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Dynamic Pit Tracker: An Iterative Heuristic Algorithm Tracing Optimized Solution for Ultimate Pit Limit and Blocks Sequencing Problem

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Abstract

One of the most critical designs in open-pit mining is the ultimate pit limit (UPL). The UPL is frequently computed initially through profit-maximizing algorithms like the Lerchs-Grossman (LG). Then, in order to optimize net present value (NPV), production planning is executed for the blocks that fall within the designated pit limit. This paper presents a mathematical model of the UPL with NPV maximization, enabling simultaneous determination of the UPL and long-term production planning. Model behavior is nonlinear. Thus, in order to achieve model linearization, the model has been partitioned into two linear sub-problems. The procedure facilitates the model solution and the strategy by decreasing the number of decision variables. Naturally, the model is NP-Hard. As a result, in order to address the issue, the Dynamic Pit Tracker (DPT) heuristic algorithm was devised, accepting economic block models as input. A comparison is made between the economic values and positional weights of blocks throughout the steps in order to identify the most appropriate block. The outcomes of the mathematical model, LG, and Latorre-Golosinski (LAGO) algorithms were assessed in relation to the DPT on a two-dimensional block model. Comparative analysis revealed that the UPLs generated by these algorithms are consistent in this instance. Utilizing the new algorithm to determine UPL for a 3D block model revealed that the final pit profit matched LG UPL by 97.95%.

1. Introduction

Various designs should be carried out for open pit mining. In general, open pit mines commonly are designed in two stages: Ultimate Pit Limit (UPL) optimization and production planning. UPL is the term used to refer to the ultimate boundary of an open pit mine that is attained upon the mine's decommissioning. In reality, the economic threshold of surface mining operations is established by the outline. This indicates that surface mining beyond the boundary is not economically viable, and miners are advised to contemplate adopting an underground approach or abandoning the operation entirely. So, UPL design itself is an economic evaluation. In this way, the most common goal of UPL determination is to maximize profits. Following this, long- and short-term production planning will be conducted to achieve the highest Net Present Value (NPV). Like other long-term economic projects with huge amounts of investments. Ideally, the criterion of ULP optimization should be maximization of NPV of the pit [1], which previously was justified by Whittle [2]. This means the extraction sequence of blocks and UPL should be determined at the same time.

In the past decades, some algorithms have been proposed for designing of the final pit with mathematical, heuristic, and meta-heuristic approaches. The Floating Cone (FC) algorithm [3] and its modified methods [4] and Krobov algorithm

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[5] are some of the heuristic algorithms for the UPL determination. The algorithms focused on undiscounted profit maximization instead of NPV maximization. Meta-heuristic methods such as the Genetic Algorithm (GA) [6] and artificial neural networks (ANN) [7] have been used to determine UPL. In this regard, Sayadi et al. [8] developed a new 3D pit optimization algorithm using the neural network and applied it in an economic classified block model. Franco-Sepulveda et al. [9] also discussed the application of ANN in open pit mining. Jodeiri et al. [10] introduced a Flashlight Algorithm (FL) as a heuristic approach to determine UPL. Unlike other heuristic algorithms, it is based on the movement from bottom to top. According to the results presented by the researchers, the FL algorithm performed better than Korobov and FC algorithms in the examined cases. Turan and Onur [11] also developed an improved floating cone algorithm to optimize open pit mine design and production planning. The mentioned algorithms have been developed based on approximate approaches which means they don't guarantee an optimal solution. In contrast, some algorithms are based on mathematical solutions. Lerchs-Grossman's algorithm (LG) [12] based on graph theory is a mathematical approach and it finds the optimal solution. Liu and Kozan [13] developed two novel graph-based algorithms based on network flow graph and conjunctive graph theory. Esmaeil et al. [14] developed a logical mathematical algorithm that considers the important designing parameters and the mining economy. The mathematical approaches still aim to design the UPL according to the profit maximization. Another problem with the mathematical algorithms is their immense complexity of understanding and programing. On the other hand, the running time of solution will considerably increase with increase in the size of the problem. Another weakness of the algorithms reviewed is that they do not provide extraction sequences for the mining blocks.

Unlike the above methods, some algorithms are also provided for the simultaneous determination of the UPL and the extraction sequence of mining blocks. Caccetta and Hill [15] and Saleki et al. [16] mathematically proved that the UPL resulted from NPV maximization is smaller than, or equal to, the one resulted from NPV maximization. This means that theoretically and ideally there is no warranty that LG's ultimate pit outline produces the maximum NPV. So, Roman [17], Gershon [18], Wang and Sevim [19, 20, 21], Askari-Nasab and Awuah-Offei [22], Latorre and Golosinski [23], and Saleki et al. [24] developed innovative methodologies and heuristic algorithms to determine the final pit with the aim of NPV maximization. The disadvantages of these methods, or some of them, include lack of mathematical proof, two-dimensionality, complexity, and non-use in industrial and commercial terms.

Another classification of approaches dealing with open pit mine design is deterministic/uncertain. During the past decade, several approaches considering economical and geological factors uncertainties in open pit mining have been developed. Dimitrakopoulos [25] considered a new mine planning paradigm integrating stochastic simulation and stochastic optimization. Benndorf and Dimitrakopoulos [26] developed a stochastic integer programming formulation (SIP) to integrate geological uncertainty described by sets of equally possible scenarios of the unknown orebody. Chatterjee et al. [27] implemented a minimum cut network flow algorithm for the optimal production phase and ultimate pit limit design under commodity price or market uncertainty. Lamghari and Dimitrakopoulos [28] introduced a new open-pit mine production scheduling problem (MPSP) formulation accounting for metal uncertainty and considering multiple destinations for the mined material, including stockpiles. They compared four different heuristics for the problem. Gilani and Sattarvand [29] developed a stochastic optimization algorithm based on ant colony optimization (ACO) approach to integrate geological uncertainty described through a series of the simulated ore bodies. Richmond [30] proposed an algorithm integrating Monte Carlo-based simulation and heuristic optimization techniques which can account explicitly for commodity price cycles and uncertainty. Upadhyay and Askari-Nasab [31] presented simulation optimization framework/tool to account for uncertainties in mining operations for robust short-term production planning and proactive decision making. Paithankar and Chatterjee [32] used the maximum flow algorithm with a genetic algorithm to generate the long-term production schedule. Rimele et al. [33] studied the combined effect of geological and commodity price uncertainty. Gilani et al. [34] used a stochastic particle swarm based model to consider geological uncertainty in long term production planning optimization. Acorn et al. [35] used a heuristic pit optimizer to manage the effect of geological uncertainty in the resources within a pit shell with multiple uncertainty rated solutions. Lagos et al. [36] also presents an adaptive optimization scheme for multi-period production scheduling in open-pit mining under geological uncertainty. Armstrong et al. [37] developed an adaptive stochastic optimisation approach for multi-period production scheduling in open-pit mines under geological uncertainty, and compared it to an existing two-stage optimisation method. Danish et al. [38] presented a Simulated Annealing based stochastic optimization algorithm to integrate geological uncertainty into the optimization process while considering stockpiling options and other relevant constraints.

Industrially, as well as practically speaking, algorithms for open pit design are the core of technical and commercial software packages widely used in the mining industry. Educationally, they are an essential component of surface mining courses for future mining engineers. These algorithms are easier to understand for educational purposes than more sophisticated methodologies, which enable students to better understand open pit mining design. The literature review indicates that in recent years there has been less emphasis on innovative heuristic algorithms. On the other hand, it has been partially neglected to develop algorithmic solutions aiming at NPV maximization as the ideal goal of UPL optimization and design. Furthermore, the topic of UPL and production planning are among challenging mathematical optimization problems in which researchers are interested and come up with new algorithms and solutions. So, new solutions to the problems can develop the area in different ways. Consequently, as the literature review shows, the new approaches can be bases to lead researchers to think up and find new ideas about open pit mining to increase the accuracy and efficiency of the approaches. In this regard, broadening the mathematical definition of the optimization goal of UPL and development of solutions to the models are some key aspects of open pit mining which need more attention. Therefore, methodologies are needed seeking novel solutions aiming at UPL and NPV optimization simultaneously. As mentioned, one of the main approaches in optimization is development of heuristics to combine UPL and blocks extraction sequence.

In this paper, an integer mathematical model is presented to determine the optimal final pit by maximizing NPV. Because the proposed model is non-linear and complicated to solve, it has been linearized in two steps. Then, a heuristic algorithm was developed to solve the model. For a 2D block model, the results of the objective function and the proposed algorithm were compared with the results of other algorithms. The algorithms have been applied to determine simultaneously the ultimate pit limit and blocks' extraction sequence of a 3D economic block model.

2. Mathematical model of ultimate pit limit to maximize NPV

The final pit limit strategy determination can be illustrated in Figure 1. In this process, after removing the dispensable blocks, the optimum order of extraction of the ore blocks will be determined.

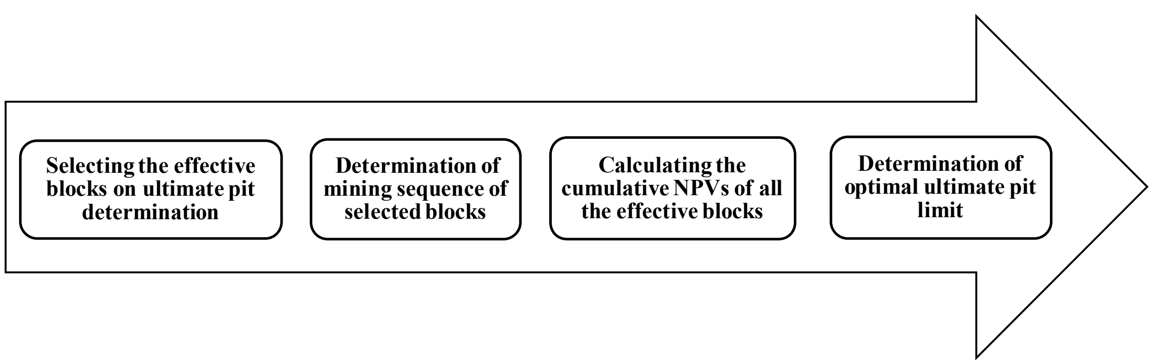


Figure 1. Suggested process of determination of the optimal ultimate pit limit

2.1. Notation

In table 1, the indices, sets and parameters related to sets are presented for use in this paper. Also, the right-handed coordinate system used in the modeling is shown in Figure 2.a. Accordingly, the x axis represents the vertical direction with index of i. The y axis is east-west, with index of j, and the z axis is north-south and its index is k. The visual definitions of the sets have been illustrated in Figure 2.b.

Table 1. The indices, sets and parameters related to sets

Explanation	Set/Index/Parameter related to sets
number of blocks inside the biggest possible pit (*BPP*)	N
set of integer numbers	$\mathbb{Z}$
set of coordinates of all blocks within the ore body	$OB \subset \mathbb{Z}^3$
set of coordinates of blocks within the *BPP*	$BPP \subset \mathbb{Z}^3$
coordinates of the blocks within *BPP* In horizon *i*	$BPP_i \subset \mathbb{Z}^3$
set of coordinates within the BPP_i that are valid in the defined conditions	$BPP_i^{jk} = \{(i,j',k') \in BPP_i \mid k' \neq k \wedge j' \neq j\}$
set of coordinates of blocks within the downward cone of block *ijk* without its coordinate	$DC_{ijk} \subset \mathbb{Z}^3$
set of coordinates of the blocks within the *BPP* and outside DC_{ijk} and below the horizon *i*	$ODC_{ijk} \subset \mathbb{Z}^3$
set of coordinates of blocks above the block *ijk* in the horizon $i-1$ that must be removed to extract it. (in 3D models 9 blocks for each block (1: 9) and for 2D models 1:3)	$UB_{ijk} = \left\{\begin{matrix}(i-1,j,k-1),(i-1,j,k),\\(i-1,j,k+1),(i-1,j-1,k-1),\\(i-1,j-1,k),(i-1,j-1,k+1),\\(i-1,j+1,k-1),(i-1,j+1,k),\\(i-1,j+1,k+1)\end{matrix}\right\} \subset \mathbb{Z}^3$

Between the above defined sets, the following relations are established:

$$BPP \subseteq OB \tag{1}$$

$$BPP_i \subset BPP \tag{2}$$

$$\bigcup_{i=1}^{i_{max}\cup} BPP_i \tag{3}$$

$$ODC_{ijk} = (BPP \backslash DC_{ijk}) \backslash BPP_i \tag{4}$$

Other parameters and decision variables are also given in Table 2.

Table 2. Parameters and Decision Variables

Explanation	Parameter/decision variable
value of block *ijk* (assuming the stability of economic parameters over time)	V_{ijk}
highest (optimal value) of the cumulative Net Present Value	$NPV_{\max}$
integer decision variable represents the optimal order of extracting the block *ijk*	$y_{ijk} \in \square$
binary variable indicates the presence or absence of block *ijk* in the ultimate optimum pit	$x_{ijk} = \begin{cases} 0 & \text{if the block is out of optimum pit limit} \\ 1 & \text{if the block is within optimum pit limit} \end{cases}$
Auxiliary binary variable to prevent equalization of the values of y_{ijk} and $y_{i'j'k'}$ decision variables	$q_{ijk,i'j'k'} \in \{0,1\}$
Discount rate for the period of the extraction of one block	c
A large integer number	M

2.2. Mathematical modeling

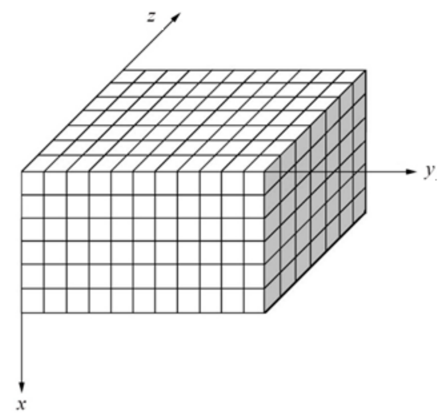
The objective function of the traditional ultimate pit limit determination for undiscounted profit maximization is shown in Eq. (5). Most of the ultimate pit limit determination algorithms have been developed to achieve a precise or approximate answer to this objective function.

$$Max \quad Z = \sum_{(i,j,k)\in OB} V_{ijk} \times x_{ijk} \tag{5}$$

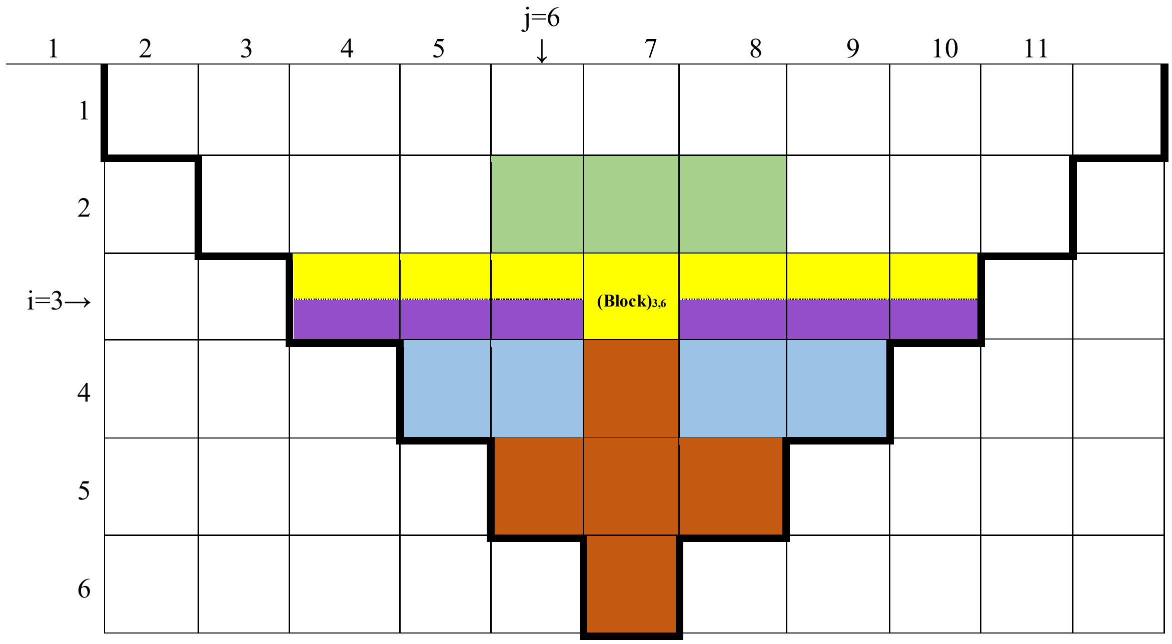
Subject to:

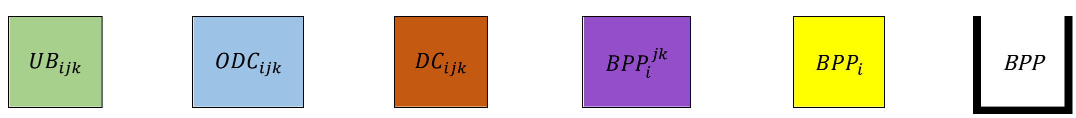
$$x_{ijk} \le x_{i'j'k'} \quad \forall (i,j,k) \in OB,\ (i',j',k') \in UB_{ijk} \tag{6}$$

In order to determine the final pit with the aim of maximizing NPV, the model should be written for all the blocks. Its objective function must have two sets of variables simultaneously, one variable that determines the optimal final pit and the other for optimal extraction sequence. So each block has two variables, one for its extraction sequence, and the other to define its existence or not within the final pit.



a) axes of *x*, *y*, *z*





b) Definition of BPP_i, BPP_i^{jk}, DC_{ijk}, ODC_{ijk}, UB_{ijk}

Figure 2. Block model of deposit and axes of *x*, *y* and *z*, and illustration of sets defined in Table 1.

Since, the search space for the problem solution includes all the blocks in the deposit, the size of the problem increases dramatically. To reduce this huge size, one of the techniques suggested by Wang and Swim [20] is used. It is the determination of a pit as the biggest possible pit (BPP) by considering the technical slope and permitted mining limits. This pit includes the deepest and horizontally farthest ore blocks (Figure 3). In fact, the set of the blocks within BPP are the blocks effective on the UPL determination. The technique also has been used before in heuristic algorithms [10, 24]. Using this technique, rubbish blocks that have no effect on the final pit determination will be eliminated from the calculations. The total space of a 2D deposit and its BPP, are shown in Figures 3a and 3b, respectively.

w	w	o	o	o	w	o	w	w	w
w	w	o	w	w	w	o	o	o	w
w	w	w	o	o	o	w	o	w	w
w	w	w	o	o	w	w	w	w	w
w	w	w	w	w	w	w	w	w	w

a) Entire search space for the whole of the ore body to design the UPL (O: ore Blocks, W: waste Blocks)

w	w	o	o	o	w	o	w	w	w
w	w	o	w	w	w	o	o	o	w
w	w	w	o	o	o	w	o	w	w
w	w	w	o	o	w	w	w	w	w
w	w	w	w	w	w	w	w	w	w

←BPP

b) BPP formation and restricting search space

Figure 3. Concept of BPP

According to the above description, the following non-linear integer programming model is presented to find the optimal final pit limit with NPV maximization. The model expresses steps presented in Figure 1.

$$Max\ \ Z = \sum_{(i,j,k)\in BPP} V_{ijk} \times \left(\frac{1}{1+c}\right)^{y_{ijk}} \times x_{ijk} \qquad (7)$$

Subject to:

$$x_{ijk} \le x_{i'j'k'} \qquad \forall (i,j,k)\in BPP,\ (i',j',k')\in UB_{ijk} \qquad (8)$$

$$y_{ijk} > y_{i'j'k'} \qquad \forall (i,j,k)\in BPP,\ (i',j',k')\in UB_{ijk} \qquad (9)$$

$$\left.\begin{array}{l} y_{ijk} - y_{ij'k'} - M \times q_{ijk,ij'k'} < 0 \\ y_{ijk} - y_{ij'k'} + M \times (1 - q_{ijk,ij'k'}) > 0 \end{array}\right\} \qquad \forall (i,j,k)\in BPP,\ (i,j',k')\in BPP_i^{jk} \qquad (10)$$

$$\left.\begin{array}{l} y_{ijk} - y_{i''j''k''} - M \times q_{ijk,i''j''k''} < 0 \\ y_{ijk} - y_{i''j''k''} + M \times (1 - q_{ijk,i''j''k''}) > 0 \end{array}\right\} \qquad \forall (i,j,k)\in BPP,(i'',j'',k'')\in ODC_{ijk} \qquad (11)$$

$$1 \le y_{ijk} \le N \qquad \forall (i,j,k)\in BPP \qquad (12)$$

The constraints (8) and (9) are resulted from the technical necessity of blocks removal order. The constraint (8) ensures removing 9 blocks immediately above each block before its extraction. Eq. (9) also states that the extraction number of each block is larger than the extraction number of its 9 immediate upper blocks.

The constraint (9) ensures the inequality of the extraction sequence number of blocks within the upward cone of the block *ijk* with the extraction sequence number of block *ijk*. So, the model needs some others constraints to prevent equality between the *ijk* block's sequence and blocks within the same level of *I* and also inside BPP but outside the *ijk* downward cone. The constraints (10) and (11), using the auxiliary binary variables, guarantee the allocation of each extraction time to only one block and each block to just one time. The "*q*" is an auxiliary binary variable combined with the big number of M (bigger than the total number of blocks). During the calculations by examining different values of *q* (0 or 1) at the moment only one of the equations in constraint (10) remains in the solution process because the other one becomes a valid term regardless of the values of "*y*" as M is a big number. For example, if *q* is zero, the left side of the second equation in (10) is always positive and valid. So this equation is negligible and only "$y_{ijk} - y_{ij'k'} < 0$" will be the effective constraint which means y_{ijk} and $y_{ij'k'}$ are not equal. The solver will examine both 0 and 1 for *q* and finds the optimal answer. Equations (10) and (11) prevent equality of variable y_{ijk} in levels, and within ODC_{ijk}, respectively.

The number of main variables of this objective function is *N* integer variables (y_{ijk}) and *N* binary ones, totally *2 × N*. The number of auxiliary variables depends entirely on the shape of the *BPP* and the sets associated with their constraints.

It should also be noted that by setting the discount rate to zero in the objective function (7), the model turns to the model of the ultimate pit limit with profit maximization. This indicates that the model of the ultimate pit limit with profit maximization is a special state of the final pit with NPV maximization.

2.2.1. Linearizing the objective function

Function (7) is a nonlinear-integer model and due to the huge number of its decision variables, it is very difficult to solve and requires powerful computer memory and processors. Usually, solver software programs find non-accurate and local answers to these kinds of problems. To solve this problem, according to the main strategy, the function is divided into two linear steps. In the first step, the optimal extraction sequence for all blocks will be obtained. Subsequently, the final pit limit will be determined. In this way, the nonlinear function is broken into two linear functions, each of which has N variables.

2.2.1.1. First Step: Determining the optimal sequencing of blocks

To solve the objective function (7), it is necessary to determine the optimal removal order of blocks. To maximize the sum of terms of function (7), each term has to be maximized in accordance with the constraints. In a technical sense, since the coefficient $1/(1+c)$ is less than 1, subsequently the ore blocks (positive terms) must be mined earlier (lowest possible y_{ijk}), and the waste blocks (negative ones) should be extracted afterward.

The mathematical model of the first stage is represented in two equivalent forms of (13) and (14):

$$Max\ \ Z = \sum_{(i,j,k)\in BPP} -1\times V_{ijk} \times y_{ijk} \tag{13}$$

or

$$Min\ \ Z = \sum_{(i,j,k)\in BPP} V_{ijk} \times y_{ijk} \tag{14}$$

Subject to:

$$y_{ijk} > y_{i'j'k'} \qquad \forall (i,j,k) \in BPP,\ (i',j',k') \in UB_{ijk} \tag{15}$$

$$\left.\begin{aligned} & y_{ijk} - y_{ij'k'} - M \times q_{ijk,ij'k'} < 0 \\ & y_{ijk} - y_{ij'k'} + M \times (1 - q_{ijk,ij'k'}) > 0 \end{aligned}\right\} \quad \forall (i,j,k) \in BPP,\ (i,j',k') \in BPP_i^{jk} \tag{16}$$

$$\left.\begin{aligned} & y_{ijk} - y_{i''j''k''} - M \times q_{ijk,i''j''k''} < 0 \\ & y_{ijk} - y_{i''j''k''} + M \times (1 - q_{ijk,i''j''k''}) > 0 \end{aligned}\right\} \quad \forall (i,j,k) \in BPP, (i'',j'',k'') \in ODC_{ijk} \tag{17}$$

$$1 \le y_{ijk} \le N \qquad \forall (i,j,k) \in BPP \tag{18}$$

The equations (13) and (14) sort the extraction sequence of blocks within BPP according to their value subject to the mining and technical constraints in a descending way to extract the blocks with higher value earlier.

2.1.1.2. Second Step: Determination of optimum pit limit

In the mathematical model of second step, the variables solved in function (13) or (14) are used as constant coefficients in determining the net present value of each block. In this step, the binary variables associated with the final pit are unknown and should be determined. It should be noted that at this stage, the discount rate parameter (c) plays an important role in determining the net present value of each block (NPV_{ijk}). The objective function of the second stage is expressed as follows:

$$Max\ \ Z = \sum_{(i,j,k) \in BPP} \underbrace{V_{ijk} \times \left(\frac{1}{1+c}\right)^{y_{ijk}}}_{NPV_{ijk}} \times x_{ijk} \tag{19}$$

Subject to:

$$x_{ijk} \le x_{i'j'k'}$$
$$\forall (i,j,k) \in BPP,\ (i',j',k') \in UB_{ijk} \tag{20}$$

To solve the objective function (Eq. 13 or 14), the cumulative NPVs of the blocks should be calculated and drawn graphically. Then the global extremum of the cumulative values is defined as the ultimate pit limit ($y_{NPV_{\max}}$). In other words, the block with order of $y_{NPV_{\max}}$ and all blocks having the extraction order less than it, are considered as the optimum final pit.

Flowing the model development, a small 2D example is explained in order to illustrate steps involved in the proposed model. At first, the objective function (7) for the two-dimensional block model assumed in Figure 4 is solved with Lingo. This solution is in one step. Then the same objective function is solved in two steps by using Equations. (13) or (14) and (19) and the final pit can be calculated. Figure 5 shows the BPP for the model

j →, i ↓

-1	-1	2	-1	-1
-1	+4	-1	-1	-1
-1	-1	+1	-1	-1

Figure 4. 2D economic block model example

-1	-1	2	-1	-1
-1	+4	-1	-1	-1
-1	-1	+1	-1	-1

(shaded) ←*BPP*

Figure 5. BPP for the two-dimensional economic block model

The results from solving with Lingo for this model by following parameters are represented in Figure 6:

$N = 9$

$c = 0.1$

M=100

-1 …….. 1,3	-1 …….. 1,2	+2 …….. 1,1	-1 …….. 0,6	-1 …….. 0,5
	+4 …….. 1,4	-1 …….. 0,7	-1 …….. 0,8	
		+1 …….. 0,9		

V_{ij} …….. x_{ij}, y_{ij}

Figure 6. Determination of UPL and optimal extraction order of blocks simultaneously in one step

Solving the model in two steps:

A) First step: Determining the optimum sequences of extraction of blocks

In this step, decision variables of the mining orders of blocks are specified. Subsequently the optimum pit should be determined. After solving this model, its global optimal solution was obtained. The obtained optimal sequence is shown in Figure 7.

-1 …….. 3	-1 …….. 2	+2 …….. 1	-1 …….. 6	-1 …….. 5
	+4 …….. 4	-1 …….. 7	-1 …….. 8	
		+1 …….. 9		

V_{ij} …….. y_{ij}

Figure 7. Determining the optimal sequences of the whole of BPP blocks

B) Second step: Ultimate pit limit determination

In the second step, determining the ultimate pit limit, the answers obtained for the decision variables of the first step (mining sequencing of blocks) are used as fixed parameters in the second stage. The results of this stage obtained from the Lingo solver are shown in Figure 8. As the results show, the final pit with an NPV of 2.972 currency units (CU) and 4 blocks has the highest NPV. The cumulative values of the NPVs are presented in Table 3. In Figure 9, the graph of the cumulative values is shown.

1 …….. 1,3	-1 …….. 1,2	+2 …….. 1,1	-1 …….. 0,6	-1 …….. 0,5
	+4 …….. 1,4	-1 …….. 0,7	-1 …….. 0,8	
		+1 …….. 0,9		

V_{ij} …….. x_{ij}, y_{ij}

Figure 8. Optimum UPL obtained in the second step

Table 3. Calculating the cumulative NPV for the example model

Block Coordinate	Optimal Sequencing	Value (CU)	NPV (CU)	Cumulative NPV (CU)
(1,3)	1	2	1.818	1.818
(1,2)	2	-1	-0.826	0.991
(1,1)	3	-1	-0.751	0.24
(2,2)	**4**	4	2.732	**2.972** (NPV_{max})
(1,5)	5	-1	-0.62	2.351
(1,4)	6	-1	-0.564	1.787
(2,3)	7	-1	-0.513	1.273
(2,4)	8	-1	-0.466	0.807
(3,3)	9	1	0.424	1.231

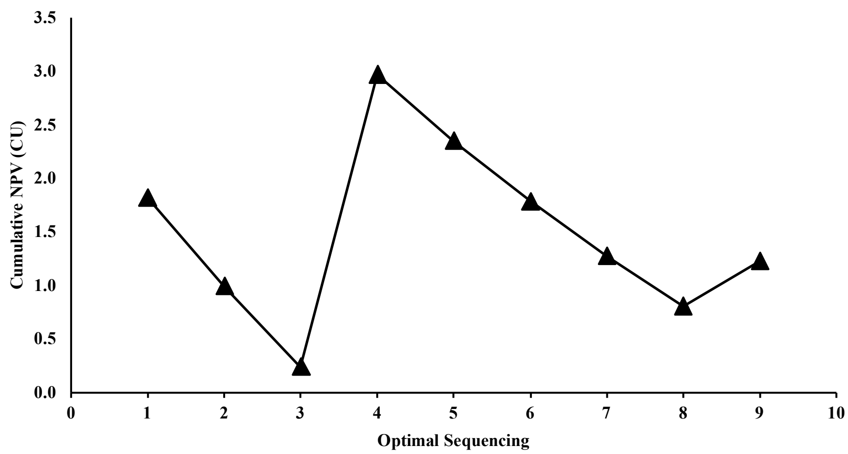


Figure 9. Cumulative chart of NPVs and its maximum value

In this example, the results of the nonlinear and linear strategies are in agreement. But one-step solution for real deposits with a large number of variables, is local, approximate, and time-consuming. It is therefore very difficult to determine its exact optimal answer. Due to the fact that each linear sub-problem contains one-half of the total number of non-linear variables, breaking the main issue into two linear sub-problems is an appropriate mathematical solution.

3. Heuristic Solution

The mathematical model developed is NP-Hard and is so difficult to solve by a mathematical solution in a reasonable time. So, one of the most used strategies to find an acceptable solution for this type of optimization problems is a heuristic algorithm. These algorithms are looking for the best answer during a simpler process which is achievable in a reasonable time.

To solve the model, the heuristic algorithm called Dynamic Pit Tracker (DPT) shown in Figure 10 has been developed. The algorithm's input is the economic block model of the ore body. To use the algorithm, the monetary value and positional weight (PW) of blocks are needed. PW is the sum of the positive values of blocks within a block's downward cone (DC). Previously, downward cones and PWs were only used in grade block models [18, 24], but in this study, they are used in economic block models as well. The concepts of DC and PW are shown in Figure 11.

The downward cone of a block identifies blocks whose extraction paths are mostly determined by its extraction. So, it indicates that extracting that block opens and facilitates access to them. In contrast, not to extract that one makes extraction of blocks inside the downward cone impossible. According to the meaning and idea of the downward cone, during mining, we want to exploit ore blocks with higher values earlier. While comparing candidates, PW's concept helps us determine the path leading to parts with higher grades and values. Consequently, the greater PW of a block means its removal leads the miner to blocks with higher values.

In each step of the algorithm, firstly candidate blocks whose extraction technically is possible are compared. The first criterion compared between the candidates is their economic values. So, the block with the highest value must be selected to lead the process to the higher NPV. Otherwise, if there are at least two blocks with the highest value, then their PWs should be compared. The logic behind the second criterion is that the higher PW leads the process to higher grade part of the ore body which logically will result in a higher NPV. The algorithm has been explained in details in the next section through a numerical example.

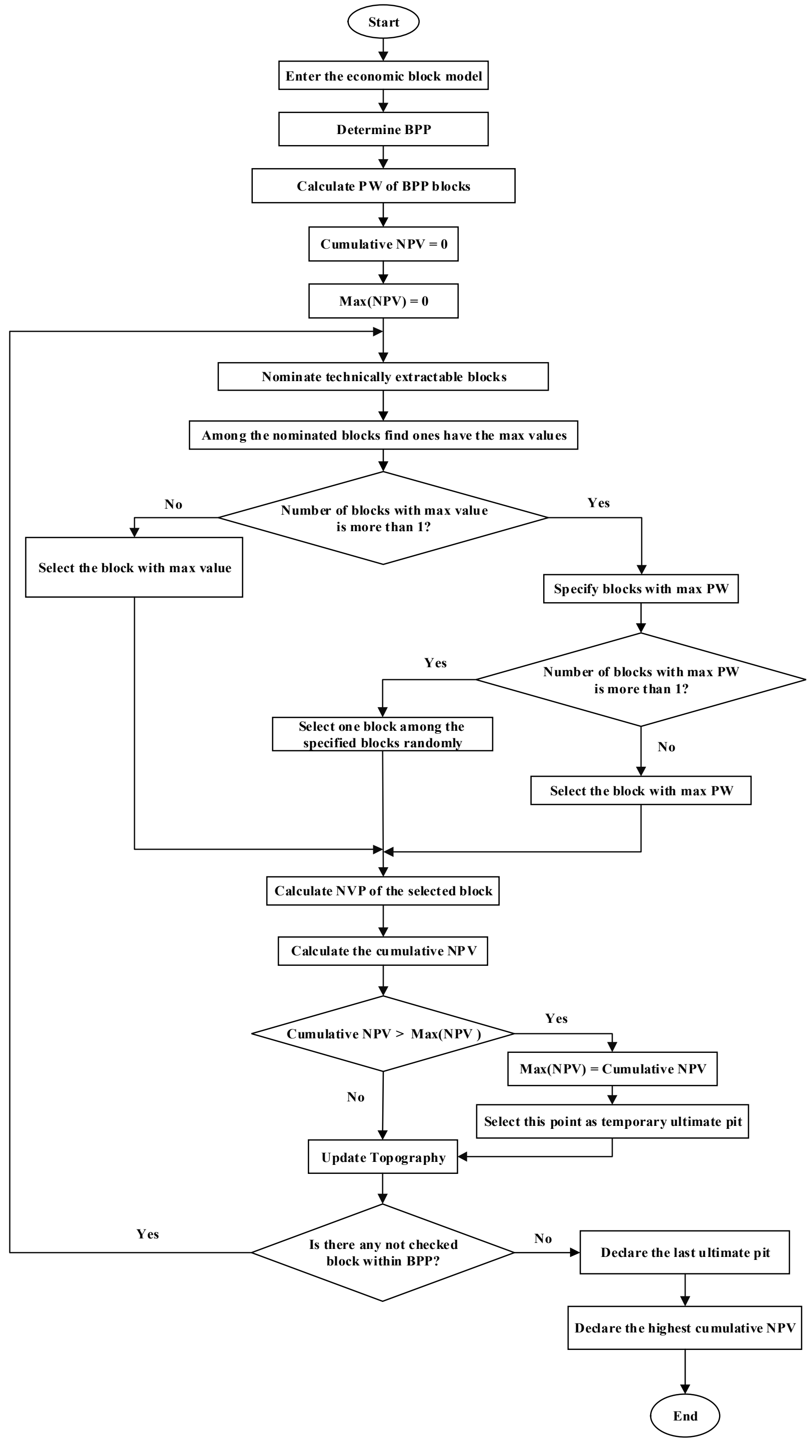


Figure 10. The developed Dynamic Pit Tracker algorithm to find UPL

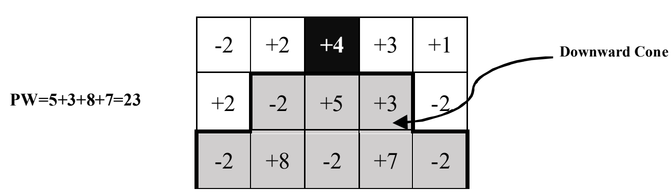


Figure 11. The concept of DC and PW for an ore block

3.1. A Numerical Example

In this section, using a simple 2D block model (Figure 12) the process of the algorithm is explained. Figure 13 shows the input data (V and PW) calculated. Assuming that the discount rate for each block is 3%.

	1	2	3	4	5	6
1	-1	-1	3	-1	-1	-1
2	-1	5	6	-1	1	-1
3	-1	-1	1	3	-1	-1

V ←BPP

Figure 12. 2D example of an economic block model

		1	2	3	4	5	6
1	V	-1	-1	3	-1	-1	-1
	PW	6	15	15	11	5	4
2	V		5	6	-1	1	
	PW		1	4	4	3	
3	V			1	3		
	PW			0	0		

V / PW ← Block Cell

Figure 13. The input data calculated for each block cell according to DPT requirements

Figure 14 shows the steps of the algorithm solution. In each step, the extractable blocks are candidates. By comparing the Vs and PWs of candidates, the most suitable block will be mined. First, their Vs are compared. If one has the highest V, it will be removed. But if some candidates have equal V, their PWs must be examined and the one with the highest PW will be selected. If there is more than 1 block with the highest PW, randomly one should be chosen. Then, the overall NPV of the mined blocks will be calculated after each selection. When one block has been selected to extract, it must be deleted from the block model to update the topography for the next turn. In each step, if the updated cumulative NPV is greater than the previous value stored, the previous value will be replaced by the updated one. This point is a local extremum and is called the temporary pit limit. Based on the calculations, the cumulative NPV was plotted. For this example, Figure 15 shows the cumulative NPV graph. A total of 5 local extrema have been calculated, of which the 5th is the maximum and the final pit value.

Based on the solution, the ultimate pit for the block model contains 12 blocks with 10.999 CU. The model also was solved by the mathematical solution which was compared with the algorithm results (Figure 16). The comparison for this example clearly shows the exact correspondence between the optimal solution and the algorithm's approximation. Scientifically, examples cannot prove the optimality of algorithms but can demonstrate their performance.

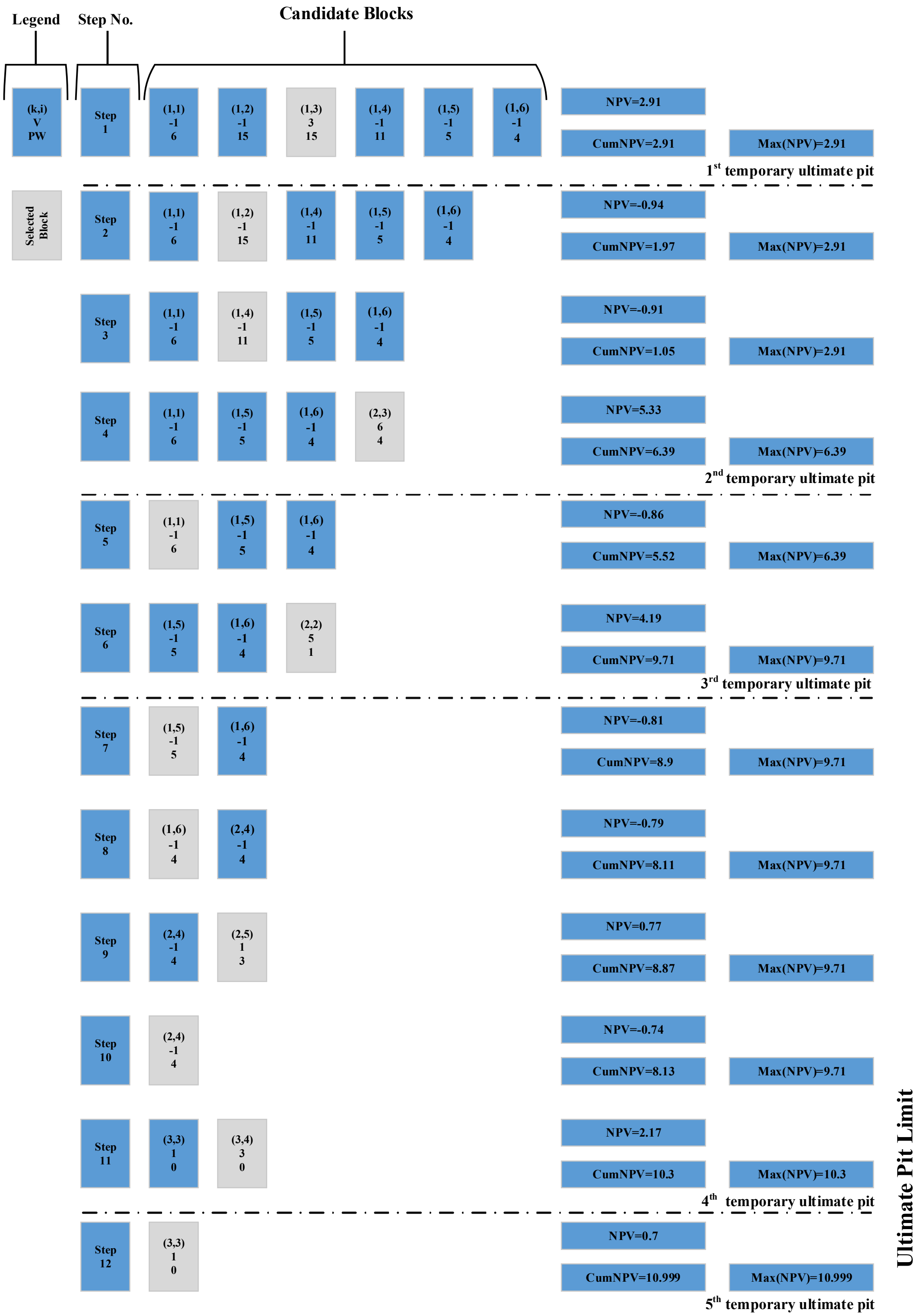


Figure 14. Steps of application of DPT calculations for the 2D example

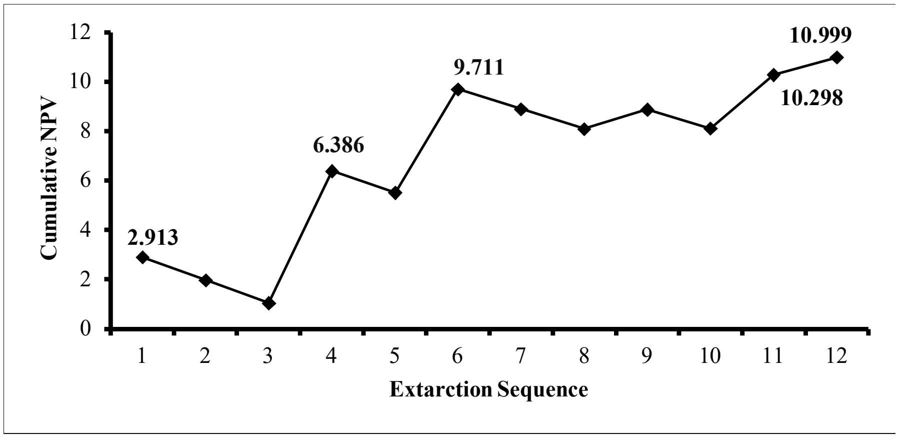


Figure 15. The graph of cumulative NPV and the local extrema

5	2	1	3	7	8
	6	4	10	9	
		12	11		

a) The extraction sequences based on algorithm's results

5	2	1	3	7	8
	6	4	10	9	
		12	11		

b) The optimal extraction sequences (mathematical solution)

Figure 16. Comparison between results of mathematical and heuristic solutions

4. A comparison between the developed algorithm and other algorithms

In this section, the heuristic algorithm has been applied to find the final pit and extraction sequence of the block model shown in Figure 17. The algorithm was compared with the results of the objective function developed, the LG algorithm, and the Latorre-Golosinski (LAGO) algorithm [23]. To do this, at first the UPL based on the undiscounted profit has been determined by LG method. So, the LG algorithm (Figure 18) values the ultimate pit at 253 CU. This result will be compared with the result of the algorithm developed at 0 discount rate to validate it.

At the second step of comparison, the answer to the objective function of this block model as an optimum mathematical solution was compared with the results of the LAGO heuristic algorithm. So, the final pit and optimum extraction sequence obtained from the objective function and the LAGO algorithm are shown in Figures. 19 and 20, respectively, for a 3% discount rate per block. As can be seen, the results of the objective function and LAGO algorithm are in agreement. The pits generated are smaller than the LG pit which has 100 blocks with an undiscounted profit of 250 CU. Consequently, their NPV at a discount rate of 3% per block is 35 CU.

As the third step, the sequence of extraction has been determined by the new algorithm. The extraction sequence has been shown in Figure 21. To determine the UPL, the cumulative NPV has been calculated. The graph of cumulative NPV is plotted and shown in Figure 22. According to the result and the procedure of the algorithm, the maximum NPV occurs in the block with 100 extraction order, which is determined as the final pit. So, the overall NPV for the ultimate pit is 29.11 CU.

Consequently, the pit boundary determined by the proposed algorithm fully agrees with the objective function and the LAGO algorithm.

Additionally, as another way of validation, the final pit determined by the algorithm developed for zero discount rate is completely consistent with the LG pit (see Figures 17 and 23).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
2	-3	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	**1**	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3	-3	-3	-3	-3
3	-3	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	**2**	**1**	**2**	**1**	**2**	**5**	-2	-3	-3	-3	-3	-3	-3	-3	-3
4	-3	-3	-3	-3	-2	-2	-2	-2	**1**	**1**	**1**	**2**	**2**	**3**	**8**	**10**	-2	-2	-1	-2	-3	-3	-3	-3	-3	-3
5	-3	-3	-3	-3	-3	-2	-2	**1**	**1**	**1**	**3**	**2**	**10**	**15**	**10**	**3**	**1**	**1**	-1	-2	-3	-3	-3	-3	-3	-3
6	-3	-3	-3	-3	-3	-3	**1**	-1	**2**	**7**	**5**	**10**	**15**	**20**	**5**	**2**	**1**	-1	-1	-3	-3	-3	-3	-3	-3	-3
7	-3	-3	-3	-3	-3	-3	**1**	**3**	**4**	-1	**7**	**15**	**20**	**15**	**10**	**2**	**2**	-2	-2	**1**	-3	-3	-3	-3	-3	-3
8	-3	-3	-3	-3	-3	-3	-3	**1**	**3**	**10**	**8**	**25**	**15**	**20**	**6**	**4**	**1**	**1**	**1**	-3	-3	-3	-3	-3	-3	-3
9	-3	-3	-3	-3	-3	-3	-3	0	**2**	**8**	**15**	**30**	**20**	-3	**8**	**5**	**1**	**1**	-3	-3	-3	-3	-3	-3	-3	-3
10	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	**10**	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3

Figure 17. The hypothesized 2D block model [23]

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1		-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3			
2			-3	-3	-3	-3	-2	-2	-2	-2	-2	**1**	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3				
3				-3	-3	-3	-2	-2	-2	-2	-2	**2**	**1**	**2**	**1**	**2**	**5**	-2	-3	-3	-3					
4					-2	-2	-2	-2	**1**	**1**	**1**	**2**	**2**	**3**	**8**	**10**	-2	-2	-1	-2						
5						-2	-2	**1**	**1**	**1**	**3**	**2**	**10**	**15**	**10**	**3**	**1**	**1**	-1							
6							**1**	-1	**2**	**7**	**5**	**10**	**15**	**20**	**5**	**2**	**1**	-1								
7								**3**	**4**	-1	**7**	**15**	**20**	**15**	**10**	**2**	**2**									
8									**3**	**10**	**8**	**25**	**15**	**20**	**6**	**4**										
9										**8**	**15**	**30**	**20**		**8**											
10												**10**														

Figure 18. The optimum pit limit with LG

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1			-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3					
2				-3	-3	-3	-2	-2	-2	-2	-2	**1**	-2	-2	-2	-2	-2	-2	-3	-3						
3					-3	-3	-2	-2	-2	-2	-2	**2**	**1**	**2**	**1**	**2**	**5**	-2	-3							
4						-2	-2	-2	**1**	**1**	**1**	**2**	**2**	**3**	**8**	**10**	-2	-2								
5							-2	**1**	**1**	**1**	**3**	**2**	**10**	**15**	**10**	**3**	**1**									
6								-1	**2**	**7**	**5**	**10**	**15**	**20**	**5**	**2**										
7									**4**	-1	**7**	**15**	**20**	**15**	**10**											
8										**10**	**8**	**25**	**15**	**20**												
9											**15**	**30**	**20**													
10												**10**														

Figure 19. Optimum pit limit with the highest NPV at 3% discount rate

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1			91	82	57	50	43	37	31	21	2	1	3	5	7	10	13	17	26	65	73					
2				92	83	58	51	44	38	32	22	4	6	8	11	14	18	27	66	74						
3					93	84	59	52	45	39	33	23	9	12	15	19	28	67	75							
4						94	85	60	53	46	40	34	24	16	20	29	68	76								
5							95	86	61	54	47	41	35	25	30	69	77									
6								96	87	62	55	48	42	36	70	78										
7									97	88	63	56	49	71	79											
8										98	89	64	72	80												
9											99	90	81													
10												100														

Figure 20. The optimum extracting sequence of the blocks of the ultimate pit limit at 3% discount rate

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	145	118	91	73	65	50	43	31	21	17	10	5	1	2	3	7	13	26	37	57	82	101	109	127	136	154
2		146	119	92	74	66	51	44	32	22	18	11	6	4	8	14	27	38	58	83	102	110	128	137	155	
3			147	120	93	75	67	52	45	33	23	19	12	9	15	28	39	59	84	103	111	129	138	156		
4				148	121	94	76	68	53	46	34	24	20	16	29	40	60	85	104	112	130	139	157			
5					149	122	95	77	69	54	47	35	25	30	41	61	86	105	113	131	140	158				
6						150	123	96	78	70	55	48	36	42	62	87	106	114	132	141	159					
7							151	124	97	79	71	56	49	63	88	107	115	133	142	160						
8								152	125	98	80	72	64	89	108	116	134	143	161							
9									153	126	99	81	90		117	135	144	162								
10												100														

Figure 21. The extraction sequence of the block model resulted from the new heuristic algorithm

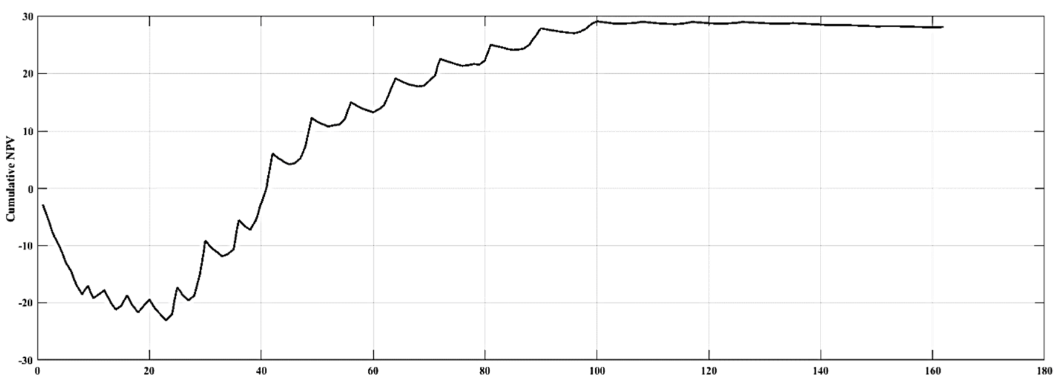


Figure 22. The cumulative NPV graph resulted from the new algorithm

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
2	-3	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	1	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3	-3	-3	-3	-3
3	-3	-3	-3	-3	-3	-3	-2	-2	-2	-2	-2	2	1	2	1	2	5	-2	-3	-3	-3	-3	-3	-3	-3	-3
4	-3	-3	-3	-3	-2	-2	-2	-2	1	1	1	2	2	3	8	10	-2	-2	-1	-2	-3	-3	-3	-3	-3	-3
5	-3	-3	-3	-3	-3	-2	-2	1	1	1	3	2	10	15	10	3	1	1	-1	-2	-3	-3	-3	-3	-3	-3
6	-3	-3	-3	-3	-3	-3	1	-1	2	7	5	10	15	20	5	2	1	-1	-1	-3	-3	-3	-3	-3	-3	-3
7	-3	-3	-3	-3	-3	-3	1	3	4	-1	7	15	20	15	10	2	2	-2	-2	1	-3	-3	-3	-3	-3	-3
8	-3	-3	-3	-3	-3	-3	-3	1	3	10	8	25	15	20	6	4	1	1	1	-3	-3	-3	-3	-3	-3	-3
9	-3	-3	-3	-3	-3	-3	-3	0	2	8	15	30	20	-3	8	5	1	1	-3	-3	-3	-3	-3	-3	-3	-3
10	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	10	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3

Figure 23. The ultimate pit resulted from the new algorithm at 0 discount rate

5. Application of the developed algorithm in a 3D block model

The algorithm proposed has been applied to a 3D economic block model. Table 4 presents the block model characteristics. There are 6533 blocks involved in the BPP, and the LG pit limit profit amounts to 26761 CU. The heuristic algorithm with zero discount rate produced a final pit limit with a value of 26213 CU. This shows 97.95% accuracy according to the LG result. If the discount rate is set at 0.1% per block the maximum NPV will be 5660 CU. The graph of cumulative NPV is shown in Figure 24. Figure 25 shows the ultimate pit limit plot.

Table 4. The characteristics of the 3D block model

Parameter	Value
Number of all blocks	20680
Blocks' dimension (m)	10×10×10
East-west (blocks)	40
South-north (blocks)	47
Vertical (blocks)	11
number of BPP blocks	6533
Number of ore blocks	1784
Number of effective waste blocks	4749
Percentage of BPP to all blocks	31.6

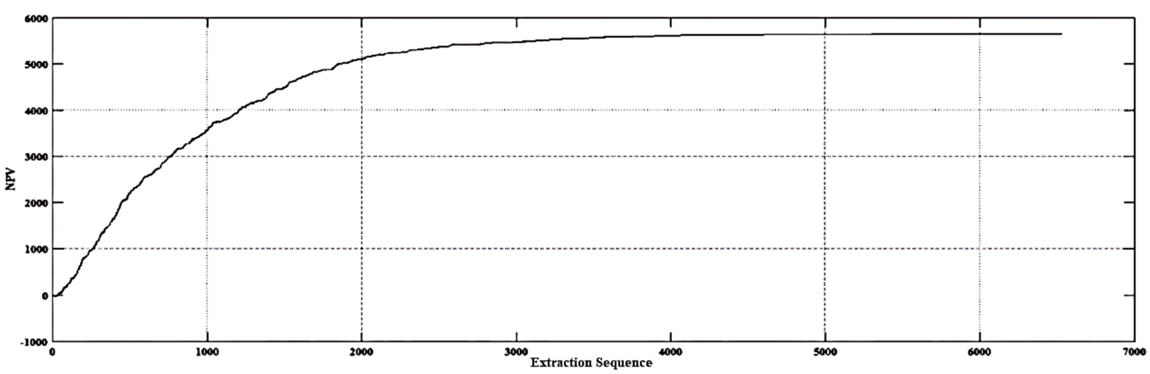


Figure 24. The cumulative NPV graph for 3D block model

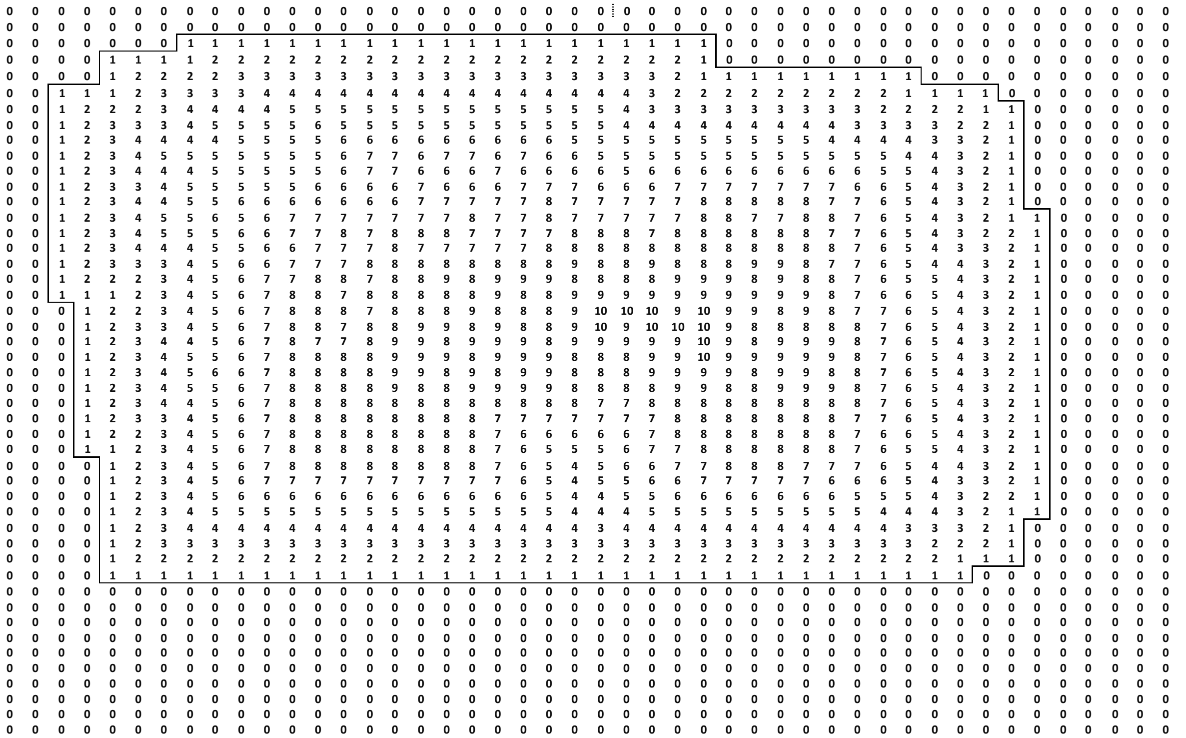


Figure 25. The plot of the ultimate pit according to the result of the new algorithm (each cell shows the number of blocks vertically belonging to the final pit in that point)

6. Conclusions

In conclusion, this paper introduces a mathematical integer model for optimizing the Ultimate Pit Limit (UPL) to maximize Net Present Value (NPV). When the discount rate in the objective function is set to zero, the model simplifies to the common UPL model focused on profit maximization. The key finding is that the primary solution for UPL lies in NPV maximization.

The developed objective function is non-linear, necessitating a two-step linearization process involving extraction sequence and UPL determination. Due to the complexity, large number of decision variables, and constraints, the model is classified as an NP-hard optimization problem. To address this challenge, a heuristic methodology named Dynamic Pit Tracker (DPT) was derived through linearization. This heuristic algorithm offers a simpler and more programmable solution than conventional complex algorithms.

The DPT algorithm takes the economic block model as input, calculating a positional weight (PW) for each block. During the algorithm's execution, the economic values and PWs of candidate blocks are compared. The one with the highest value or PW will be selected at each step. This strategy aims to maximize NPV throughout the mine's lifespan. The results demonstrate that the algorithm can yield an acceptable solution within a reasonable timeframe.

Comparisons with other algorithms validate DPT effectiveness, showing agreements on pit boundaries and total pit values. In a 3D block model, DPT achieved 97.95% accuracy compared to the LG algorithm. Notably, DPT, in addition to UPL determination, provides a mining extraction plan resulting in an NPV of 5660 CU for the orebody. It should be noticed also that for heuristic algorithms the accuracy cannot be predicted based on the previous studies and cases, but the logic of the algorithms and the results of the studies can show the overall trend of the accuracy.

Future studies should focus on enhancing accuracy and developing commercial software packages for practical application. Improvements in comparison criteria, incorporation of additional production planning constraints and targets, consideration of uncertain parameters, and comparison between the new algorithm and other solutions are recommended to make the model more comprehensive and applicable in diverse mining scenarios.

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الگوریتم جستجوگر پویا: یک الگوریتم ابتکاری برای حل مساله تعیین همزمان محدوده بهینه نهایی و ترتیب استخراج بلوک‌های معادن روباز

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چکیده:

یکی از طراحی‌های اصلی در معدن‌کاری روباز، محدوده نهایی است. این محدوده معمولاً ابتدا از طریق الگوریتم‌های بهینه‌سازی با هدف حداکثرسازی سود مانند الگوریتم لرچ-گروسمن تعیین می‌شود. سپس، برای بهینه‌سازی ارزش حال حاضر خالص فعلی (NPV)، برنامه‌ریزی تولید برای استخراج بلوک‌هایی که در محدوده نهایی تعیین شده قرار دارند، اجرا می‌شود. در این مقاله مدل ریاضی طراحی معادن روباز با هدف بهینه‌سازی NPV ارائه شده است که امکان تعیین همزمان محدوده نهایی و برنامه‌ریزی تولید بلندمدت را فراهم می‌سازد. مدل ریاضی ارائه شده غیرخطی است. بنابراین، به منظور خطی‌سازی مدل، مدل به دو زیرمسأله خطی تقسیم شده است. با استفاده از این روش، فرآیند حل مدل و استراتژی به کار گرفته شده با کاهش تعداد متغیرهای تصمیم‌گیری ساده تر می شود. طبیعت مدل ریاضی طراحی محدوده نهایی به علت تعداد بالای متغیرها و محدودیت ها و نیز پیچیدگی‌های مساله از نوع NP-Hard است. در نتیجه، به منظور حل این مسأله، در این تحقیق الگوریتم جستجوگر پویا (DPT) به عنوان یک راه حل ابتکاری توسعه داده شده است که مدل بلوکی اقتصادی داده‌های ورودی آن است. در هر مرحله از محاسبات این الگوریتم مقایسه‌هایی ابتدا بین ارزش‌های اقتصادی و سپس بین وزن مکانی بلوک‌ها در طول مراحل تعیین محدوده انجام می‌شود تا بلوک مناسب‌تر برای استخراج در هر مرحله شناسایی شود. در این تحقیق، نتایج حاصل از بهینه‌سازی بر اساس مدل ریاضی، الگوریتم‌های لرچ-گروسمن و لاتوری-گلوسینسکی با DPT بر روی یک مدل بلوک دوبعدی با هم مقایسه شد. تحلیل مقایسه‌ای نشان داد که محدوده‌های نهایی تعیین شده توسط این الگوریتم‌ها در مدل بلوکی استفاده شده دارای تطابق هستند. استفاده از الگوریتم جدید برای تعیین محدوده نهایی در یک مدل بلوک سه‌بعدی نیز سود نهایی محدوده را با دقت ۹۷/۹۵ درصد تطابق با محدوده حاصل از الگوریتم لرچ-گروسمن به دست آورد.

کلمات کلیدی: معادن روباز، محدوده بهینه نهایی، ارزش خالص فعلی، برنامه ریزی عدد صحیح، الگوریتم ابتکاری