otimes_\alpha [z]_{\Theta_1} = [y]_{\Theta_1} \underset{\beta}{\otimes} [z]_{\Theta_1}$, for all $[z]_{\Theta_1} \in [G_1[G_2] : \Theta_1]$. Hence, $[x \otimes_\alpha z]_{\Theta_1} = [y \underset{\beta}{\otimes} z]_{\Theta_1}$. This implies that $(x \underset{\alpha}{\otimes} z)\overline{\Theta_1}(y \underset{\beta}{\otimes} z)$. We conclude that $(x \underset{\alpha}{\otimes} z)\overline{\Theta_2}(y \underset{\beta}{\otimes} z)$, because $\Theta_1 \subseteq \Theta_2$. Hence, $[(x \underset{\alpha}{\otimes} z)]_{\Theta_2} = [(y \underset{\beta}{\otimes} z)]_{\Theta_2}$. We obtain $[x]_{\Theta_2} \otimes_\alpha [z]_{\Theta_2} = [y]_{\Theta_2} \underset{\beta}{\otimes} [z]_{\Theta_2}$. Then,

$$[([([x]_{\Theta_2})]_{\Theta_1/\Theta_2}, \alpha)]_\rho = [([([y]_{\Theta_2})]_{\Theta_1/\Theta_2}, \beta)]_\rho.$$

So Ψ is one-to-one. It is easy to see that Ψ is onto. Suppose that $[([([x]_{\Theta_2})]_{\Theta_1/\Theta_2}, \alpha)]_\rho = [([([y]_{\Theta_2})]_{\Theta_1/\Theta_2}, \beta)]_\rho$. Then,

$$[([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \otimes_\alpha [([z]_{\Theta_2})]_{\Theta_1/\Theta_2} = [([y]_{\Theta_2})]_{\Theta_1/\Theta_2} \underset{\beta}{\otimes} [([z]_{\Theta_2})]_{\Theta_1/\Theta_2},$$

for all $[([z]_{\Theta_2})]_{\Theta_1/\Theta_2} \in [[G_1[G_2] : \Theta_2] : \Theta_1/\Theta_2]$. We conclude that

$$[([x]_{\Theta_2} \otimes_\alpha [z]_{\Theta_2})]_{\Theta_1/\Theta_2} = [([y]_{\Theta_2} \underset{\beta}{\otimes} [z]_{\Theta_2})]_{\Theta_1/\Theta_2}.$$

Hence, $[x \otimes_\alpha z]_{\Theta_2}\overline{\Theta_1/\Theta_2}[y \underset{\beta}{\otimes} z]_{\Theta_2}$. By the definition of Θ_1/Θ_2, we obtain $x\underset{\alpha}{z}\overline{\Theta_1}y \underset{\beta}{\otimes} z$. Therefore, $[x\underset{\alpha}{z}]_{\Theta_1} = [y \underset{\beta}{\otimes} z]_{\Theta_1}$. This implies

$$[x]_{\Theta_1} \otimes_\alpha [z]_{\Theta_1} = [y]_{\Theta_1} \underset{\beta}{\otimes} [z]_{\Theta_1}.$$

So, $[([x]_{\Theta_1}, \alpha)]_\rho = [([y]_{\Theta_1}, \beta)]_\rho$ and Ψ is well-defined. □

Theorem 3.20. *Suppose that G_1 and G_2 are Γ-semihypergroups such that $G_1 \cap G_2 = \emptyset$ and $\Theta_1 \subseteq \Theta_2$ is a regular relations on $G_1[G_2]$. Then,*

$$\overline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2}) = [\overline{Apr}_{\Theta_1}(A)]_{\Theta_2}.$$

Proof. Let $[x]_{\Theta_2} \in \overline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2})$. Hence, $[([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \cap [A]_{\Theta_2} \neq \emptyset$. There exists $[z]_{\Theta_2} \in [([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \cap [A]_{\Theta_2}$. Then, $[z]_{\Theta_2} \in [([x]_{\Theta_2})]_{\Theta_1/\Theta_2}$ and $[z]_{\Theta_2} \in [A]_{\Theta_2}$. This implies that $[z]_{\Theta_2}\Theta_1/\Theta_2[x]_{\Theta_2}$. Then, $z\Theta_1 x$, by the definition of Θ_1/Θ_2. We have $z \in A$ and $z \in [x]_{\Theta_1}$ implies that $[x]_{\Theta_1} \cap A \neq \emptyset$. Hence, $x \in \overline{Apr}_{\Theta_1}(A)$. We obtain $[x]_{\Theta_2} \in [(\overline{Apr}_{\Theta_1}(A))]_{\Theta_2}$ and

$$\overline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2}) \subseteq [\overline{Apr}_{\Theta_1}(A)]_{\Theta_2}\ .$$

Conversely, suppose that $[x]_{\Theta_2} \in [\overline{Apr}_{\Theta_1}(A)]_{\Theta_2}$. Then, $x \in \overline{Apr}_{\Theta_1}(A)$. This means that $[x]_{\Theta_1} \cap A \neq \emptyset$. There exists $z \in [x]_{\Theta_1} \cap A$. So $z \in A$ and $z \in [x]_{\Theta_1}$. So, $z\Theta_1 x$ and $[z]_{\Theta_2} \in [A]_{\Theta_2}$. Then, $[z]_{\Theta_2}\Theta_1/\Theta_2[x]_{\Theta_2}$. We have $[z]_{\Theta_2} \in [([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \cap [A]_{\Theta_2}$ and $[([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \cap [A]_{\Theta_2} \neq \emptyset$. This means that $[x]_{\Theta_2} \in \overline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2})$ and we obtain the inclusion

$$[\overline{Apr}_{\Theta_1}(A)]_{\Theta_2} \subseteq \overline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2}).$$

□

Theorem 3.21. *If G_1 and G_2 are Γ-semihypergroups, $G_1 \cap G_2 = \emptyset$ and $\Theta_1 \subseteq \Theta_2$ is a regular relations on $G_1[G_2]$. Then,*

$$\underline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2}) = [\underline{Apr}_{\Theta_1}(A)]_{\Theta_2}.$$

Proof. Let $[x]_{\Theta_2} \in \underline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2})$. Then, $[([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \subseteq [A]_{\Theta_2}$. Suppose that $z \in [x]_{\Theta_1}$. This means that $z\Theta_1 x$. Hence,

$$[z]_{\Theta_2}\Theta_1/\Theta_2[x]_{\Theta_2}.$$

So $[z]_{\Theta_2} \in [([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \subseteq [A]_{\Theta_2}$. We conclude that $[z]_{\Theta_2} \in [A]_{\Theta_2}$. Then, $z \in A$ and we obtain $[x]_{\Theta_1} \subseteq A$. This means that $x \in \underline{Apr}_{\Theta_1}(A)$. Hence, $[x]_{\Theta_2} \in [\underline{Apr}_{\Theta_1}(A)]_{\Theta_2}$.

Conversely, suppose that $[x]_{\Theta_2} \in [(\underline{Apr}_{\Theta_1}(A))]_{\Theta_2}$. Then, $x \in \underline{Apr}_{\Theta_1}(A)$ and $[x]_{\Theta_1} \subseteq A$. Let $[z]_{\Theta_2} \in [([x]_{\Theta_2})]_{\Theta_1/\Theta_2}$. Then, $[z]_{\Theta_2}\overline{\Theta_1/\Theta_2}[x]_{\Theta_2}$. This implies that $z\Theta_1 x$. We have $z \in [x]_{\Theta_1} \subseteq A$. So $z \in A$ and $[z]_{\Theta_2} \in [A]_{\Theta_2}$. We conclude that $[([x]_{\Theta_2})]_{\Theta_1/\Theta_2} \subseteq [A]_{\Theta_2}$. We obtain $[x]_{\Theta_2} \in \underline{Apr}_{\Theta_1/\Theta_2}([A]_{\Theta_2})$. □

Proposition 3.22. *Suppose that G_1 and G_2 are Γ-semihypergroups. Then, $(\mathcal{P}(\widehat{G_1}[\widehat{G_2}]), \circ)$ is a semihypergroup.*

Proof. We know that $(\widehat{G_1}[\widehat{G_2}], \circ)$ is a semihypergroup. Suppose that $A, B, C \in \mathcal{P}(\widehat{G_1}[\widehat{G_2}])$. Then,

$$\begin{aligned}(A \circ B) \circ C = &\bigcup_{[(x,\alpha)]_\rho \in A, [(y,\beta)]_\rho \in B, [(z,\gamma)]_\rho \in C} ([(x,\alpha)]_\rho \circ [(y,\beta)]_\rho) \circ [(z,\gamma)]_\rho \\ = &\bigcup_{[(x,\alpha)]_\rho \in A, [(y,\beta)]_\rho \in B, [(z,\gamma)]_\rho \in C} [(x,\alpha)]_\rho \circ ([(y,\beta)]_\rho) \circ [(z,\gamma)]_\rho) \\ = &A \circ (B \circ C).\end{aligned}$$

□

Proposition 3.23. *Suppose that* G_1 *and* G_2 *are* Γ*-hyperideals such that* $G_1 \cap G_2 = \emptyset$. *If* (F, A) *is a soft set over* $\widehat{G_1}[\widehat{G_2}]$ *such that* F *is an onto function. Then,* $(F^{'}, A^{'})$ *is a soft set over* $G_1[G_2]$.

Proof. We define $F^{'} : A^{'} \longrightarrow G_1[G_2]$ as follows:

$$F^{'}(x) = (F([(x, \Gamma)]_\rho))^{'} \quad forall x \in A^{'}.$$

We show that $F^{'}$ is well-defined. Let $x = y$ for $x, y \in A^{'}$. Then, $[(x, \Gamma)]_\rho = [(y, \Gamma)]_\rho$. From the assumption, $F([(x, \Gamma)]_\rho) = F([(y, \Gamma)]_\rho)$, because F is well-defined. We obtain

$$(F([(x, \Gamma)]_\rho))^{'} = (F([(y, \Gamma)]_\rho))^{'}.$$

This implies that $F^{'}(x) = F^{'}(y)$ and $F^{'}$ is well-defined. □

Definition 3.24. Suppose that (F, A) is a soft set over $\widehat{G_1}[\widehat{G_2}]$. We denote the soft set $(F^{'}, A^{'})$ by $(F, A)^{'}$ and we define

$$supp((F, A)^{'}) = \{x \in A^{'} | F^{'}(x) \neq \emptyset\}.$$

Proposition 3.25. *Suppose that* A *is a right hyperideal of semihypergroup* E. *Hence, we have*

$$x \in supp((F, A)^{'}) \Longleftrightarrow [(x, \Gamma)]_\rho \subseteq supp(F, A).$$

Proof. Let $x \in supp(F, A)^{'}$. Then, $x \in A^{'}$ and $F^{'}(x) \neq \emptyset$. There exists $\alpha \in \Gamma$ such that $[(x, \alpha)]_\rho \in A$. We have $F^{'}(x) = F([(x, \Gamma)]_\rho)^{'}$. We know that $F^{'}(x) \neq \emptyset$. So, $F([(x, \Gamma)]_\rho)^{'} \neq \emptyset$ and there exists $y \in F([(x, \Gamma)]_\rho)^{'}$ and $\beta \in \Gamma$ such that $[(y, \beta)]_\rho \in F([(x, \Gamma)]_\rho)$. So, $F([(x, \Gamma)]_\rho) \neq \emptyset$. We conclude that $[(x, \Gamma)]_\rho \subseteq supp(F, A)$.

Conversely, let $[(x, \Gamma)]_\rho \subseteq supp(F, A)$. For every $\alpha \in \Gamma$, we have $[(x, \alpha)]_\rho \in supp(F, A)$. Therefore, $[(x, \alpha)]_\rho \in A$ and $F([(x, \alpha)]_\rho) \neq \emptyset$. Then, $x \in A^{'}$ and $F([(x, \alpha)]_\rho)^{'} \neq \emptyset$. We conclude that $x \in A^{'}$ and $F([(x, \Gamma)]_\rho)^{'} \neq \emptyset$. We know that $F^{'}(x) = F([(x, \Gamma)]_\rho)^{'}$. So $F^{'}(x) \neq \emptyset$ and $x \in A^{'}$ which implies that $x \in supp((F, A)^{'})$. □

Theorem 3.26. *Let* A *be a right hyperideal over semihypergroup* E. *Then,*

$$(supp(F, A))^{'} = supp(F^{'}, A^{'}).$$

Proof. Suppose that $x \in (supp(F, A))^{'}$. There exists $\alpha \in \Gamma$ such that $[(x, \alpha)]_\rho \in supp(F, A)$. Then, $[(x, \alpha)]_\rho \in A$ and $F([(x, \alpha)]_\rho) \neq \emptyset$. Hence, $x \in A^{'}$ and $F([(x, \alpha)]_\rho)^{'} \neq \emptyset$. We conclude that $x \in A^{'}$ and $F^{'}(x) \neq \emptyset$. This implies that $x \in supp(F^{'}, A^{'})$. We obtain $(supp(F, A))^{'} \subseteq supp(F^{'}, A^{'})$.

Conversely, let $x \in supp(F', A')$. Then, $x \in A'$ and $F'(x) \neq \emptyset$. We conclude that $F([(x,\Gamma)]_\rho) \neq \emptyset$ and for every $\alpha \in \Gamma$, we have

$$[(x,\Gamma)]_\rho \subseteq A.$$

We obtain $[(x,\Gamma)]_\rho \subseteq supp(F,A)$. So, $x \in (supp(F,A))'$. □

Proposition 3.27. *Suppose that A is a semihypergroup and (F,A) is a soft set over $\widehat{G_1}[\widehat{G_2}]$. Then,*

$$(F([(x,\Gamma)]_\rho))' \subseteq F'(x).$$

Proof. Let $y \in (F([(x,\Gamma)]_\rho))'$. By the assumption, there exist $\alpha \in \Gamma$ where $[(y,\alpha)]_\rho \in F([(x,\Gamma)]_\rho)$. Besides, there exist $\beta \in \Gamma$ such that $[(y,\alpha)]_\rho \in F([(x,\beta)]_\rho) \subseteq [(F'(x),\beta)]_\rho$. So, There exist $t \in F'(x)$ such that $[(y,\alpha)]_\rho = [(t,\beta)]_\rho$. We have $y \otimes_\alpha z = t \underset{\beta}{\otimes} z$, for every $z \in G_1[G_2]$. We set $z = e_\alpha$. we obtain $y \otimes_\alpha e_\alpha = t \underset{\beta}{\otimes} e_\alpha$. This implies that

$$y \in t \underset{\beta}{\otimes} e_\alpha \subseteq F'(x) \underset{\beta}{\otimes} G_1[G_2] \subseteq F'(x).$$

We conclude that $y \in F'(x)$ and $(F([(x,\Gamma)]_\rho))' \subseteq F'(x)$. □

There is a connection between subsemihypergroups of semihypergroup $\widehat{G_1}[\widehat{G_2}]$ and Γ-subsemihypergroups of associated Γ-hypergroupoid $G_1[G_2]$ as follows:

Proposition 3.28. *Let A be a non-empty subset of $\widehat{G_1}[\widehat{G_2}]$ and A be a subsemihypergroup of $\widehat{G_1}[\widehat{G_2}]$. Then, A' is a Γ-subsemihypergroup of $G_1[G_2]$.*

Proof. Let x and y in A'. Then, there exist α and β in Γ such that $[(x,\alpha)]_\rho, [(y,\beta)]_\rho \in A$. Then, by the assumption, we have

$$[(x,\alpha)]_\rho \circ [(y,\beta)]_\rho \subseteq A.$$

This implies that $[(x \otimes_\alpha y, \beta)]_\rho \subseteq A$. This means that $x \otimes_\alpha y \subseteq A'$. □

Definition 3.29. We say that $(\widehat{G_1}[\widehat{G_2}], \Theta)$ is a Pawlak approximation space, where $\widehat{G_1}[\widehat{G_2}]$ is a semihypergroup and Θ is an equivalence relation over $\widehat{G_1}[\widehat{G_2}]$.

Definition 3.30. Let $(\widehat{G_1}[\widehat{G_2}], \Theta)$ be a Pawlak approximation space and let $G = (F,A)$ be a soft set over $\widehat{G_1}[\widehat{G_2}]$. Then, $\underline{U}(\Theta, G)$ is called a lower rough soft semihypergroup w.r.t Θ of $\widehat{G_1}[\widehat{G_2}]$ if $\underline{Apr}_{\Theta}F([(x,\alpha)]_\rho)$ is a subsemihypergroup of $\widehat{G_1}[\widehat{G_2}]$, for every $[(x,\alpha)]_\rho \in A$. Also, we can define lower rough soft hyperideal and lower rough soft prime hyperideal in this way. Moreover, G is called a rough soft semihypergroup w.r.t Θ of $\widehat{G_1}[\widehat{G_2}]$ if $\underline{Apr}_{\Theta}F([(x,\alpha)]_\rho)$ and $\overline{Apr}_{\Theta}F([(x,\alpha)]_\rho)$ are subsemihypergroups of $\widehat{G_1}[\widehat{G_2}]$, for all $[(x,\alpha)]_\rho \in A$.

Example 3.31. In Example 3.3, $(\widehat{G_1}[\widehat{G_2}], \widehat{\Theta})$ is a Pawlak approximation space. Suppose that $G = (F, A)$ be a soft set over $\widehat{G_1}[\widehat{G_2}]$, where $A = \{[(m,\alpha)]_\rho, [(n,\alpha)]_\rho\}$ and

$$F([(m,\alpha)]_\rho) = \widehat{G_2}, \quad F([(n,\alpha)]_\rho) = \{[(a,\alpha)]_\rho, [(b,\beta)]_\rho\}.$$

Therefore,

$$\underline{F}_{\widehat{\Theta}}([(m,\alpha)]_\rho) = \overline{F}_{\widehat{\Theta}}([(m,\alpha)]_\rho) = \widehat{G_2},$$
$$\underline{F}_{\widehat{\Theta}}([(n,\alpha)]_\rho) = \emptyset, \overline{F}_{\widehat{\Theta}}([(n,\alpha)]_\rho) = \widehat{G_1}.$$

Thus, $G = (F, A)$ is a rough soft semihypergroup over $\widehat{G_1}[\widehat{G_2}]$.

Proposition 3.32. *Suppose that $\widehat{G_1}[\widehat{G_2}]$ is a semihypergroup with unit and (F, A) is a soft right hyperideal over $\widehat{G_1}[\widehat{G_2}]$ such that A is a semihypergroup. Then,*

$$(F([(x,\Gamma)]_\rho))^{'} = F^{'}(x).$$

Proof. Let $y \in F^{'}(x)$. Then, $[(y,\Gamma)]_\rho \in [(F^{'}(x),\Gamma)]_\rho = F([(x,\Gamma)]_\rho)$. So $[(y,\Gamma)]_\rho \subseteq F([(x,\Gamma)]_\rho)$. This implies that $y \in (F([(x,\Gamma)]_\rho))^{'}$. We obtain, $F^{'}(x) \subseteq (F([(x,\Gamma)]_\rho))^{'}$. Now, let $y \in (F([(x,\Gamma)]_\rho))^{'}$. There exist $\alpha \in \Gamma$ such that $[(y,\alpha)]_\rho \in F([(x,\Gamma)]_\rho) = [(F^{'}(x),\Gamma)]_\rho$. We conclude that $y \in F^{'}(x)$ and $(F([(x,\Gamma)]_\rho))^{'} \subseteq F^{'}(x)$. □

4. Relation between decision-making algorithm of rough soft semihypergroups and associated rough soft sets

In this section first, we obtain the decision-making algorithm of rough soft semihypergroups $\widehat{G_1}[\widehat{G_2}]$. We find a relation between the decision-making algorithm of this rough soft semihypergroup and its associated rough soft set.

To add to this, Zhan et al. [50] presented a decision-making method by using a fuzzy set and constructed a congruence relation to define lower and upper approximations in soft n-semigroups. In this section, we use an equivalence relation which is regular, to construct approximations of soft semihypergroups as a generalization of Zhan's concept and prove that by replacing any regular relation with the congruence relation defined by Zhan the decision-making algorithm base on the minimum parameter is effective and accurate.

Decision making method I: Let $\widehat{G_1}[\widehat{G_2}]$ be a semihypergroup and E be a set of related parameters. Then, $G = (F, A)$ is called original description soft set over $\widehat{G_1}[\widehat{G_2}]$, where

$$A = \{[(e_1, \alpha_1)]_\rho, [(e_2, \alpha_2)]_\rho, \cdots, [(e_m, \alpha_m)]_\rho\} \subseteq E,$$

let Θ be an equivalence relation of $\widehat{G_1}[\widehat{G_2}]$ and let $(\widehat{G_1}[\widehat{G_2}], \Theta)$ be a Pawlak approximation space. Then, we present the decision algorithm for rough soft sets on semihypergroups as follows:

step 1: Input the original description semihypergroup $\widehat{G_1}[\widehat{G_2}]$, soft set G and Pawlak approximation space $(\widehat{G_1}[\widehat{G_2}], \Theta)$.

step 2: Compute the lower and upper rough soft approximations $\underline{Apr}_\Theta(G)$ and $\overline{Apr}_\Theta(G)$ on G, respectively.

step 3: Compute the different values of $\| F([e_i, \alpha_i]_\rho) \|$, where

$$\| F([e_i, \alpha_i]_\rho) \| = \frac{| \overline{Apr}_\Theta(F([e_i, \alpha_i]_\rho)) | - | \underline{Apr}_\Theta(F([e_i, \alpha_i]_\rho)) |}{| F([e_i, \alpha_i]_\rho) |}.$$

step 4: Find the minimum value $\| F([e_k, \alpha_k]_\rho) \|$ of $\| F([e_i, \alpha_i]_\rho) \|$, where

$$\| F([e_k, \alpha_k]_\rho) \| = \min_{1 \leq i \leq n} \{\| F([e_i, \alpha_i]_\rho) \|\}.$$

step 5: The decision is $F([e_k, \alpha_k]_\rho)$.

Example 4.1. Consider Example 3.31, we define a soft set $G = (F, A)$ over $\widehat{G_1}[\widehat{G_2}]$, where $A = \{[(m, \alpha)]_\rho, [(n, \alpha)]_\rho, [(l, \alpha)]_\rho\}$ such that

$$\begin{aligned}
F([(m, \alpha)]_\rho) &= \{[(a, \alpha)]_\rho, [(a, \beta)]_\rho, [(c, \alpha)]_\rho, [(c, \beta)]_\rho, \\
F([(n, \alpha)]_\rho) &= \{[(b, \beta)]_\rho, [(c, \alpha)]_\rho, [(b, \alpha)]_\rho, [((1\ 2\ 3), \cdot)]_\rho, [((1\ 3\ 2), \cdot)]_\rho, \\
F([(l, \alpha)]_\rho) &= \{[((1\ 3\ 2), \cdot)]_\rho, [(a, \alpha)]_\rho\}.
\end{aligned}$$

We obtain,

$$\begin{aligned}
\underline{Apr}_{\widehat{\Theta}} F([(m, \alpha)]_\rho) &= \emptyset, \\
\overline{Apr}_{\widehat{\Theta}} F([(m, \alpha)]_\rho) &= \widehat{G_1}, \\
\underline{Apr}_{\widehat{\Theta}} F([(n, \alpha)]_\rho) &= \{[((1\ 2\ 3), \cdot)]_\rho, [((1\ 3\ 2), \cdot)]_\rho\}, \\
\overline{Apr}_{\widehat{\Theta}} F([(n, \alpha)]_\rho) &= \widehat{G_1} \cup \{[((1\ 2\ 3), \cdot)]_\rho, [((1\ 3\ 2), \cdot)]_\rho\}, \\
\underline{Apr}_{\widehat{\Theta}} F([(l, \alpha)]_\rho) &= \{[((1\ 3\ 2), \cdot)]_\rho\},
\end{aligned}$$

$$\overline{Apr}_{\widehat{\Theta}}F([(l,\alpha)]_\rho) = \widehat{G_1} \cup \{[((1\ 3\ 2),\cdot)]_\rho\}.$$

Then,

$$\| F([(m,\alpha)]_\rho) \| = \frac{6-0}{4} = 1.5,$$
$$\| F([(n,\alpha)]_\rho) \| = \frac{8-2}{5} = 1.2,$$
$$\| F([(l,\alpha)]_\rho) \| = \frac{7-1}{2} = 3.$$

The decision is $F([(n,\alpha)]_\rho)$.

Theorem 4.2. *Let (F,A) be a rough soft hyperideal over $\widehat{G_1}[\widehat{G_2}]$ w.r.t Θ and Θ be a regular relation. Then, for every $[(e_i,\alpha_i)]_\rho \in A$ such that $F([(e_i,\alpha_i)]_\rho) \neq \emptyset$, we have*

$$(\underline{Apr}_\Theta F([(e_i,\alpha_i)]_\rho))^{'} \subseteq \underline{Apr}_{\Theta'}(F([(e_i,\alpha_i)]_\rho))^{'}.$$

Proof. By the assumption $F([(e_i,\alpha_i)]_\rho) \neq \emptyset$ is a hyperideal over $\widehat{G_1}[\widehat{G_2}]$ w.r.t Θ, for every $[(e_i,\alpha_i)]_\rho \in A$. Then, By Theorem 3.9, we conclude that $(\underline{Apr}_\Theta F([(e_i,\alpha_i)]_\rho))^{'} \subseteq \underline{Apr}_{\Theta'}(F([(e_i,\alpha_i)]_\rho))^{'}$. □

Theorem 4.3. *Let (F,A) be a non-null rough soft set over $\widehat{G_1}[\widehat{G_2}]$ w.r.t Θ. Then, for every $[(e_i,\alpha_i)]_\rho \in A$ such that $F([(e_i,\alpha_i)]_\rho) \neq \emptyset$, we have*

$$(\overline{Apr}_\Theta F([(e_i,\alpha_i)]_\rho))^{'} = \overline{Apr}_{\Theta'}(F([(e_i,\alpha_i)]_\rho))^{'}.$$

Proof. By Theorem 3.11, we obtain the equation. □

Proposition 4.4. *Suppose that $(\widehat{G_1}[\widehat{G_2}],\Theta)$ is a Pawlak approximation space. Then, $(\widehat{G_1}[\widehat{G_2}],\Theta)^{'}$ is also a Pawlak approximation space.*

Proof. By the assumption, Θ is a regular relation on semihypergroup $\widehat{G_1}[\widehat{G_2}]$ and by Proposition 3.16, $\Theta^{'}$ is a regular relation on $G_1[G_2]$. So, $(G_1[G_2],\Theta^{'})$ is a Pawlak approximation space. □

Proposition 4.5. *Let Θ be a regular relation on $\widehat{G_1}[\widehat{G_2}]$. Then,*

$$\underline{Apr}_\Theta F([(e_i,\Gamma)]_\rho) \subseteq [(\underline{Apr}_{\Theta'}F^{'}(e_i),\Gamma)]_\rho.$$

Proof. First, we prove that $F([(e_i,\Gamma)]_\rho) \subseteq [(F^{'}(e_i),\Gamma)]_\rho$. Suppose that $[(x,\alpha)]_\rho \in F([(e_i,\Gamma)]_\rho)$. Then, $x \in F([(e_i,\Gamma)]_\rho)^{'} \subseteq F^{'}(e_i)$. Thus, $x \in F^{'}(e_i)$. This implies that $[(x,\alpha)]_\rho \in [(F^{'}(e_i),\Gamma)]_\rho$. We conclude that

$$F([(e_i,\Gamma)]_\rho) \subseteq [(F^{'}(e_i),\Gamma)]_\rho.$$

Now, let $[(x,\alpha)]_\rho \in \underline{Apr}_\Theta F([(e_i,\Gamma)]_\rho)$. Then, $[([(x,\alpha)]_\rho)]_\Theta \subseteq F([(e_i,\Gamma)]_\rho)$. Also, $t \in [x]_{\Theta'}$, implies that $t\Theta' x$ and $[(t,\Gamma)]_\rho \overline{\Theta}[(x,\Gamma)]_\rho$. So, for $\alpha \in \Gamma$, there exists $\beta \in \Gamma$ such that $[(t,\beta)]_\rho \Theta [(x,\alpha)]_\rho$. We conclude that

$$[(t,\beta)]_\rho \in F([(e_i,\Gamma)]_\rho) \subseteq [(F'(e_i),\Gamma)]_\rho.$$

So, $t \in F'(e_i)$. We obtain $[x]_{\Theta'} \subseteq F'(e_i)$. So, $x \in \underline{Apr}_{\Theta'} F'(e_i)$ and $[(x,\alpha)]_\rho \in [(\underline{Apr}_{\Theta'} F'(e_i),\Gamma)]_\rho$. Therefore,

$$\underline{Apr}_\Theta F([(e_i,\Gamma)]_\rho) \subseteq [(\underline{Apr}_{\Theta'} F'(e_i),\Gamma)]_\rho.$$

□

Theorem 4.6. *Let A and $\widehat{G_1}[\widehat{G_2}]$ be semihypergroups with unit and (F,A) be a soft right hyperideal over $\widehat{G_1}[\widehat{G_2}]$ and Θ be a regular relation on $\widehat{G_1}[\widehat{G_2}]$. Then, for every $e_i \in A'$, we have*

$$\underline{Apr}_{\Theta'}(F([(e_i,\Gamma)]_\rho))' = \underline{Apr}_{\Theta'} F'(e_i).$$

Proof. The proof obtained by Proposition 3.32. □

Theorem 4.7. *Let A and $\widehat{G_1}[\widehat{G_2}]$ be semihypergroups with unit such that (F,A) be a soft right hyperideal over $\widehat{G_1}[\widehat{G_2}]$ and Θ be a regular relation on $\widehat{G_1}[\widehat{G_2}]$. Then, for every $e_i \in A'$, we have*

$$\overline{Apr}_{\Theta'}(F([(e_i,\Gamma)]_\rho))' = \overline{Apr}_{\Theta'} F'(e_i).$$

Proof. The proof obtained by Proposition 3.32. □

Now, we obtain the decision-making algorithm of associated rough soft set as follows:

step 1: Input the associated set $(G_1[G_2],\otimes_\alpha)$, soft set $G' = (F,A)'$, Pawlak approximation space $(G_1[G_2],\Theta')$ where Θ' is a regular relation of $G_1[G_2]$.

step 2: For all $[(e_i,\alpha_i)]_\rho \in A$, we conclude that $e_i \in A'$. Thus, we obtain the lower and upper rough soft approximations $\underline{Apr}_{\Theta'}(F'(e_i))$ and $\overline{Apr}_{\Theta'}(F'(e_i))$.

step 3: Compute the different values of $\| F'(e_i) \|$ as follows:

$$\begin{aligned}\| F'(e_i) \| &= \frac{|\overline{Apr}_{\Theta'}(F'(e_i))| - |\underline{Apr}_{\Theta'}(F'(e_i))|}{|F'(e_i)|}\\ &\leq \frac{|\overline{Apr}_{\Theta'}(F([(e_i,\Gamma)]_\rho))'| - |\underline{Apr}_{\Theta'}(F([(e_i,\Gamma)]_\rho))'|}{|F'(e_i)|}\\ &= \frac{|(\overline{Apr}_\Theta F([(e_i,\Gamma)]_\rho))'| - |(\overline{Apr}_\Theta F([(e_i,\Gamma)]_\rho))'|}{(F([(e_i,\Gamma)]_\rho))'}\end{aligned}$$

$$= || \ (F([(e_i,\Gamma)]_\rho))^{'} \ || \ .$$

step 4: Find the minimum value of $|| \ F^{'}(e_k) \ ||$, where

$$\begin{aligned} || \ F^{'}(e_k) \ || &= \min_{1\leq i\leq n} || \ F^{'}(e_i) \ || \\ &= \min_{1\leq i\leq n} || \ (F([(e_i,\Gamma)]_\rho))^{'} \ || \\ &= || \ (F([(e_k,\Gamma)]_\rho))^{'} \ || \ . \end{aligned}$$

step 5: The decision is $F^{'}(e_k)$

Example 4.8. Consider examples 3.31 and 4.1. We have $G^{'} = (F, A)^{'}$ such that $A^{'} = \{m, n, l\}$ and we obtain

$$F^{'}(m) = \{a, c\}, \quad F^{'}(n) = \{b, c, (1\ 2\ 3), (1\ 3\ 2)\}, \quad F^{'}(l) = \{(1\ 3\ 2), a\}.$$

Also, $(\widehat{\Theta})^{'}$ is a regular relation on $G_1[G_2]$. So, we have

$$[a]_{(\widehat{\Theta})^{'}} = [b]_{(\widehat{\Theta})^{'}} = [c]_{(\widehat{\Theta})^{'}} = G_1,$$

$$[(1\ 2\ 3)]_{(\widehat{\Theta})^{'}} = \{(1\ 2\ 3)\}, [(1\ 3\ 2)]_{(\widehat{\Theta})^{'}} = \{(1\ 3\ 2)\}, [(1)]_{(\widehat{\Theta})^{'}} = \{(1)\}.$$

Now, we obtain the lower and upper rough soft approximations as follows:

$$\begin{aligned} &\underline{Apr}_{(\widehat{\Theta})^{'}}F^{'}(m) = \emptyset, \\ &\overline{Apr}_{(\widehat{\Theta})^{'}}F^{'}(m) = G_1. \\ &\underline{Apr}_{(\widehat{\Theta})^{'}}F^{'}(n) = \{(1\ 2\ 3), (1\ 3\ 2)\}, \\ &\overline{Apr}_{(\widehat{\Theta})^{'}}F^{'}(n) = G_1 \cup \{(1\ 2\ 3), (1\ 3\ 2)\}. \\ &\underline{Apr}_{(\widehat{\Theta})^{'}}F^{'}(l) = \{(1\ 3\ 2)\}, \\ &\overline{Apr}_{(\widehat{\Theta})^{'}}F^{'}(l) = G_1 \cup \{(1\ 3\ 2)\}. \end{aligned}$$

We compute the difference values as follows:

$$\| \ F^{'}(m) \ \| = \frac{3-0}{2} = 1.50,$$

$$\| \ F^{'}(n) \ \| = \frac{5-2}{4} = 0.75,$$

$$\| \ F^{'}(l) \ \| = \frac{4-1}{2} = 1.50.$$

The decision is $F^{'}(n)$. Moreover, in the Example 4.1, we conclude that the appropriate parameter is $[(n, \alpha)]_\rho$.

5. Conclusions

The concept of rough soft sets introduced by Feng [14] by combining rough and soft sets. Also, rough soft sets to algebraic structures hemirings investigated by Zhan [43]. They gave some characterizations of rough soft hemirings. In the present paper, we proposed a novel rough soft algebraic hyperstructure extension semihypergroups induced by operators. Hence, we try to investigate the relation between decision-making algorithm of rough soft semihypergroups and associated rough soft sets. The concept of decision-making has found very important in an imprecise environment. Hence, we try to put decision-making approaches based on rough soft extension semihypergroups induced by operators, and the motivation for decision algorithm extensions semihypergroups induced by operators to find which is the best parameter of a given soft set. We hope it would be served as a foundation of rough soft set theory and other decision-making methods in different areas, such as theoretical computer sciences, information sciences and intelligent systems, and so on.

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Journal of Algebraic Systems

APPLICATIONS OF ROUGH SOFT TO EXTENSIONS SEMIHYPERGROUPS INDUCED BY OPERATORS AND CORRESPONDING DECISION-MAKING METHODS

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نیم‌اَبَرگروه‌های تعمیم یافته نرم ناهموار و کاربردهای آن‌ها در نظریه تصمیم‌گیری

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در این مقاله، مفهوم مجموعه نرم ناهموار را به یک اَبَرساختار جبری خاص اعمال کرده و مفهوم نیم‌اَبَرگروه نرم ناهموار ارائه می‌شود. مفهوم تقریب‌های پایینی و بالایی را برای یک نیم‌ابرگروه خاص ارائه داده و برخی از خواص آن را به دست می‌آید. بعلاوه، یک رابطه بین تقریب پایینی (بالایی) نیم‌اَبَرگروه خاص ارائه شده و Γ-اَبَرگروه‌وار متناظر آن در نظر گرفته شده است. در بخش آخر این پژوهش، الگوریتم تصمیم‌گیری نیم‌اَبَرگروه‌های نرم ناهموار را مورد بحث قرار داده و سپس یک رابطه بین الگوریتم تصمیم‌گیری نیم‌اَبَرگروه‌های نرم ناهموار و Γ-اَبَرگروه‌وارهای نرم ناهموار متناظر آن برای نیم‌ابرگروه خاص مورد نظر به‌دست می‌آید.

کلمات کلیدی: Γ-نیم‌اَبَرگروه، رابطه منظم، فضای تقریب، مجموعه نرم ناهموار، الگوریتم تصمیم‌گیری.