Journal of Mining and Environment (JME) Vol. 15, No. 4, 2024, 1491-1508

Shahrood University of Technology

Journal of Mining and Environment (JME)

Journal homepage: www.jme.shahroodut.ac.ir

Iranian Society of Mining Engineering (IRSME)

# Optimizing Long-Term Production Scheduling in Open Pit Mining under Commodity Price Uncertainty: A Two-Stage Stochastic Programming Approach

Elham Lotfi[1], Javad Gholamnejad[1]*, Mehdi Najafi [1], MohammadSadegh Zamani[2]

1. Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran
2. Department. of Mathematical Sciences, Yazd University, Yazd, Iran

***Article Info***

*Received 18 November 2023*

*Received in Revised form 11 May 2024*

*Accepted 6 June 2024*

*Published online 6 June 2024*

DOI: 10.22044/jme.2024.13814.2576

***Keywords***

*Long-term production planning*

*Open-pit mining*

*Metal price uncertainty*

*Dynamic cut-off grades*

*Two-stage stochastic programming*

***Abstract***

In the context of open pit mining operations, long-term production scheduling faces significant challenges due to inherent uncertainties, particularly in commodity prices. Traditional mathematical models often adopt a single-point estimation strategy for commodity price, leading to suboptimal mine plans and missed production targets. The simultaneous effect of commodity price uncertainty on the cut-off grade and long-term production scheduling is less considered. This paper introduces a novel model for optimizing open pit mine long-term production scheduling under commodity price uncertainty considering a dynamic cut-off grade strategy, based on a two-stage Stochastic Production Programming (SPP) framework. The presented model seeks to identify optimal mining block sequences, maximizing total discounted cash flow while penalizing deviations from production targets. To illustrate the model's efficiency, it was implemented in a copper mine. First, the Geometric Brownian Motion (GBM) model is used to quantify the future commodity price. Then, both deterministic and SPP models were solved using GAMS software. The results showed that the practical NPV obtained from the SPP model is approximately 3% higher than the DPP model, while all constraints are satisfied.

## 1. Introduction

The long-term production planning (LTPP) of open-pit mines poses a complex optimization problem, aiming to identify an optimum life-of-mine (LOM) production schedule that maximizes the Net Present Value (NPV) while adhering to various technical and operational constraints. The primary input for LTPP is an economic block model, which consists of a collection of mining blocks representing the ore body and the surrounding rock. Each block is assigned a net economic value based on the revenue generated from the recoverable metal content and subtracting all associated operating costs, including mining, processing, refining, and selling costs. In conventional approaches, geological, financial, and other relevant parameters are often assumed to be fixed. However, in reality, these factors are subject to uncertainty and may vary throughout the mine's life.

Among the uncertain factors, commodity prices play a significant role, in influencing future annual schedules [1–3]. Due to their high volatility, commodity prices are beyond the control of mine operators and investors [4]. Moreover, they directly impact income, and their effects cannot be mitigated through additional exploration or investigation efforts. Additionally, cut-off grades are employed as a criterion to differentiate between ore and waste throughout the entire orebody. The determination of how individual blocks should be processed during scheduling is influenced by the prevailing commodity prices. Henceforth, a new scheduling model based on two-stage stochastic programming should be developed to take into account commodity price scenarios with dynamic

✉ Corresponding author: j.gholamnejad@yazd.ac.ir (J. Gholamnejad)

corresponding cut-off grades in the mathematical model simultaneously. As a result, the extraction sequences of blocks lead to the determination of the optimal destination allocation for blocks. The LTPP problem in open pit mines can be categorized into deterministic and uncertainty-based approaches [5, 6]. While deterministic approaches assume fixed values for input parameters, the reality is that these parameters are uncertain. This literature review explores various mathematical, heuristic, and meta-heuristic methods proposed to solve the LTPP problem.

Deterministic approaches, such as Lagrangian relaxation and branch and cut, have been widely used by many researchers to obtain exact optimal solutions for the LTPP problem [7–11]. However, these methods suffer from limitations when applied to instances of realistic size. Recognizing the complexity of open-pit production scheduling, researchers have applied aggregation-disaggregation and heuristic approaches to address the LTPP problem [5, 12–15]. Clustering techniques and heuristic algorithms have been utilized to reduce problem size and find near-optimal solutions; nevertheless, optimality cannot be guaranteed with these approaches. Meta-heuristic approaches, including Genetic Algorithms (GA), Simulated Annealing (SA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Imperialist Competitive Algorithm (ICA), and Tabu Search (TS), have gained attention in solving the LTPP problem [6, 16–21]. These algorithms do not guarantee optimality but can generate good solutions within a reasonable time frame compared to exact optimization methods. While deterministic approaches dominate commercial optimizers, the importance of uncertainty in mine planning optimization cannot be ignored. Geological uncertainty and commodity price uncertainty significantly impact the profitability of mining operations.

Considerable endeavors have been dedicated to formulating methodologies that integrate geological uncertainty. Among these approaches, a prominent one involves minimizing deviations from production targets, thereby generating schedules that optimize Net Present Value (NPV) while minimizing penalties associated with failure to meet tonnage, grade, or quality objectives. Two-stage stochastic programming has also been implemented to involve recourse actions when new information arrives [22, 23]. For instance, Tahernejad et al. (2018) explores the impact of grade uncertainty on the technical and financial aspects of mine planning by comparing Sequential Gaussian Simulation (SGS) and Ordinary Kriging (OK) methods [24]. Armstrong et al. (2021) proposed an adaptive stochastic optimization approach for multi-period production scheduling under geological uncertainty in open-pit mines which updates the geological model each period as new information becomes available [25]. Jelvez et al. (2023) addressed a multistage methodology for long-term open-pit mine production planning under grade uncertainty. They stated that incorporating uncertainty helps reduce the risk of losses due to failure to meet production targets as compared with the fully deterministic case [23]. Most stochastic production scheduling approaches pose a significant computational challenge. For this reason, an extensive range of algorithms, heuristics, and metaheuristics have been developed to offer reasonable computation times for the problem of long-term production planning [17, 18, 26]. A complete review of models and algorithms that have been performed in the last two decades to address the integration of uncertainty in mine planning can be found at [27–30]

Previous research has primarily focused on integrating geological uncertainty, while commodity price uncertainty remains to be further investigated. Existing literature has predominantly focused on the application of real options to mine project valuations as a means of addressing commodity price uncertainty. Numerous scholars have explored the integration of commodity price uncertainty into long-term mine planning [1, 2, 31–35]. However, despite these efforts, further research is still required in this domain. In recent times, several alternative approaches have emerged for incorporating commodity price scenarios into open-pit mine optimization. These approaches encompass reactive valuation, robust stochastic optimization, stochastic programming with recourse, and multi-criteria decision-making models. Each approach presents distinct advantages and considerations in addressing the challenge of commodity price uncertainty. In the subsequent sections, we will provide a brief overview of these approaches, highlighting their unique characteristics and implications.

Meagher et al. (2010) employed a parametric minimum cut algorithm to integrate geological and market uncertainties into open-pit mine planning. In this particular approach, the selection of blocks as either ore or waste is determined by prioritizing those with a higher probability of being ore, rather than relying on the average amount of realizations. This approach may be too conservative in terms of evaluating a block that has a probability higher than 50% of being more valuable than another block, but also a greater likelihood of being classified as waste material [2]. Kumral (2010) developed a robust stochastic optimization (RSO) model to address uncertainties in grades and commodity prices for long-term mine planning. However, this approach provides insensitive solutions to different uncertainty realizations and is applies only to a small number of realizations [36]. Kumral (2011) recommended two approaches involving a stochastic programming problem with recourse, and a maximin model considering uncertainties in financial and geo-metallurgical variables. Although the solution obtained from stochastic programming with recourse was robust, and a maximin problem led to extracting more uniform metal quantity in periods to coincide with mill requirements, the major limitation is applying three price scenarios as low, most likely, and high prices. Therefore, a reliability analysis should be integrated with the proposed planning method [37]. Similarly, Asad and Dimitrakopoulos (2013) applied Lagrangian relaxation of production capacity constraints to address production-phase design and ultimate pit limit under commodity price and geological uncertainties. One of the advantages of this approach compared to previous similar cases, is the consideration of the discounted economic value of the blocks in price realizations. However, the major weakness of this approach still lies in incorporating the average realizations for the integration of multiple block models [35]. Rahmanpour and Osanloo (2015) obtained an optimal solution for long-term mine production planning by formulating it as a multi-criteria decision-making problem, considering maximum upside potential, minimum downside risk, and value at risk. The noticeable capability of this approach is to determine the optimum extraction sequence and destination for each block. However, it is limited when applied to larger-scale cases [38]. Kumral and Sari (2017) proposed a fuzzy possibilistic MIP model for open-pit mine production scheduling, considering grade and financial uncertainties. Although, the solution obtained was robust, considering a triangular possibility distribution for all uncertain parameters may not always be accurate or appropriate [39]. Mokhtarian and Sattarvand (2016) introduced a novel approach for integrating commodity price uncertainty into long-term mine production planning by constructing a series of economic block models based on realizations from the commodity price distribution function. The sampling method that has been used from the commodity price cumulative distribution function, Median Latin hypercube sampling (MLHS), tends to prioritize sampling points around the median, which can introduce a bias towards the center of the distribution. indeed, it has not been clarified to determine the decision-making process for blocks with equal selection probabilities over multiple periods in this approach [40]. Bakhtavar et al. (2017) presented a chance-constrained programming-based model to optimize production strategies considering grade, metal price, and operational capacities (mining, processing, and refinery) in bimetallic deposit open-pit mines. According to the unrealistic assumptions associated with the use of chance-constrained programming, the results of this approach may be not reliable [41]. Alipour et al. (2017) and Alipour et al. (2020) proposed robust box counterpart and ellipsoidal set-based counterpart programming approaches, respectively, considering block economic value as the objective function coefficient and block weights and operational capacities (mining and processing) as constraint coefficients for open-pit mine production scheduling under uncertainty [42, 43]. Kumral and Asil Sari (2017) introduced an extraction sequencing approach incorporating financial and geological uncertainties by combining chance-constrained programming and Monte Carlo simulation [44]. Mokhtarian and Sattarvand (2018) suggested a novel stochastic optimization procedure for long-term mine production planning using the imperialist competitive algorithm (ICA) under commodity price conditions [19]. Tahernejad et al. (2018) developed an approach based on the Information Gap Decision Theory (IGDT) to hedge the NPV expectation of mine projects against the risk associated with the information gap between forecasted and actual prices [45]. Rim'el'e et al. (2020) devised an algorithm for open-pit mine planning incorporating geological and commodity price uncertainties. They utilized a combination of a two-stage stochastic integer program and a stochastic dynamic programming algorithm to determine the optimal policy for metal production targets based on the evolution of commodity prices [46]. Shenavar et al. (2021) developed a procedure to evaluate the effects of the grade and price uncertainties simultaneously on the stope optimization and underground mine evaluation [47]. Liu et al. (2023) expanded a dynamic optimization method for mine production scheduling by incorporating fuzzy mining quantity and fuzzy stripping quantity into the production plan, which is determined using economic time series prediction of production cost and coal sales price, combined with fuzzy structural elements [48].

According to the literature, the uncertainty in final product prices and geological uncertainty regarding the grade and tonnage of mining blocks are recognized as the most crucial factors influencing the profitability of mining operations. The previous research on long-term production planning in open-pit mines under uncertainty only considered price scenarios or a combination of price, grade, and tonnage, disregarding that price scenarios change the cut-offs in each period. Neglecting this relationship would result in incorrect discretization of blocks for their optimal destinations. Therefore, in this study, the long-term production planning problem is formulated considering commodity price scenarios and dynamic cut-off grades. Other variables such as operating costs, grade, tonnage, and geo-metallurgical variables are assumed to be fixed. The subsequent sections present the stochastic forecasting model for quantifying commodity price uncertainty. Metal price scenarios are then generated based on the suggested stochastic forecasting model in section 2. A detailed description of the proposed novel approach is provided in section 3. Section 4 presents an application of the new model to a case study, exploring practical aspects and related concepts. The final section concludes the paper and provides recommendations.

## 1. Methodology

The represented methodology utilizes a combination of statistical analysis and stochastic programming to address metal price uncertainty in the context of stochastic mine scheduling. The methodology focuses specifically on copper prices and employs the Mean-Reverting Process (MRP) model to capture the mean-reverting behavior observed in copper price data. The MRP model is estimated using the Maximum Likelihood Estimation (MLE) method, which allows for the determination of the model's parameters, including the reversion speed and standard deviation. The estimated MRP model parameters provide insights into the behavior of copper prices and their mean-reverting characteristics.

To incorporate metal price uncertainties into the stochastic mine scheduling optimization model, a two-stage stochastic programming approach is utilized. This approach considers both geological uncertainty and commodity price uncertainties in the decision-making process. In the first stage of the stochastic programming model, decisions regarding the extraction periods of mining blocks are made based on available geological information. The objective is to determine an extraction schedule that maximizes the expected value of the objective function under uncertainty. In the second stage, decisions regarding the destinations of the blocks are formulated as second-stage decision variables. The optimization model aims to provide a life-of-mine production schedule that is scenario-independent concerning geological uncertainty but scenario-dependent to commodity price uncertainties and dynamic cut-off grades.

The methodology also includes the generation of multiple scenarios for future copper prices based on the estimated MRP model. These scenarios provide insights into the range of potential copper price paths over a specified time horizon. Statistical analysis is conducted to assess the confidence level of the price changes within these scenarios. Overall, this methodology allows for the incorporation of metal price uncertainties into the stochastic mine scheduling optimization model, providing a more robust and adaptive approach to decision-making in the mining industry.

### 2.1. Metal Price Uncertainty Quantification

To quantify metal price uncertainty, the Mean-Reverting Process (MRP) model is utilized, which is suitable for reverting variables like copper prices [49]. The MRP model is represented by the following equation:

$$\frac{dP}{P_0} = \kappa\,(\mu - lnP)dt + \sigma dz_t \tag{1}$$

Where P is the price of the metal, $P_0$ is the initial price, κ is the reversion speed, μ is the long-term equilibrium log price, σ is the standard deviation, dz is an increment in a standard Wiener process, and dt is an increment of time.

### 2.2. Maximum Likelihood Estimation (MLE)

MLE is a statistical approach used to estimate the parameters of a given model by maximizing the likelihood function. In the case of the MRP model, the likelihood function is constructed based on the observed historical data of copper prices.

The MLE method aims to find the parameter values that maximize the probability of observing the historical price data given the MRP model assumptions. This estimation process allows us to obtain the most likely set of parameter values that characterize the mean-reverting behavior of copper prices.

The Yuima package in the R software is employed for the estimation of the model's coefficients. The Yuima package is a comprehensive toolkit in R for simulating and analyzing stochastic differential equations (SDEs). It provides a wide range of functions and tools specifically designed for the estimation and analysis of stochastic processes, making it suitable for estimating the parameters of the MRP model.

The parameters of the MRP model, namely κ, μ, and σ, are estimated using the Maximum Likelihood Estimation (MLE) method. The estimated parameters for the MRP model are $\kappa = 0.12$ and $\sigma = 0.281$.

Therefore, the equation of the mean reversion process was formulated to estimate copper price scenarios as follows:

$$\frac{dP}{P_0} = 0.12\,(8.788 - lnP)dt + 0.2811dz_t \tag{2}$$

### 2.3. Normalized Annual Copper Price

Figure 1 illustrates the normalized annual copper price over the period 1960 to 2022 in current US Dollars. The nominal copper price data are obtained from Indexmundi [50], and inflation is adjusted using the US Consumer Price Index (CPI) data from the US Bureau of Labor Statistics (2022) shown in Figure 2.

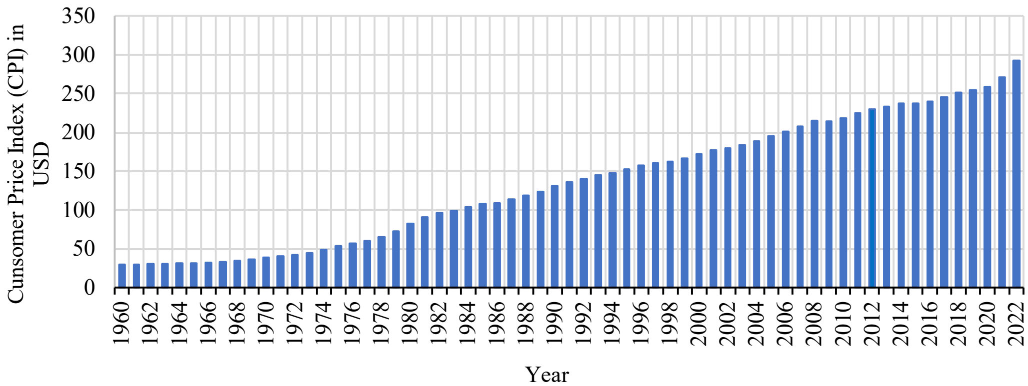


**Figure 1. The US Consumer Price Index (CPI)** [51]

The normalization process involves indexing the prices to December for better comparison and analysis.

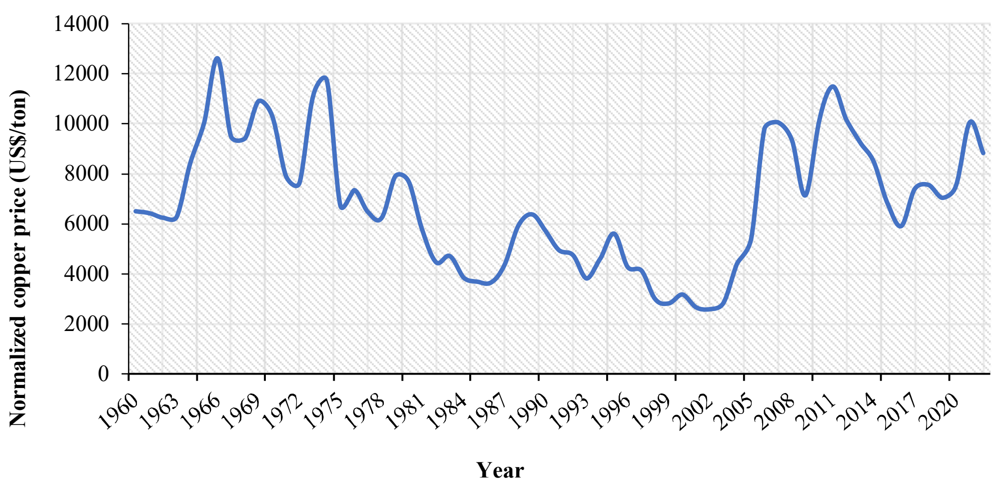


**Figure 2. Normalized copper price from 1960 to 2022 (indexed to December 2022, inflation-adjusted))**

According to the estimated parameters, the MRP model has been obtained to generate copper price scenarios as shown in Equation 1. The equally probable multiple scenarios of future copper prices are estimated.

### 2.4. Copper Price Paths for the Next 7 Years

Figure 3 shows twenty paths over seven years. Each path represents a possible scenario for future copper prices. The analysis at the statistical confidence level of at least 95% confirms the copper price changes between about 8000 and 11000 US$ per ton during the next seven years.

By analyzing the market prices of copper and estimating the parameters of the MRP model, we gain insights into the behavior of copper prices and their mean-reverting characteristics. These insights are crucial for incorporating metal price uncertainties into the stochastic mine scheduling optimization model.

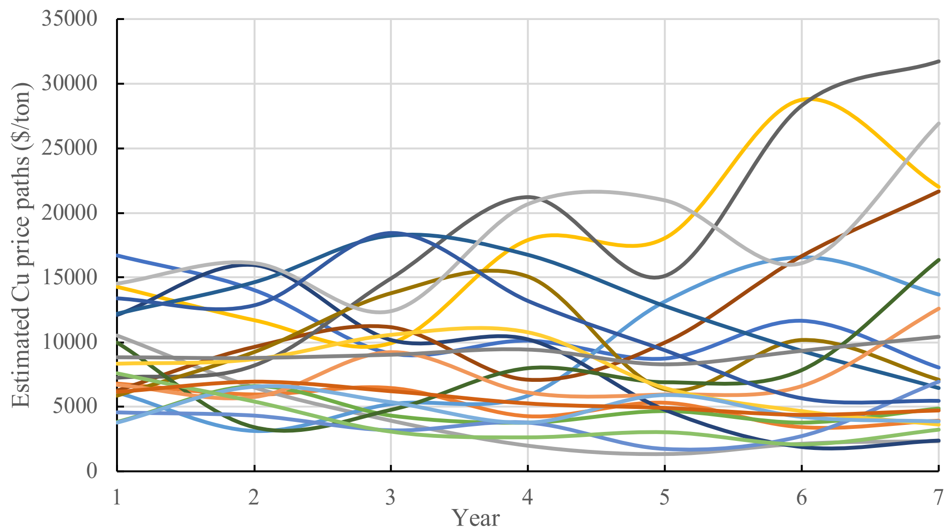


**Figure 3. Copper price paths for the next 7 years**

## 2. Mathematical model

In this section, the mathematical notation and formulation of the new two-stage stochastic production programming (SPP) are stated as follows:

### 3.1. Indices and Sets

**i:** Block index in direction x, i={1,...,I}.
**j:** Block index in direction y, j={1,...,J}.
**k:** Block index in direction z, k={1,...,K}.
**t:** Time period index, t={1,...,T}.
**s:** Scenario index, s={1,...,S}.
**$\Gamma$ijk:** the set of overlaying blocks that must be removed before mining block ijk to satisfy the slope constraint.

### 3.2. Parameters

**I, J, K:** the maximum number of blocks in each direction of x,y, and z, respectively.

**T:** the number of periods over which blocks are being scheduled (periods).

**S:** the number of scenarios generated to capture the price uncertainty.

**d1:** the financial discount rate.

**d2:** the risk discount rate.

**$Pr^{t}_{s}$:** metal price during period t under scenario s.

**$\pi$s:** the probability of occurrence of scenario s with equally probable occurrence, ($\sum\pi s=1$).

**$T_{(ijk)}$ :** the total weight of block ijk

**$g_{(ijk)}$:** the average grade of block ijk.

**$R_e$:** the extraction recovery

**RT:** the total metal recovery of extracted material sent to the mill.

**R'T:** the total metal recovery of extracted material sent to the leach pad.

**$C_w$:** mining cost per ton of waste.

**$C_r$:** smelting, refining, and freight cost per ton of metal produced.

**$C_{eh}$:** mining cost per ton of ore sent to the mill.

**$C_{el}$:** mining cost per ton of ore sent to the leach pad.

**$C_p$:** processing cost per ton of ore sent to mill.

**$C'_p$:** processing cost per ton of ore sent to the leach pad.

**C′r:** cost of SX-EW and cathode freight to the market in terms of a/per ton of metal produced.

**TEmax:** the maximum weight of waste and ore material that can be extracted in each period (maximum mining capacity).

**TM min:** the minimum weight of ore material that must be processed by the mill in each period (minimum processing capacity).

**TMmax:** the maximum weight of ore material that can be processed by the mill during each period (maximum processing capacity).

**TLmax:** the maximum weight of ore material that can be processed by the leach pad during each period (maximum leaching capacity).

***gwl$^t_s$*:** the cut-off grade for discretization between waste and leach ore under the price scenario s.; it can be expressed as:

$$gwl_s^t = \frac{(C_{el} - C_w) + C'_p}{R'_T \times (P_s^t - C'_r)} \quad (3)$$

***glm$^t_s$*:** the cut-off grade for discretization between leach ore and mill ore under the price scenario s.; it can be expressed as:

$$glm_s^t = \frac{(C_{eh} - C_{el}) + (C_p - C'_p)}{R_T \times (P_s^t - C_r) - R'_T \times (P_s^t - C'_r)} \quad (4)$$

$$if \begin{cases} g_{(ijk)} < gwl_s^t & \gamma^t{}_{(ijk),s} = 1, \alpha^t{}_{(ijk),s} = \beta^t{}_{(ijk),s} = 0 \\ gwl_s^t \le g_{(ijk)} < glm_s^t & \beta^t{}_{(ijk),s} = 1, \alpha^t{}_{(ijk),s} = \gamma^t{}_{(ijk),s} = 0 \\ g_{(ijk)} \ge glm_s^t & \alpha^t{}_{(ijk),s} = 1, \beta^t{}_{(ijk),s} = \gamma^t{}_{(ijk),s} = 0 \end{cases} \quad (5)$$

$$\gamma^t{}_{(ijk),s} + \alpha^t{}_{(ijk),s} + \beta^t{}_{(ijk),s} = 1 \quad \forall i,j,k,s \quad (6)$$

**$\alpha^t_{(ijk),s}$, $\beta^t_{(ijk),s}$, and $\gamma^t_{(ijk),s}$ :** parameters that determine the processing method of each block according to their grade and cut-offs during period t under the price scenario s.

**$NV^t_{(ijk),s}$:** the net value of block ijk under scenario s; that is:

$$\begin{aligned} NV_{(ijk),s}^t &= \alpha^t{}_{(ijk),s} \times TR_{(ijk)}\left[g_{(ijk)} \times R_T \times (P_s^t - C_r) - C_{eh} - (R_e \times C_p)\right] \\ &+\beta^t{}_{(ijk),s} \times TR_{(ijk)}\left[g_{(ijk)} \times R'_T \times (P_s^t - C'_r) - C_{el} - (R_e \times C'_p)\right] \\ &-\gamma^t{}_{(ijk),s} \times (TR_{(ijk)} \times C_w) \end{aligned} \quad (7)$$

**Gmin:** the minimum average grade of the ore material to be processed by the mill during period t.

**Cl:** the undiscounted cost per unit of shortage ore material ($Mdsh^t_s$) for the mill during period t.

**Cu:** the undiscounted cost per unit of surplus material ($Mdsu^t_s$) for the mill during period t.

**Cu′:** the undiscounted cost per unit of surplus material ($Ldsu^t_s$) for the leach pad during period t.

**Cl′:** the undiscounted cost per unit of shortage in metal content sent to the mill ($gdsh^t_s$) during period t.

### 3.3. Variables

The decision variables to formulate the model are as follows:

**$x^t_{ijk}$ :** the binary decision variable is associated with each block ijk for each period t as below:

$$x_{ijk}^t = \begin{cases} 1 & \text{if block ijk is mined during period t;} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

**$Mdsu^t_s$:** a continuous variable denotes the surplus of ore material sent to the processing plant during period t if scenario s occurs.

**$Mdsh^t_s$:** a continuous variable denotes the shortage of ore material sent to the processing plant during period t if scenario s occurs.

**$Ldsu^t_s$:** a continuous variable measures the surplus of ore material sent to the leach pad during period t if scenario s occurs.

**$gdsh^t_s$:** a continuous variable denotes the deficient amount of metal content sent to the processing plant during period t if scenario s occurs

### 3.4. Objective Function

The corresponding objective function of the problem depicted as follows can be expressed by:

$$Max\frac{1}{S}\left\{\left(\sum_{t=1}^{T}\frac{1}{(1+d_1)^t}\sum_{i=1}^{I}\sum_{j}^{J}\sum_{k}^{K}\sum_{s}^{S}NV_{ijk,s}^{t}\times x_{(ijk)}^{t}\right) - \left(\sum_{t=1}^{T}\frac{1}{(1+d_2)^t}\sum_{s=1}^{S}[(C_l\times Mdsh_s^t)+(C_u\times Mdsu_s^t)+(C'_u\times Ldsu_s^t)+(C'_l\times gdsh_s^t)]\right)\right\} \quad (9)$$

The objective function of the model is constructed as the maximization of the expected NPV of the mine minus the total cost of deviations from production targets. The objective function is formulated in two parts: part 1 represents the profit that can be generated if all the mined ore blocks are processed immediately. Part 2 consists of economic risk management as presented in Ramazan and Dimitrakopoulos (2013), and is added to minimize the deviations from production targets [52].

### 3.5. Constraint formulation

As mentioned in the previous section, the constraints are considered stochastic and non-stochastic constraints in the present model.

#### 3.5.1. Scenario independent constraints (first-stage constraints):

1. Precedence constraints: These constraints ensure that a block can only be extracted during some time period t if all of its predecessor blocks have been mined completely before or during period t as shown in Figure 4.

$$x_{ijk}^{t} \leq \sum_{\tau=1}^{t} x_{o}^{\tau}\ \forall ijk \quad o \in \Gamma_{ijk}\ and\ t \quad (10)$$

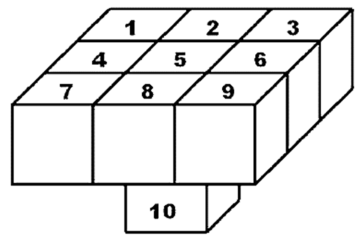


**Figure 4: Sequencing rules on the removal of nine blocks above a given block, block 10 (right)**

2. Reserve constraint: These constraints let each block be mined at most once during the scheduling horizon.

$$\sum_{t}^{T} x_{ijk}^{t} \leq 1 \quad \forall(i,j,k) \quad (11)$$

3. Mining capacity constraint: The total rock tonnage of ore and waste blocks mined per period is bounded by these constraints:

$$\sum_i\sum_j\sum_k (TR_{ijk})\times x_{ijk}^t \leq TE^t{}_{max} \quad \forall t \quad (12)$$

#### 3.5.2. Scenario dependent constraints (second-stage constraints):

4. Mill capacity constraints: These constraints control the maximal and minimal amounts for the total ore tonnage sent to the processing plant per each period.

$$\sum_i\sum_j\sum_k \left(\alpha^t{}_{(ijk),s}\times TR_{ijk}\right)\times x_{ijk}^t - Mdsu_s^t \leq TM^t{}_{max} \quad \forall\ (t,s) \quad (13)$$

$$\sum_i\sum_j\sum_k \left(\alpha^t{}_{(ijk),s}\times TR_{ijk}\right)\times x_{ijk}^t + Mdsh_s^t \geq TM^t{}_{min} \quad \forall\ (t,s) \quad (14)$$

5. Leaching capacity constraints: These constraints restrict the maximal amounts of ore tonnage sent to the leach pad per each period.

$$\sum_i\sum_j\sum_k \beta_{(ijk),s}^{t}\left(TR_{ijk}\right)\times x_{ijk}^t - Ldsu_s^t \leq TL_{max}^t \quad \forall\ (t,s) \quad \textbf{(15)}$$

6. Grade blending constraints: these constraints ensure that the average grade of the material sent to the mill respects a given lower bound. (Stochastic constraints related to grade blending are used to satisfy the grade requirement at the mill. This constraint can be expressed by:

$$\sum_{i}\sum_{j}\sum_{k}\left(g_{ijk}-G_{min}\right)\left(\alpha_{(ijk),s}^{t}\times TR_{ijk}\right)\times x_{ijk}^{t}+\ gdsh_{s}^{t}\geq 0 \qquad \forall\ (t,s) \tag{16}$$

After defining the indices, parameters, variables, and objective functions, and calling the data files, the production planning problem was modeled in GAMS software using both deterministic and stochastic approaches. In this study, since the long-term open-pit production planning problem belongs to the category of Mixed Integer Programming (MIP), the solver CPLEX involving branch-and-cut algorithm (BC) was used on the GAMS and 64-bit optimization routines. All programs were run on an Intel core i7-8550U (1.8 GHz) with 16 GB of RAM.

## 3. Case study

The proposed model and solution approaches have been implemented in the Miduk copper mine. The case study is one of the biggest copper ore mine located in Kerman. The general information and scheduling parameters have been displayed in Table 1.

**Table 1. General information of the deposit and scheduling parameters**

| Description | values |
|---|---|
| Number of blocks | 15391 |
| Horizon (y) | 7 |
| Block dimensions (m) | 15×15×15 |
| Pit Slope Angle | 45° |
| Total mining capacity (Mt) | 40 |
| Minimum of Processing/mill target capacity (Mt) | 13 |
| Maximum of Processing/mill target capacity (Mt) | 14 |
| Leaching capacity (Mt) | 3.5 |
| Average/Mean grade (%) | 0.4 |
| Copper price scenarios ($/ton) | 140 |
| Financial discount factor (%) | 10 |
| price discount factor (%) | 10 |

The final pit is divided into several pushbacks. The mined material is loaded from the mining area by shovels and a front-end loader and sent to the processing plant, leaching pad, or waste dumps depending on the material type. Trucks are used to carry material from the pit to processing stream flows or dumps. The location of the pits can be seen in Figure 5.

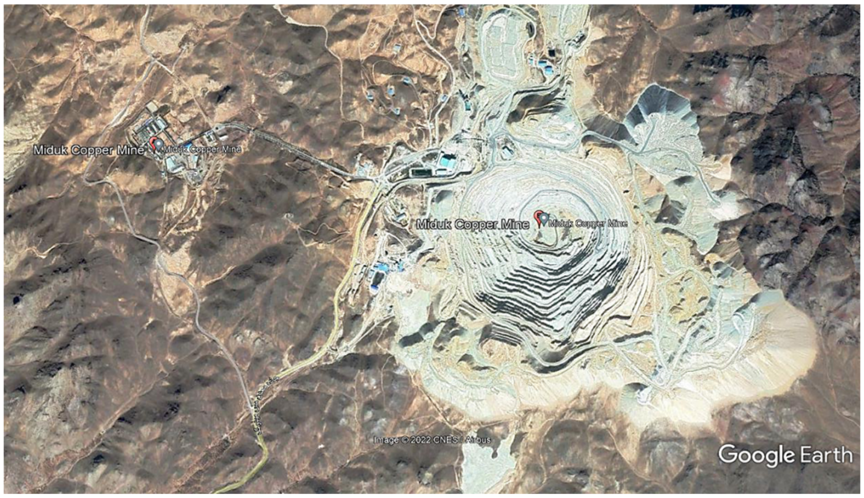


**Figure 5. The layout of the Miduk copper mine on the satellite picture (Google Earth- September 2012)**

The first pushback is selected to apply the presented model. There are 15391 blocks within the first push-back. In the schedules, the total mining capacity of the shovel–truck fleet was 40 million tonnes (Mt), maximum and minimum processing plant capacities were 14 Mt and 13 Mt of mill ore, and a maximum leaching capacity was 3.5 Mt of leach ore per annum. The cut-off grades and corresponding schedules were produced using 140 simulated copper price realizations obtained from the MRP model described in the previous section. To benchmark the proposed two-stage stochastic model, two schedules were produced; one using the proposed Stochastic Production Planning (SPP) and another one referred to as Deterministic Production Planning (DPP). In the DPP approach, the copper price was assumed to be US$8829/tonne and cut-off grades for discrimination between mill ore, leach ore, and waste, were assumed at 0.2% and 0.15% respectively. This part of the deposit would take about 7 years to mine considering the capacity constraints/operational capacities.

## 4. Discussion

### 5.1. Numerical results

As mentioned before, to model the uncertainty of data, we applied two-stage stochastic programming by using a finite number of scenarios. The relevant probabilities of these scenarios are assumed to be equal to 1/S. In the current study, we utilized MRP to produce commodity price scenarios. Table 2 summarizes the results of the schedules obtained using these two models of DPP and SPP. The term practical net present value refers to to the intersection of two models, DPP and SPP, which is equivalent to the first component of the objective function in the SPP model without considering penalty costs. The term theoretical net present value represents the sum of the first and second components of the objective function in the SPP model.

**Table 2. Results of implementation of DPP and SPP model for case study**

| Description | DPP | SPP |
|---|---|---|
| Practical NPV (US$) | 2,477,700,372 | 2,553,435,182 |
| Penalty costs (US$) | - | -131,173,491 |
| Theoretical NPV(US$) | 2,477,700,372 | 2,422,261,691 |
| Gap (%) | <2 | <2 |
| Time (h:m:s) | 52:08:40 | 53:40:15 |
| NO. Scheduled Blocks | 14695 | 15391 |
| Mill Ore (ton) | 97,956,337 | 114,153,300 |
| Leach Ore (ton) | 7,289,494 | 5,413,331 |
| Waste (ton) | 20,910,319 | 11,142,225 |
| Total (ton) | 126,156,150 | 130,708,856 |
| Stripping Ratio | 0.199 | 0.105 |

It should be noted that the objective function of both models is formulated based on the maximization of NPV, which captures the trade-offs between production targets and associated costs. In the SPP model, penalty costs are explicitly included, in contrast to DPP models where penalty costs are not considered. By excluding the penalty costs from the results of the SPP model, the NPV in the SPP model will exceed that of the DPP model. Therefore, the SPP model provides the NPV value equivalent to $2,553,435,182 representing a higher value and more accurate of the real-world system by considering the potential risks and associated penalties. Figure 6 and Figure 7 demonstrate the block sequencing of DPP and SPP models.

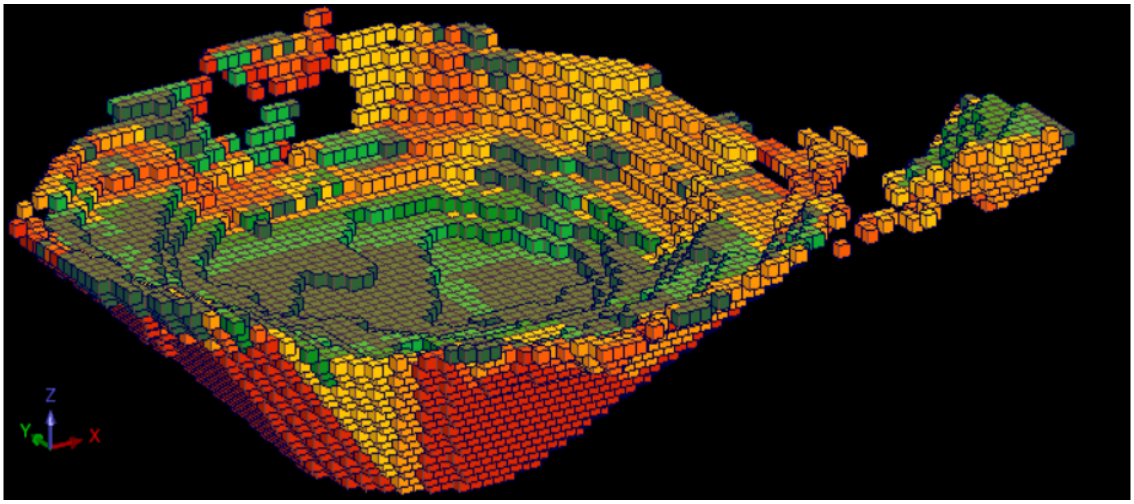


**Figure 6. Block sequencing in the DPP model**

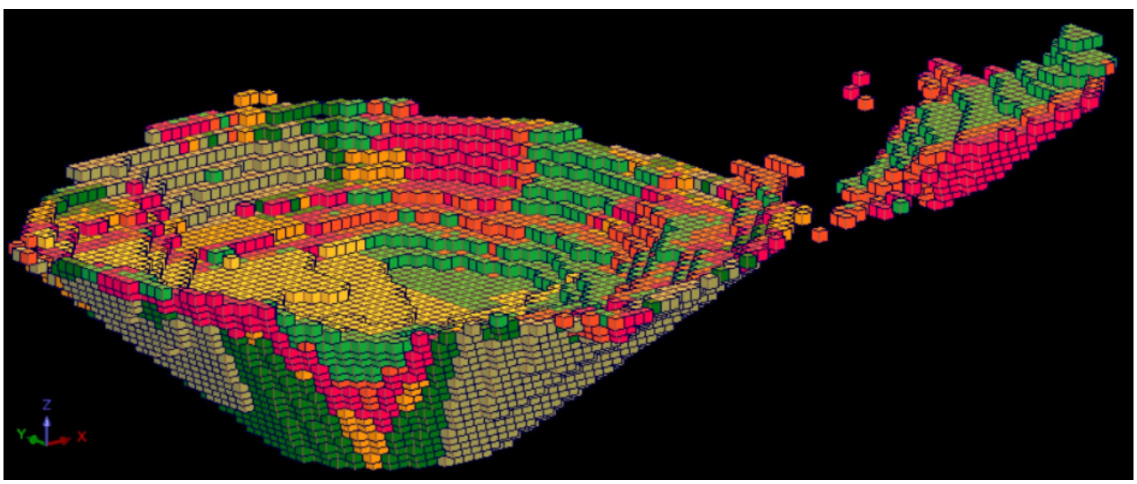


**Figure 7. Block sequencing in the SPP model**

According to Figure 8, the tonnage of the mineral sent to the processing plant and leaching follows the trend of copper metal price changes appropriately, indicating the effectiveness of the proposed model in adapting to the conditions of copper price fluctuations and dynamic cut-off grades. In other words, as the price increases and consequently the dynamic cut-off grades decrease in the stochastic model, more blocks are sent to the processing plant and leaching, demonstrating the adaptability of the stochastic model to conditions of declining copper prices and increasing cut-off grades. In the first year of operation, the amount of mineral in the plant is much higher than the annual plant capacity limit and is close to the total annual extraction amount. This can be attributed to the low penalty cost for exceeding production capacities, as the risk discount rate allows for higher penalties in the earlier periods compared to the later periods, resulting in the extraction of a higher number of blocks to maximize the net present value.

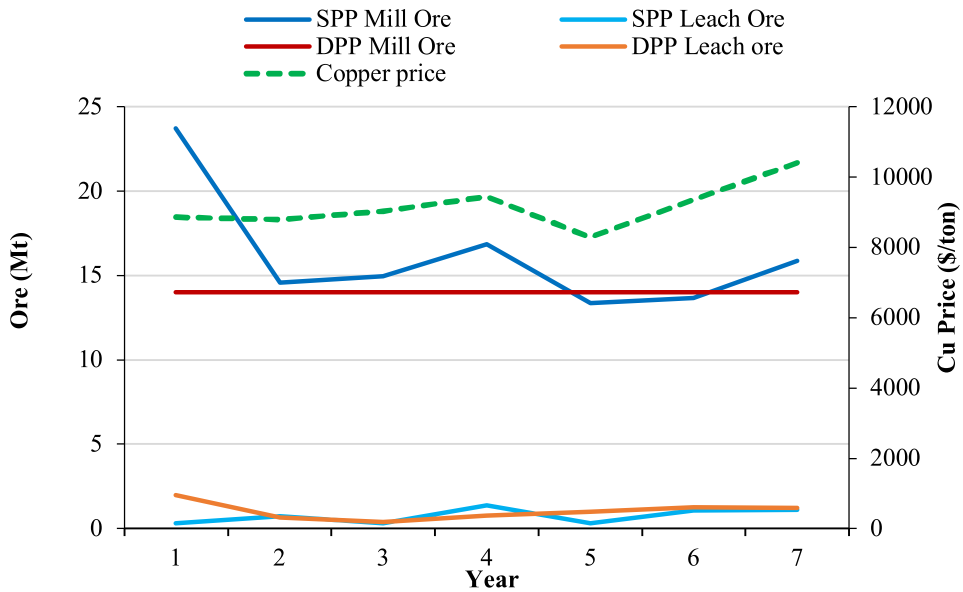


**Figure 8. Ore tonnage output**

As seen in Figure 9, the metal content sent to the mill process is higher in earlier periods than in later periods, meaning that the profit made by processing the ore will be higher. This is how the model reacts to the application of a discount factor being applied. Additionally, it shows that the risk of the metal content sent to the mill is smaller at the beginning than later as it begins to be higher in year 5 in SPP and year 6 in the DPP model.

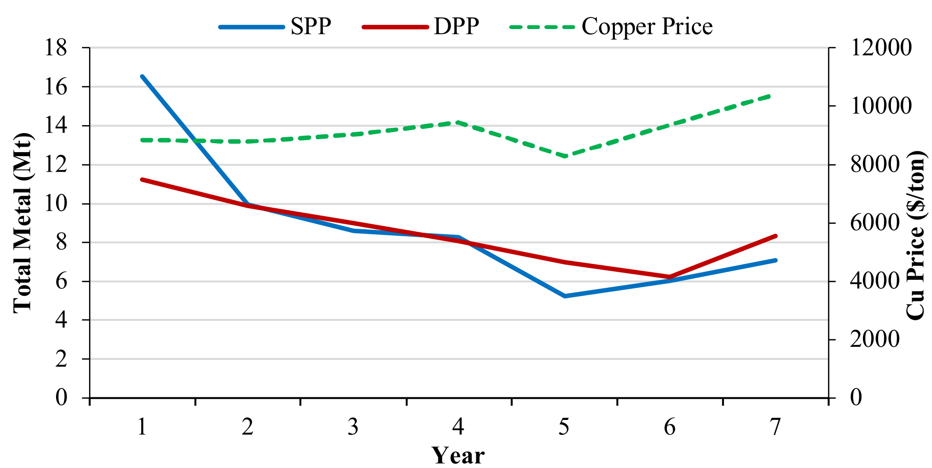


**Figure 9. Metal tonnage output**

Combined with the previous graph, this means that the risk is pushed to the later periods for this process.

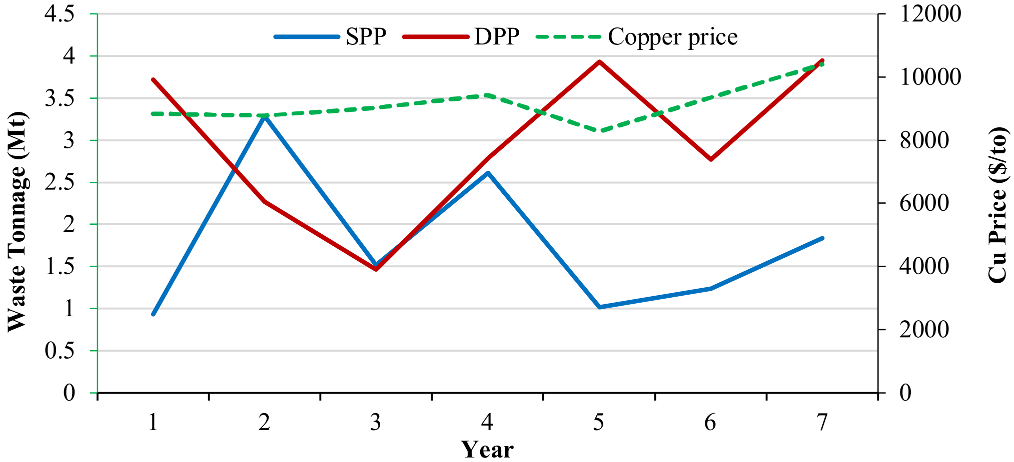


**Figure 10. Waste tonnage output**

Figure 10 exhibits the variations in waste tonnage over the 7-year mining operation. In the DPP model, regardless of price and cut-off grade changes, the waste materials have the highest amount in the first year and show an increasing trend over time. However, in the SPP model, parallel to the extraction of maximum mineral in the first year and to achieve a higher amount of metals as observed in the previous diagrams, the waste tonnage has the minimum amount in the first year and exhibits a decreasing trend over the 7-year mining operation.

The cumulative cash flow diagram of the DPP and SPP models has been plotted in Figure 11. The purple dashed line represents the difference in the cumulative cash flows between the deterministic and stochastic models.

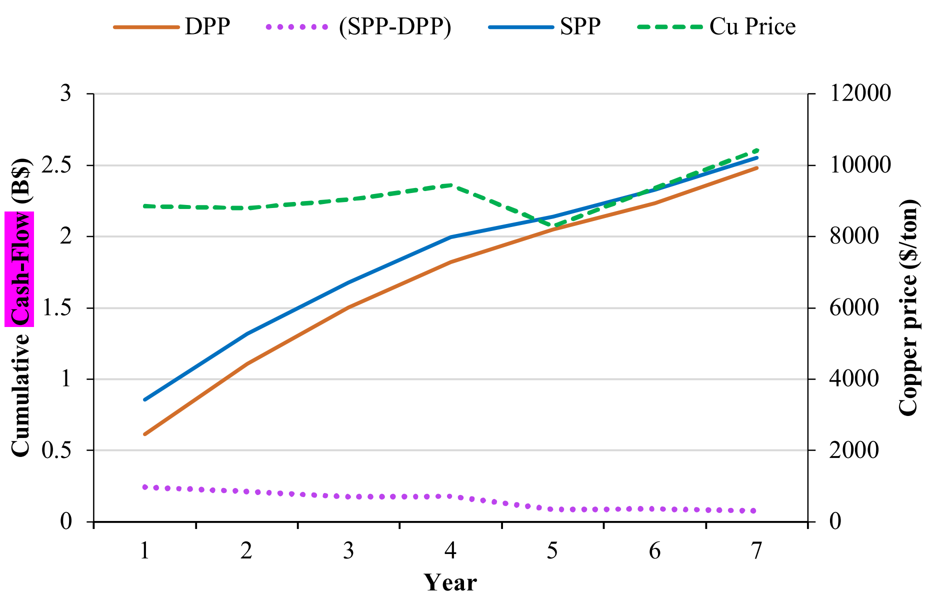


**Figure 11. Cumulative Cash flow output**

As observed, despite price fluctuations and changes in cut-off grades, the SPP model consistently maintains a higher level than the DPP model throughout the periods. Both models progress with almost the same slope until the fourth period. Due to a price decrease in the fourth period, the slope of the SPP diagram slightly decreases, and it continues with a new slope corresponding to a price increase in the fifth period. In contrast, the DPP diagram continues with the same previous slope, disregarding price variations. Therefore, the SPP model has successfully adapted to price uncertainty and changes in cut-off grades, and throughout all periods, it has consistently exhibited higher cumulative cash flow values.

## 5.2. Comparisons

Based on the results of production planning using DPP and SPP models, the practical NPV was estimated to be $2,477,700,372 and $2,553,435,182, respectively, with a difference of $75,734,810 higher for the stochastic model compared to the deterministic model. The financial loss resulting from price fluctuations and changes in cut-offs in the SPP model was estimated to be $131,173,491. Indeed, examining the impact of copper price scenarios on the results of the DPP model indicates a potential financial loss of $46,537,702 for production planning and operations based on the DPP model, which will be incurred during project operations.

Furthermore, the new SPP model, due to the prioritization of block scheduling based on price scenarios simultaneously with dynamic cut-offs, exhibits better performance and efficiency in terms of the selection of high-grade ore blocks. This can be clarified when comparing the tonnages in both models. Thus, based on the SPP model with a gap of less than 2 percent, a total of 114,153,300 tons of high-grade ore, 5,413,331 tons of low-grade ore, and 11,142,225 tons of waste material were extracted from the mining area, resulting in a total extraction of 130,708,856 tons. In the DPP model, out of the total 126,156,150 tons of rock extraction, 97,956,337 tons were high-grade ore, 7,289,494 tons were low-grade ore, and 20,910,319 tons were waste material. Overall, the stripping ratio in the SPP model was found to be 0.105, while in the DPP model, it was 0.199.

The results highlight the need for implementing risk mitigation strategies and adopting stochastic planning methodologies. By incorporating the SPP model and considering metal market uncertainties and corresponding cut-offs, the project can enhance its ability to adapt to changing conditions and mitigate the risk of substantial financial losses.

## 5. Conclusions

This paper presents a novel approach to open-pit mine planning that addresses the commodity price uncertainty. Traditional production scheduling models often overlook the dynamic nature of commodity prices and their impact on cut-off grades, leading to suboptimal plans.

The proposed model is constructed on a two-stage stochastic production programming (SPP) framework that optimizes long-term mine production schedules while considering both commodity price uncertainty and dynamic cut-off grades. The key innovation of this model lies in its ability to its simultaneous consideration of dynamic cut-off grades that adapt to price fluctuations within various operational scenarios, allowing for more flexible and robust mine plans.

The case study conducted at the Miduk copper mine located in Kerman, the largest province in southeast Iran, demonstrates the effectiveness of the proposed approach. By using commodity price scenarios generated through an MRP model, the SPP model outperformed the DPP model in terms of NPV and adaptability to changing conditions. The SPP model consistently maintained a higher level of cash flows than the DPP model, showcasing its ability to capture the value created by managing commodity price uncertainty.

Integrating the economic factors into mine scheduling, such as commodity price scenarios and dynamically adjusting cut-off grades, enables mining companies to make informed decisions, optimize profitability, and effectively respond to market volatility. Further academic research in this domain could delve into additional uncertainty factors influencing mine planning and refine stochastic modeling techniques to enhance the precision and reliability of results. By expanding the scope of investigation, scholars can contribute to advancing the understanding and application of advanced mine planning methodologies.

## References

[1]. Abdel Sabour, A., Dimitrakopoulos, R. (2011). Incorporating geological and Mine, market uncertainties and operational flexibility into open-pit mine design. *Journal of Mining Science*, *47(2)*, :191–201.

[2]. Abdel Sabour, S. A., Dimitrakopoulos, R. G., & Kumral, M. (2008). Mine design selection under uncertainty. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *117*(2), 53–64. https://doi.org/10.1179/174328608X343065.

[3]. Alipour, A, Khodaiari, A. A., Jafari, A., & Tavakkoli-Moghaddam, R. (2017). Robust production scheduling in open-pit mining under uncertainty: a box counterpart approach. *Journal of Mining and Environment*, *8*(2), 255–267. https://doi.org/10.22044/jme.2017.849.

[4]. Alipour, Aref, Khodaiari, A. A., Jafari, A., & Tavakkoli-Moghaddam, R. (2020). Robustness price of open-pit mine production scheduling. *International Journal of Mining and Geo-Engineering*, *54*(2), 117–122. https://doi.org/10.22059/IJMGE.2019.267417.594762.

[5]. Amponsah, S. Y., Takouda, P. M., & Ben-Awuah, E. (2023). A Multiple Objective Genetic Algorithm Approach for Stochastic Open Pit Production Scheduling Optimisation. *International Journal of Mining, Reclamation and Environment*, *37*(6), 460–487. https://doi.org/10.1080/17480930.2023.2196918.

[6]. Armstrong, M., Lagos, T., Emery, X., Homem-de-Mello, T., Lagos, G., & Sauré, D. (2021). Adaptive open-pit mining planning under geological uncertainty. *Resources Policy*, *72*, 102086. https://doi.org/https://doi.org/10.1016/j.resourpol.2021.102086.

[7]. Asad, M. W. A., & Dimitrakopoulos, R. (2013). Implementing a parametric maximum flow algorithm for optimal open pit mine design under uncertain supply and demand. *Journal of the Operational Research Society*, *64*(2), 185–197. https://doi.org/10.1057/jors.2012.26.

[8]. Bakhtavar, E., Jafarpour, A., & Yousefi, S. (2017). Optimal production strategy of bimetallic deposits under technical and economic uncertainties using stochastic chance-constrained programming. *Journal of Mining and Environment*, *8*(3), 475–485. https://doi.org/10.22044/jme.2017.934.

[9]. Bienstock, D., & Zuckerberg, M. (2010). Solving LP relaxations of large-scale precedence constrained problems. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, *6080 LNCS*, 1–14. https://doi.org/10.1007/978-3-642-13036-6_1.

[10]. Bley, A., Boland, N., Fricke, C., & Froyland, G. (2010). A strengthened formulation and cutting planes for the open pit mine production scheduling problem. *Computers & Operations Research*, *37*(9), 1641–1647. https://doi.org/https://doi.org/10.1016/j.cor.2009.12.008.

[11]. Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., & Rubio, E. (2012). A New Algorithm for the Open-Pit Mine Production Scheduling Problem. *Operations Research*, *60*(3), 517–528. http://www.jstor.org/stable/23260150.

[12]. Consuegra, F. R. A., & Dimitrakopoulos, R. (2010). Algorithmic approach to pushback design based on stochastic programming: Method, application and comparisons. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *119*(2), 88–101. https://doi.org/10.1179/037178410X12780655704761.

[13]. Del Castillo, F., & Dimitrakopoulos, R. (2014). Joint effect of commodity price and geological uncertainty over the life of mine and ultimate pit limit. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *123*(4), 207–219. https://doi.org/10.1179/1743286314Y.0000000069.

[14]. Dimitrakopoulos, R. G., & Abdel Sabour, S. A. (2007). Evaluating mine plans under uncertainty: Can the real options make a difference? *Resources Policy*, *32*(3), 116–125. https://doi.org/10.1016/j.resourpol.2007.06.003.

[15]. Dimitrakopoulos, R., & Lamghari, A. (2022). Simultaneous stochastic optimization of mining complexes - mineral value chains: an overview of concepts, examples and comparisons. *International Journal of Mining, Reclamation and Environment*, *36*(6), 443–460. https://doi.org/10.1080/17480930.2022.2065730.

[16]. Eduardo S. Schwartz. (1997). stochastic behaviour of commodity prices.pdf. In *The Journal of Finance* (Vol. 52, Issue 3, p. 51). https://static.twentyoverten.com/593e8a9e7299b471eaecf644/ H1tGPLaXM/The-Stochastic-Behavior-of-Commodity-Prices-Implications-for-Valuation-and-Hedging.pdf.

[17]. Eivazy, H., & Askari-Nasab, H. (2012a). A hierarchical open-pit mine production scheduling optimisation model Hesameddin Eivazy and Hooman Askari-Nasab *. *Int. J. Mining and Mineral Engineering*, *2*.

[18]. Eivazy, H., & Askari-Nasab, H. (2012b). A mixed integer linear programming model for short-term open pit mine production scheduling. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *121*(2), 97–108. https://doi.org/10.1179/1743286312Y.0000000006.

[19]. Fathollahzadeh, K., Waqar, M., Asad, A., & Mardaneh, E. (2021). Review of Solution Methodologies for Open Pit Mine Production Scheduling Problem. *International Journal of Mining, Reclamation and Environment*, *00*(00), 1–36. https://doi.org/10.1080/17480930.2021.1888395.

[20]. Franco-Sepulveda, G., Campuzano, C., & Pineda, C. (2017). NPV risk simulation of an open pit gold mine project under the O'Hara cost model by using GAs. *International Journal of Mining Science and Technology*, *27*(3), 557–565. https://doi.org/10.1016/j.ijmst.2017.03.004.

[21]. Gilani, S.-O., Sattarvand, J., Hajihassani, M., & Abdullah, S. S. (2020). A stochastic particle swarm based model for long term production planning of open pit mines considering the geological uncertainty. *Resources Policy*, *68*, 101738. https://doi.org/https://doi.org/10.1016/j.resourpol.2020.101738.

[22]. https://inflationdata.com/Inflation/Consumer_Price_Index/HistoricalCPI.aspx?reloaded=true. (n.d.).

[23]. INDEXMUNDI. 2018. Copper, grade A cathode, LME spot price. World Bank. http://www.indexmundi.com/commodities/?commodity = copper&months=300. (n.d.).

[24]. Jelvez, E., Ortiz, J., Varela, N. M., Askari-Nasab, H., & Nelis, G. (2023). A Multi-Stage Methodology for Long-Term Open-Pit Mine Production Planning under Ore Grade Uncertainty. *Mathematics*, *11*(18). https://doi.org/10.3390/math11183907.

[25]. Khan, A., Asad, M. W. A., & Topal, E. (2023). A heuristic method for production scheduling of an open pit mining operation. *International Journal of Mining, Reclamation and Environment*, 1–13. https://doi.org/10.1080/17480930.2023.2281201.

[26]. Kumral, M. (2011). Incorporating geo-metallurgical information into mine production scheduling. *Journal of the Operational Research Society*, *62*(1), 60–68. https://doi.org/10.1057/jors.2009.174.

[27]. Kumral, Mustafa. (2010). Robust stochastic mine production scheduling. *Engineering Optimization*, *42*(6), 567–579. https://doi.org/10.1080/03052150903353336.

[28]. Kumral, Mustafa, & Sari, Y. A. (2017a). An application of possibilistic programming to production sequencing of mining parcels. *International Journal of Management Science and Engineering Management*, *12*(2), 79–88. https://doi.org/10.1080/17509653.2016.1159147.

[29]. Kumral, Mustafa, & Sari, Y. A. (2017b). Simulation-based mine extraction sequencing with chance constrained risk tolerance. *Simulation*, *93*(6), 527–539. https://doi.org/10.1177/0037549717692415.

[30]. Lambert, W. B., & Newman, A. M. (2014). Tailored Lagrangian Relaxation for the open pit block sequencing problem. *Annals of Operations Research*, *222*(1), 419–438. https://doi.org/10.1007/s10479-012-1287-y.

[31]. Lamghari, A., & Dimitrakopoulos, R. (2012). A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. *European Journal of Operational Research*, *222*(3), 642–652. https://doi.org/10.1016/j.ejor.2012.05.029.

[32]. Laroche-Boisvert, M., & Dimitrakopoulos, R. (2021). An application of simultaneous stochastic optimization at a large open-pit gold mining complex under supply uncertainty. *Minerals*, *11*(2), 1–16. https://doi.org/10.3390/min11020172.

[33]. Liu, G., Guo, W., Fu, E., Yang, C., & Li, J. (2023). Dynamic optimization of open-pit coal mine production scheduling based on ARIMA and fuzzy structured element. *Frontiers in Earth Science*, *10*(January), 1–16. https://doi.org/10.3389/feart.2022.1040464.

[34]. Lotfian, R., Gholamnejad, J., & Mirzaeian Lardkeyvan, Y. (2021). Effective solution of the long-term open pit production planning problem using block clustering. *Engineering Optimization*, *53*(7), 1119–1134. https://doi.org/10.1080/0305215X.2020.1771703.

[35]. Meagher, C., Sabour, S. a A., & Dimitrakopoulos, R. (2009). Pushback Design of Open Pit Mines Under Geological and Market Uncertainties. In *Orebody Modelling and Strategic Mine Planning* (Vol. 17, Issue March, pp. 16–18).

[36]. Mohammadi, S., Kakaie, R., Ataei, M., & Pourzamani, E. (2017). Determination of the optimum cut-off grades and production scheduling in multi-product open pit mines using imperialist competitive algorithm (ICA). *Resources Policy*, *51*(November 2016), 39–48. https://doi.org/10.1016/j.resourpol.2016.11.005.

[37]. Mokhtarian Asl, M., & Sattarvand, J. (2016). Commodity price uncertainty propagation in open-pit mine production planning by Latin hypercube sampling method. *Journal of Mining and Environment*, *7*(2), 215–227. https://doi.org/10.22044/jme.2016.541.

[38]. Mokhtarian Asl, M., & Sattarvand, J. (2018). Integration of commodity price uncertainty in long-term open pit mine production planning by using an imperialist competitive algorithm . In *Journal of the Southern African Institute of Mining and Metallurgy* (Vol. 118, pp. 165–172). scieloza .

[39]. Moreno, E., Espinoza, D., & Goycoolea, M. (2010). Large-scale multi-period precedence constrained knapsack problem: A mining application. *Electronic Notes in Discrete Mathematics*, *36*, 407–414. https://doi.org/https://doi.org/10.1016/j.endm.2010.05.052.

[40]. Osanloo, M., Gholamnejad, J., & Karimi, B. (2008). Long-term open pit mine production planning: A review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, *22*(1), 3–35. https://doi.org/10.1080/17480930601118947.

[41]. Rahmanpour, M., & Osanloo, M. (2016). Determination of value at risk for long-term production planning in open pit mines in the presence of price uncertainty. *Journal of the Southern African Institute of Mining and Metallurgy*, *116*, 229–236. http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S2225-62532016000300007&nrm=iso.

[42]. Ramazan, S. (2007). The new Fundamental Tree Algorithm for production scheduling of open pit mines. *European Journal of Operational Research*, *177*(2), 1153–1166. https://doi.org/https://doi.org/10.1016/j.ejor.2005.12.035.

[43]. Ramazan, S., & Dimitrakopoulos, R. (2013). Production scheduling with uncertain supply: A new solution to the open pit mining problem. *Optimization and Engineering*, *14*(2), 361–380. https://doi.org/10.1007/s11081-012-9186-2.

[44]. Rimélé, A., Dimitrakopoulos, R., & Gamache, M. (2020). A dynamic stochastic programming approach for open-pit mine planning with geological and

commodity price uncertainty. *Resources Policy*, *65*, 101570. https://doi.org/https://doi.org/10.1016/j.resourpol.2019.101570.

[45]. Sabour, S. A. and Wood, G. (2009). Modelling financial risk in open pit Decision-making, mine projects: implications for strategic. *J. South. Afr. Inst. Min. Metall., 109,* 169–175. http://www.scielo.org.za/scielo.php?script=sci_abstract&pid=S2225-62532009000300004&lng=en&nrm=iso.

[46]. Shenavar, M., Ataee-Pour, M., & Rahmanpour, M. (2021). Evaluation of Underground Mineable Reserve in Presence of Grade and Commodity Price Uncertainties. *Journal of Mining and Environment*, *12*(2), 385–396. https://doi.org/10.22044/jme.2021.10301.1974.

[47]. Tabesh, M., & Askari-Nasab, H. (2011). Two-stage clustering algorithm for block aggregation in open pit mines. *Mining Technology*, *120*(3), 158–169. https://doi.org/10.1179/1743286311Y.0000000009.

[48]. Tahernejad, M., Ataei, M., & Khalokakaie, R. (2018). A practical approach to open-pit mine planning under price uncertainty using information gap decision theory. *Journal of Mining and Environment*, *9*(2), 527–537. https://doi.org/10.22044/jme.2017.6220.1439.

[49]. Tahernejad, M. M., Khalo Kakaei, R., & Ataei, M. (2018). Analyzing the effect of ore grade uncertainty in open pit mine planning; A case study of Rezvan iron mine, Iran. *International Journal of Mining and Geo-Engineering*, *52*(1), 53–60. https://ijmge.ut.ac.ir/article_66251.html.

[50]. Tolouei, K., Moosavi, E., Bangian Tabrizi, A. H., Afzal, P., & Aghajani Bazzazi, A. (2020). A comprehensive study of several meta-heuristic algorithms for open-pit mine production scheduling problem considering grade uncertainty. *Journal of Mining and Environment*, *11*(3), 721–736.

[51]. Tolouei, K., Moosavi, E., Tabrizi, A. H. B., Afzal, P., & Bazzazi, A. A. (2021). An optimisation approach for uncertainty-based long-term production scheduling in open-pit mines using meta-heuristic algorithms. *International Journal of Mining, Reclamation and Environment*, *35*(2), 115–140.

[52]. Zeng, L., Qiang, S., Kozan, E., Corry, P., & Masoud, M. (2021). A comprehensive interdisciplinary review of mine supply chain management. *Resources Policy*, *74*(January), 102274. https://doi.org/10.1016/j.resourpol.2021.102274.

# بهینه سازی برنامه‌ریزی تولید بلند مدت در معدنکاری روباز تحت عدم قطعیت قیمت محصول: رویکردی مبتنی بر برنامه ریزی تصادفی دو مرحله‌ای

الهام لطفی[1]، جواد غلام نژاد[1*]، مهدی نجفی[1] و محمدصادق زمانی[2]

۱. دانشکده مهندسی معدن و متالوژی، دانشگاه یزد، یزد، ایران
۲. دانشکده علوم ریاضی، دانشگاه یزد، یزد، ایران

ارسال ۲۰۲۳/۱۱/۱۸، پذیرش ۲۰۲۴/۰۶/۰۶

* نویسنده مسئول مکاتبات: j.gholamnejad@yazd.ac.ir

**چکیده:**

درعملیات معدن‌کاری روباز، برنامه زمان¬بندی تولید بلندمدت با چالش‌های قابل توجهی به دلیل عدم قطعیت‌های ذاتی به ویژه در قیمت محصـول مواجه اسـت. اتخاذ اسـتراتژی تخمین تک نقطه¬ای برای قیمت محصـول در مدل‌های ریاضـی سـنتی منجر به برنامه‌های غیربهینه و عدم دسـتیابی به اهداف تولید می‌شـود. در تحقیقات پیشـین تأثیر همزمان برآورد تاثیر عدم قطعیت  قیمت محصـول بر عیارهای حد و برنامه ریزی تولید بلندمدت کمتر مورد توجه قرار گرفته اسـت. بر این اســاس در این تحقیق، مدلی نوآورانه برای بهینه‌ســازی برنامه ریزی تولید بلندمدت معدن روباز تحت عدم قطعیت قیمت محصــول با در نظر گرفتن اســتراتژی عیارهای حد پویا و در چارچوب برنامه‌ریزی تولید تصـادفی دو مرحله‌ای (SPP) ارایه شـده اسـت. این مدل درصـدد شـناسـایی توالی اسـتخراج بهینه برای بلوک‌های معدنی، بیشینه¬سازی جریان¬ های نقدینگی تنزیل یافته همزمان با کمینه¬سازی جرایم انحراف از اهداف تولید است. برای بررسـی کارآیی مدل، پیاده‌سازی آن برای برنامه¬ریزی در یک معدن مس انجام گرفته اسـت. در ابتدا، مدل حرکت براونی هندسـی (GBM) جهت برآورد قیمت محصـول در آینده اسـتفاده شـده اسـت. سـپس، هر دو مدل قطعی و تصـادفی (SPP) با اسـتفاده از نرم‌افزار GAMS کدنویسـی وحل شـدند. نتایج حاکی ازآن اسـت که سـود خالص عملیاتی در مدل SPP نسبت به مدل DPP تقریباً ۳٪ بیشتر است.

**کلمات کلیدی:** برنامه‌ریزی تولید بلند مدت، معدنکاری روباز، عدم قطعیت قیمت فلز، عیارهای حد پویا، برنامه‌ریزی تصادفی دو مرحله‌ای.