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A Profit-based Mixed Integer Programming Model for Stope Boundary Optimization and Implementation in Indian Copper Deposits

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Abstract

The optimal layout of the stope (stope boundary) in an underground metal mine maximizes the profit of a deposit, subject to the geotechnical and operational mining constraints such as stope length, stope width, stope height. Various approaches have been introduced to address the stope boundary optimization problem, but due to the computational complexity and numerous practical constraints, the existing models offer partial solutions to the problem. In the present work, a mixed integer programming model has been developed by incorporating mining constraints in a three-dimensional framework. This model is developed based on profit maximization. The sensitivity analysis applied in a case study mine indicates that the model is efficient in assessing the upside potential and downside risk of profit under fluctuating metal prices and mining costs. Additionally, it can be applied at different stages of mine design to facilitate resource appraisal, selection of stoping methods, and comprehensive mine planning. In a practical application on a real orebody, it shows that the proposed model can generate upto 37.32% more profit compared to current stope design practice in the mines.

1. Introduction

The demand for mineral production has been increasing more quickly due to fast rise of industrialization. The easily accessible shallow-depth mineral deposits are exhausted rapidly day by day. Because of this, the mining industry is bound to mine deeper deposits through underground mining methods. It is well-realized that planning and design tools for underground mines must be optimized, to ensure optimal natural resource recovery at the lowest possible mining costs. [1, 2]. Therefore, mine planning engineers need advanced underground optimization tools that can provide the optimal solution for mine design-related problems. Stope boundary optimization is one of the important components of underground mine optimization for optimum extraction of ore, as well as maximization of profit. Unlike opencast mining, much less research has been published on stope boundary optimization in underground metal mining [3, 4, 5]. Several reasons are there for not having enough research work in this domain. These include generality, complexity, and acceptability [6].

The algorithms available for stope boundary optimization comprise the dynamic programming algorithm, downstream geostatistical approach, octree division approach, floating stope algorithm, and branch and bound technique, maximum value neighborhood method, simulated annealing-based algorithm, etc. [7, 8]. Riddle [9] developed a dynamic programming model to optimize mine boundaries, that is applicable to the block-caving

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method. This method is one of the first attempts to optimize the boundaries of underground mines. Initially, the algorithm considers that there is no pillar in the process, and maximizes the profit without considering pillar. Then it includes pillar in the operational zone, and studies the profitability of all feasible solutions for the zone. However, the approach generates optimum designs in two dimensions only, with a limitation to the block-caving method. The application of image analysis has been proposed by Deraisme et al. [10], to delineate the outline of the minable ore in underground mining. They build a 2D-sectional model of the ore body, and define the outline of the lens to be mined. The approach considers grade uncertainty in the optimization process. However, it is limited by the stope geometry; hence, it does not take into consideration the economic aspects. Therefore, true optimality is not achieved in three-dimensions. Serra (1982) proposed a mathematical approach, which is used to convert the image of ore blocks above the cut-off grade to one more image, ensuring the stope geometry constraints [11]. The Octree division algorithm is presented by Cheimanoff et al. [12]. The algorithm produces a solution for stope geometry in 3-D, but the limitation of the algorithm is that it does not analyse the sub-volume jointly. As a result, more waste is added to the final mine layout. It includes sub-volumes in the final layout, without considering the waste volume. Hence, the algorithm produces sub-optimal solution of stope layouts. A commercially available stope boundary optimization tool that applies a floating stope algorithm is developed by Alfrod [13]. The cut-off grade is the initial input parameter used to differentiate ore and waste blocks based on economic assumptions. This algorithm is used in the Datamine CAE Studio 3 software [14]. However, the algorithm produces stopes within the ore zone that overlap with each other, and it involves user adjustment of the optimization model [15, 10, 16]. Also, it does not consider the geotechnical and mining constraints. As a result, it produces a sub-optimal solution. An extension of the floating stope algorithm has also been proposed by Cawrse [17]. The algorithm is known as the Multiple Pass Floating Stop Process (MPFSP). The algorithm finds stope envelopes within the ore zone. However, it does not address the limitation of overlapping stope of the "floating Stope" algorithm. Branch and bound algorithms are used by Ovanic and Young [18] for the optimization of stope boundaries. The algorithm uses linear cumulative functions, and selects the start-point and end-point of the stope for the optimal design. The model is applicable for both uniform and non-uniform deposits. However, the restriction of the approach is that it considers only one dimension for generating an optimized solution. Thus, the algorithm generates an incomplete optimal solution to the problem. Another new algorithm known as the Maximum Value Neighborhood (MVN) algorithm was proposed by Ataee-Pour [3, 7]. The algorithm finds a possible neighborhood for every individual block within the ore body block model, and on the basis of the economic value, the algorithm selects the optimal neighborhood. In the case of a specific starting-block, the approach produces a better solution to the optimization problem. However, the constraint is that the value of the optimal solution varies depending on the selection of the starting-block [19]. A Mixed Integer Programming (MIP)-based optimization model is proposed by Griceo and Dimitrakopoulos [20]. The approach first divides the block model into several layers, panels, and rings, and then optimizes the number of rings, and maximizes the metal in the ore body. However, depending on the dimensions of the mining blocks, the model considers the block into predefined rings. Therefore, the model differs from optimality. Apart from these, a novel heuristic approach by Sandanayake et al. [21], Sens and Topal [22, 23], Copland and Nehring [24], and graph theory analysis by Bai et al. [25] have been presented.

Few studies consider the geotechnical and physical mining constraints in three-dimensional spaces. In most cases, post- design manual adjustment is required before mining the stope. Therefore, the model is limited to a partial solution to the problem. According to Sandanayake et al. [21], the approaches do not ensure an optimal solution in three-dimensional space. Thus, the algorithm offers an incomplete solution to the stope layout problem [6]. According to Ataee-Pour [26], the approaches do not ensure the true 'optimum' stope layout. Furthermore, in previous studies, potential tonnage, and grade recovery, as well as profit from a stope with varying mining costs and uncertain metal price scenarios, are not addressed properly.

In the present work, the authors concentrate on developing a profit-based optimization model based on MIP for stope boundary optimization in three-dimensional space. The physical mining constraints are considered in the model such as the maximum and minimum length, width, and height of the stope. Acknowledging the significance of other sources of risk, the present study analyses the risk associated with grade uncertainty, uncertainty in the metal price, and varying mining costs. The aim is to support mine management in making business decisions with respect to the upside potential and downside risk of an ore body, while designing the stope in the mine planning stage.

2. Present Practice of Stope Boundary Delineation in Indian Copper Mines and its Limitations

Usually, In Indian mining scenarios, the present industry practice is to perform the stope boundary delineation process manually. This may lead to substantial losses in resources, as well as profit. During the mine planning stage, the author experiences an immense challenge in selecting the optimal stope boundary in an Indian copper mine, where the grade distribution is very erratic throughout the mine with a 2.5 km strike length. In this mine, exploration drilling was initially conducted from the surface. However, to delineate individual ore lenses and, subsequently, design stope boundaries, underground exploration has been carried out from the footwall drives of each level. This fan-shaped drilling is carried out from the mine's main levels at 25 m intervals along the strike from a drilling pocket in the footwall drive. An example of surface and underground exploration drilling is shown in Figure 1.

Ore lens outlining is primarily performed, considering the geological cut-off grade of the deposit. This is the minimum grade of ore that will meet the variable cost of mining, and this is the dividing line between ore and waste. Here, in this mine, the geological cut-off grade is 0.50% copper (Cu). After the ore lens is defined, the stope boundary is prepared on the basis of the pay limit of an ore grade. The "pay limit" may be defined as the grade of ore that will meet the overall cost. The pay limit considering the present mining cost and metal price scenario is 0.80 % Cu for this mine. After the stope boundary in a drill section is selected, the minable reserve is estimated for the stope, considering the dimensions of 25 m along the length (12.5 m on both sides of the drill section) and 60 m along the height (level interval vertically of the stope) and real ore body width (from 5–45 m between H/W and F/W) of the stope. Here, the average in situ specific gravity of the ore zone is considered 3.00 for reserve estimation. A drill hole section delineating the ore body boundary is given in Figure 2.

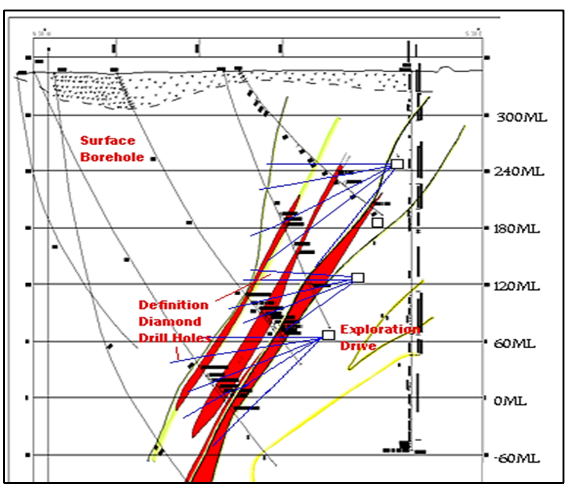

Figure 1. Orebody delineated based on surface and underground exploration drilling.

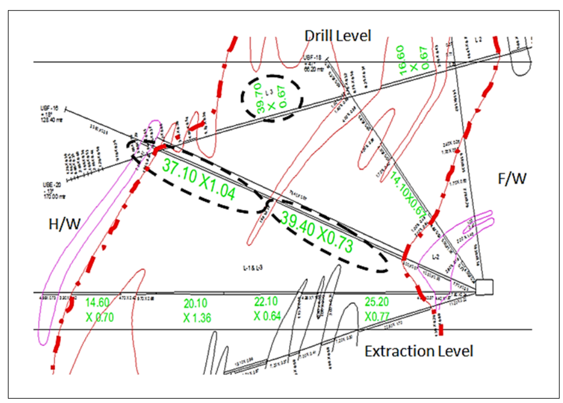

Figure 2. Drill hole section delineating the ore body based on the geological cut-off grade.

The geological section shows that the distribution of the Cu grade is very erratic. On the hanging wall (H/W) side, higher-grade materials (1.04% Cu), and on the F/W side, lower-grade (0.73% and 0.6%Cu) materials are observed. In the upper portion of the ore body, a lower grade (0.67% Cu) material is noticed. Here, only 1.04% of the Cu grade portion would be inside the stope boundary in the present manual practice of the stope boundary selection process, considering a fixed cut-off grade of 0.80% Cu. However, the present practice is not sufficient for the selection of the optimal stope boundary in three dimensions. Moreover, a small variation in metal price or mining cost has a considerable effect on ore tonnage, average grade, and profit, which is not possible to evaluate in the present practice. To overcome this limitation, instead of cut-off grade, a profit-based MIP model is proposed for stope boundary optimization. It also incorporates the impact of variations in various economic parameters on the stope boundary and profit to make a business decision during the mine planning stage.

3. Materials and Methods

3.1. Optimization model formulation

Mathematical model development is a critical step for solving stope boundary optimization problems. The block model is required for any work on stope boundary selection. Here, the block model is prepared, covering the entire mineralization zone irrespective of the metal grade in the drilled area, for which we are interested in optimizing the stope boundary. The conventional techniques of using a geological cut-off grade or pay limit grade are not considered for selecting minable ore boundaries, but profit-based optimization models are developed for boundary optimization. The ore body model and grade estimation technique are well-established, and it can be accessed through various commercial mine planning softwares. On the basis of the tonnage and estimated grade of each mining block and economic values (e.g. mining cost, ore processing cost, and metal price), a mining block value is determined, and it is used as an input to the successive mine planning process. Here, the stope boundary optimization is done by selecting from each row of the orebody, starting-point, and ending-point for which, the profit will be maximum. This proposed mathematical model (Mixed Integer Programming (MIP)) uses both continuous and integer decision variables simultaneously. The solving of the MIP consists of a combination of simplex-derived methods, as well as Integer Programming (IP) problem-solving techniques such as branch and bound and cutting planes [27]. This MIP technique generates an optimal stope boundary by maximizing the total economic value (profit) subject to various mining constraints. The IP techniques are applied here, because they can effectively manage multi-constrained problems to satisfy a realistic optimal stope boundary. The model is defined in the following paragraphs.

The stope boundary optimization is done based on the given ore body model in 3-D space along the strike, the width, and the height of the ore body. The optimal ore boundary is determined for each dimension, considering the start-points and end-points of each row.

3.1.1. Subscript notation

i: Block reference name along strike: i = 1, 2,.. n.

j: Block reference name along width: j = 1, 2,.. m.

k: Block reference name along the height: k = 1, 2..., l.

31.2. Parameters

$\boldsymbol{V_B}$: Each block volume.

$\boldsymbol{G_{ijk}}$: Grade value of each block in the ore body model.

$\boldsymbol{SG}$: Specific gravity of the ore.

$\boldsymbol{PM}$: Price per tonne of metal.

$\boldsymbol{FC}$: Mining fixed cost per tonne of ore production from mines.

$\boldsymbol{VC}$: Mining variable cost per tonne of ore production from mines.

$\boldsymbol{MC}$: Plant processing cost per tonne of ore processed.

$\boldsymbol{PC_{ijk}}$: Cumulative profit for each block combination.

$\boldsymbol{W}$: Width of the ore body.

$\boldsymbol{R_{ijk}}$: Coordinate (X cor. Y cor. Z cor.) of block

$\boldsymbol{M}$: Large, valued number to be used with the Boolean variable.

$\boldsymbol{SLmin, SLmax}$: Minimum and maximum stope lengths, respectively.

$\boldsymbol{SWmin, SWmax}$: Minimum and maximum stope widths, respectively.

$\boldsymbol{SHmin, SHmax}$: Minimum and maximum stope heights, respectively.

3.1.3. Pre-processing

Some pre-processing activities are performed to obtain the data in the required format. Here, the cumulative profit factor is calculated, which simplifies the objective function, and reduces the complexity of the mathematical model. The cumulative profit factor is determined on the basis of the metal price, mining cost, and processing cost. Equation (1) represents the calculation of the cumulative profit factor.

$$PC_{ijk} = \sum_{i=1}^{i} (V_B * SG * G_{ijk} * PM - V_B * SG * FC - V_B * SG * VC - V_B * SG\ MC) \tag{1}$$

Where,

$\boldsymbol{V_B * SG * G_{ijk} * PM}$: Revenue of the metal from the respective block.

$\boldsymbol{V_B * SG * FC}$: Fixed cost for extracting the ore from the respective block.

$\boldsymbol{V_B * SG * VC}$: Variable cost for extracting the ore from the respective block.

$\boldsymbol{V_B * SG * MC}$: Mineral processing cost of extracted ore from the respective block.

3.1.4. Decision variables

$\boldsymbol{x_{ijk}}$: Starting-point (tailing end) of the stope for each row of the ore body.

$\boldsymbol{y_{ijk}}$: Ending-point (leading end) of the stope for each row of the ore body.

$\boldsymbol{b_{jk}}$: Starting-point coordinate of the boundary.

$\boldsymbol{c_{jk}}$: Ending-point coordinate of the boundary.

$\mathbf{r_{jk}}$, $\mathbf{q_{jk}}$: Boolean variables used for maintaining continuity of rows along the width of the ore body.

$\mathbf{t_{jk}}$, $\mathbf{s_{jk}}$: Boolean variables used for maintaining the continuity of rows along the height of the ore body.

3.1.5. Objection function

The objective function for ore boundary optimization is given in Equation 2. This function maximizes the profit from the orebody by subtracting mining and processing cost from revenue.

$$\text{Maximize: } \left(\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l}\left(PC_{ijk} * (y_{ijk} - x_{ijk})\right)\right) \quad (2)$$

The objective function is to be solved subject to certain constraints, which are discussed below.

$$\sum_{i=1}^{n} x_{ijk} \leq 1 \;\forall j,k \quad (3)$$

$$\sum_{i=1}^{n} y_{ijk} \leq 1 \;\forall j,k \quad (4)$$

$$\sum_{i=1}^{n} R_{ijk} * x_{ijk} - b_{jk} = 0 \;\forall j,k \quad (5)$$

$$\sum_{i=1}^{n} R_{ijk} * y_{ijk} - c_{jk} = 0 \;\forall j,k \quad (6)$$

$$\sum_{i=1}^{n} (R_{ijk} * y_{ijk} - R_{ijk} * x_{ijk}) \geq 0 \;\forall j,k \quad (7)$$

$$(\sum_{i=1}^{n} x_{ijk} + M * r_{jk} \geq \sum_{i=1}^{n} x_{i(j-1)k} \;\; \forall j,k \quad (8)$$

$$\sum_{i=1}^{n} x_{ijk} - M * q_{jk} \leq \sum_{i=1}^{n} x_{i(j-1)k} \;\; \forall j,k \quad (9)$$

$$r_{jk} + q_{jk} = 1 \;\forall j,k \quad (10)$$

$$r_{jk} \geq 1 - M * \left(1 - r_{(j-1)k}\right) \forall j,k \quad (11)$$

$$\sum_{i=1}^{n} x_{ijk} + M * t_{jk} \geq \sum_{i=1}^{n} x_{ij(k-1)} \;\; \forall j,k \quad (12)$$

$$\sum_{i=1}^{n} x_{ijk} - M * s_{jk} \leq \sum_{i=1}^{n} x_{ij(k-1)} \;\; \forall j,k \quad (13)$$

$$t_{jk} + s_{jk} = 1 \;\forall j,k \quad (14)$$

$$t_{jk} \geq 1 - M * \left(1 - t_{(j-1)k}\right) \forall j,k \quad (15)$$

$$SLmin \leq \left(c_{jk} - b_{jk}\right) \leq SLmax \;\; \forall j,k \quad (16)$$

where: $\left(c_{jk} - b_{jk}\right)$ indicates the length of the stope.

$$SWmin \leq \sum_{j=1}^{m} (x_{ijk} * block\ width) \leq SWmax \;\; \forall i,k \quad (17)$$

where: $\sum_{j=1}^{m}(x_{ijk} * block\ width)$ indicates the width of the stope.

$$SHmin \leq \sum_{k=1}^{l} (x_{ijk} * block\ height) \leq SHmax \;\; \forall i,j \quad (18)$$

where: $\sum_{k=1}^{l}(x_{ijk} * block\ height)$ indicates the height of the stope.

$$x_{ijk} = (1,0), y_{ijk} = (1,0), b_{jk} \geq 0, c_{jk} \geq 0, r_{jk} = (1,0), q_{jk} = (1,0), t_{jk} = (1,0), s_{jk} = (1,0) \quad (19)$$

Equation (3) selects the starting-point for all the rows, and Equation (4) selects the ending-point of each row of the ore body. By using Equations 5 and 6, the coordinates of the starting boundary and ending boundary are assigned for the width and height of the ore body. These constraints create the actual boundary coordinate for width and height using the position of the block and binary variables. Equation 7 ensures that the starting boundary should always be before the ending boundary. This is a logical constraint that ensures that the ending boundary always comes after the starting boundary for each row.

Equations 8, 9, 10, and 11 ensure that the row should remain continuous along the width of the optimal ore body. These constraints ensure that the optimal boundary will include continuous rows and layers and will not allow leaving any row (s) in between along the width of the ore body. This is important from a practical mining perspective since mining cannot be possible by leaving a block in the optimal stope boundary. During the optimization process, it may appear that any intermediate row or rows may not be profitable for mining. However, during mining, the stope block, those intermediate rows must be taken for mining the other profitable rows within the optimal boundary. Therefore, although the rows are not individually profitable, the remaining rows may jointly become profitable, and maximize the profit. Likewise, Equations 12, 13, 14, and 15 ensure that the layer should remain continuous along with the height of the ore body between the bottom and top of the stope in the optimal ore body. All these constraints ensure that the optimal boundary will include continuous layers, and will not allow leaving any layers in between along the height of the ore body.

The stope dimension is also important, concerning the method of working, mechanization adopted, and stability of the excavation. For this purpose, the stope dimensions are defined before the design of the stope. The stope dimension constraints are presented in equations 16, 17, and 18. Equation (16) is used to select the stope length between the defined maximum and minimum lengths of the stope. Similarly, Equation 17 represents the stope width constraints, for which the stope width will be between the defined maximum and minimum limits of the stope, and Equation 18 represents the stope height constraints, which are bounded by the maximum and minimum heights of the stope. Equation 19 represents the nonnegative constraints used in this model. The objective-oriented source code is developed and solved in Python Solver.

3.2. Implementation of Optimization Model

3.2.1. Block-grade model

The proposed MIP model is applied to a stope design in the Indian copper mine. The deposit is in the Jhunjhunu district of Rajasthan state, having a latitude of 28°0'54" north and a longitude of 75°46'32" east. The sub-level stoping method is used for mining this deposit. A total of 12 exploratory drill hole data is collected in 4 sections over a strike length of 120 m. The drill holes are sampled at intervals of 1.0 m in core length and rock types in each interval are documented for each core. In addition to the exploratory drill holes, other relevant information related to geology, current planning practices, cost, and the metal price is also procured from the mine.

The block model is prepared, considering a standardized block dimension of 5 m × 5 m × 5 m along the x, y, and z directions, respectively. The total number of blocks in the model is 7200. The block grade model has been prepared based on the exploratory data using the DATAMINE CAE Studio 3 software. The block grade model and reserve estimation are done by using the Geostatistical estimation technique. The grade-tonnage curve and cut-off grade vs. average grade curve of the ore body constructed from the estimated block grade is shown in Figure 3. It can be seen that when the cut-off grade varies from 0.1 % Cu to 1.3% Cu, the ore tonnage significantly reduces from 1.21 Mt to 0.05 Mt. It is also observed that a slight variation in grade (% Cu) in the range of 0.4% to 0.7% results in a substantial change in ore tonnage (from 1.18 Mt to 0.64 Mt). Thus, optimal cut-off grade selection in this ore body is a critical task. The cut-off grade vs. average grade plot indicates that the average grade is a steadily increasing trend with an increase in cut-off grade.

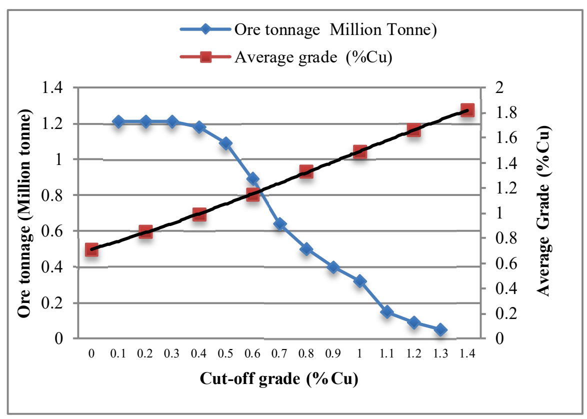

Figure 3. Cut-off grade vs. ore tonnage and cut-off grade vs. average grade curve of the ore body.

3.2.2. Preparation of economic block model

The block grade model is converted into an economic block model by applying the cost and revenue data collected from mines. Table 1 summarizes the cost, price, and other relevant data, which are collected from the mines and used for the preparation of economic block models. Table 2 indicates the stope dimension, considering the physical mining constraints. The following expression is used to calculate the economic value of a block.

$$E_i = V_i \times D_i \times MR_i \times CR_i \times (P_i - C_i) \quad (20)$$

where:

$\boldsymbol{E_i}$**:** block economic value in dollars.

$\boldsymbol{V_i}$**:** Volume of block i in m^3

$\boldsymbol{D_i}$: Density of block i in tonnes/m^3

$\boldsymbol{MR_i}$**:** Mining recovery factor of block i

$\boldsymbol{CR_i}$**:** Concentrator recovery factor of block i.

$\boldsymbol{P_i}$**:** Selling price of metal in one tonne of ore for block i, in dollars

$\boldsymbol{C_i}$**:** Cost of producing one tonne of ore for block i, in dollars.

Table 1. Revenue and cost parameters used in optimization.

Sl. No	Different input parameter	Unit	Value
1	Rock density	--	3.0
2	Fixed cost per ton of ore mining.	$	16.24
3	Variable cost per ton of ore mining.	$	7.04
4	Concentrator cost per tonne ore	$	16.16
5	Minimum width of the ore body	m	8
6	Mining recovery	%	90
7	Recovery of concentrator	%	89
8	Selling price per tonne of metal (Cu) in concentrate	$	5436

Table 2. Stope dimension based on physical mining constraints.

Input parameter	Unit	Minimum	Maximum
Stope length (SL)	m	12	50
Stope width (SW)	m	8	25
Stope height (SH)	m	15	60

4. Results and Discussion

4.1. Stope boundary optimization results

The proposed stope boundary optimization model is applied in the block grade model, and the optimal stope boundary, along with the ore tonnage, average Cu%, metal content, and profit inside the optimal boundary is determined. The optimal solution information and other details are given in Table 3. The ore boundaries before optimization and the optimal stope boundaries after optimization are given in Figure 4 and Figure 5. The other relevant quantities are summarized in Table 4. It can be noted from the tables that the optimal boundary is estimated to have 21750 tonnes of copper ore with an average grade of 1.27% Cu. It is worthy to mention here that this ore body is smaller in size, and scattered from other lens ore bodies, making it suitable for preparing a single stope. The profit that can be realized from this stope is around 6,45,122 dollars with the present mining cost and metal selling price.

Table 3. Optimization solution information and other details.

Sl. No	Model parameters and execution details	Unit	Value
1	Frac Gap	--	0.00001
2	Software used		Python
3	Optimization solver used		CBC
4	Type of computer used		Windows
5	RAM of the computer	GB	4
6	Number of decision variables		52156
7	Number of constraints		54809
8	Execution time	Sec	1908

Table 4. Results of optimal orebody boundary.

Output parameters	Unit	Value
Ore tonnage	Tonne	21,750
Metal	Tonne	276.50
Grade (Cu)	%	1.27
Profit	$	645122

An exercise has also been performed to determine the relationships among the cut-off grade, average grade, and ore tonnage at the optimal stope boundary. These relationships are important for economic assessments of deposits. This assessment also indicates that a required mill head grade from mining this block even trades off the optimality of the stope boundary. To perform this exercise, the cut-off grade is used as a constraint in the model. For this purpose, cut-off grade vs. ore tonnage, ore tonnage vs. average grade, and cut-off grade vs. average grade plots are generated for both the in-situ ore body and optimal stope boundary and are presented in Figure 6, Figure 7, and Figure 8, respectively. It should be noted that both plots (in-situ ore and optimal ore bodies) are shown in the same graph to compare the trends in the optimal ore body to those in the in-situ ore body.

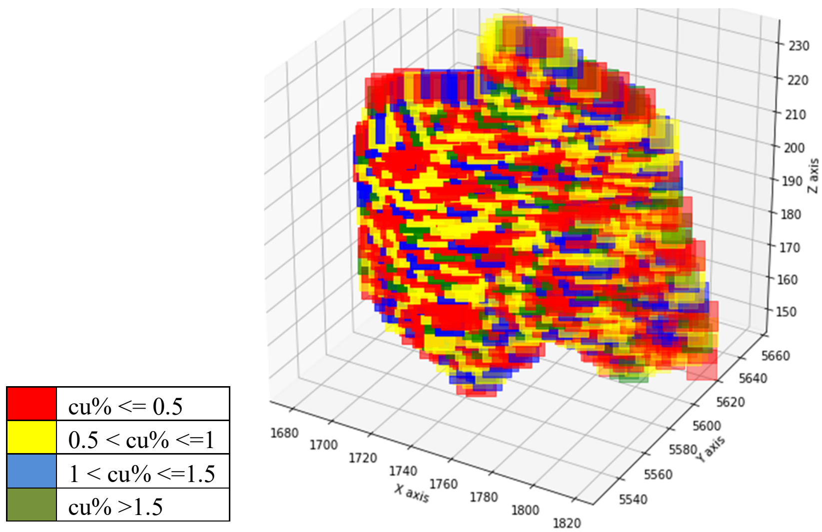

Figure 4. Original orebody obtained from the block grade model.

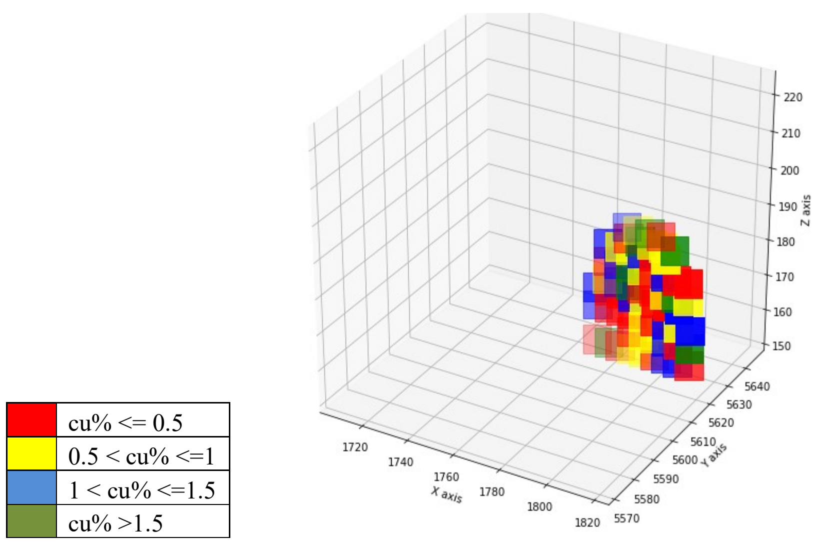

Figure 5. Optimal orebody obtained from the optimization model.

The cut-off grade vs. ore tonnage curve (Figure 6) indicates that the ore tonnage in the optimal stope boundary is significantly lower than that in the in-situ ore body. The lower-graded blocks are large in number in the in-situ ore body. Hence, the stope boundary optimization output produces less tonnage in various cut-off grades. Notably, the ore tonnage plots for both cases are not on the same scale, intending to plot both cases in the same graph. The ore tonnage vs. average grade (Figure 7) also reveals that the average grade is higher in the case of the optimal stope boundary than in the in-situ ore body. Again, the cut-off grade vs. average grade plot (Figure 8) also indicate that the average grade of the optimal stope boundary is higher in all the cases than in the in-situ ore body. For all the cases, the average grade of the optimal stope boundary is greater than that of the in- situ ore body. This average grade of the block is fundamental for production planning. As the mine ore is fed to the concentrator, the concentrator feed grade needs to be maintained. Therefore, depending on the average grade, a blending or planned dilution plan is taken during production planning.

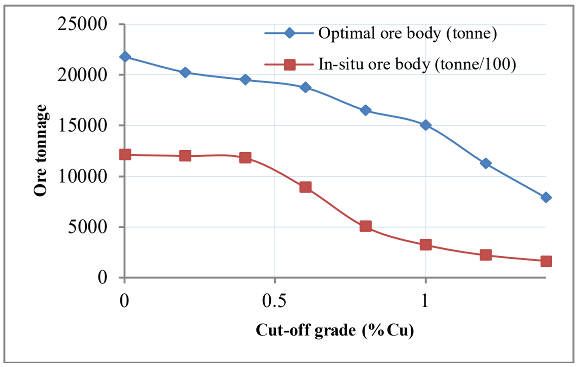

Figure 6. Cut-off grade vs. ore tonnage curve for in-situ ore and the optimal ore body.

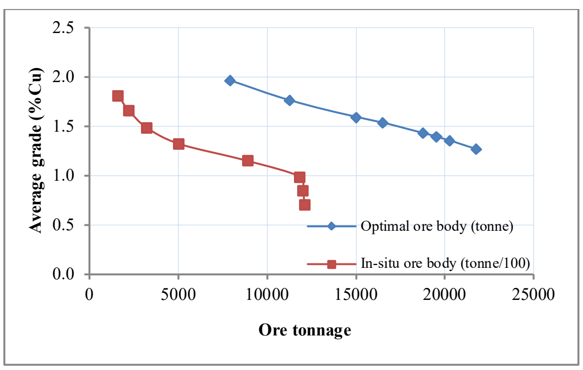

Figure 7. Ore tonnage vs. average grade curve for in situ ore and the optimal ore body.

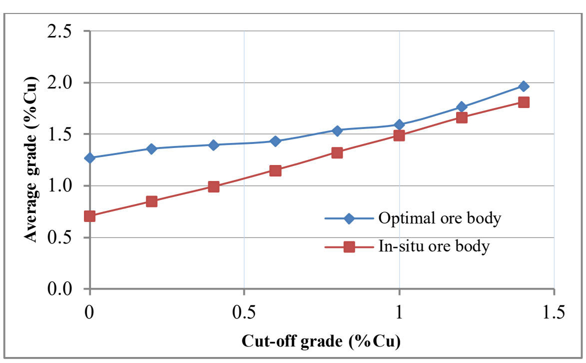

Figure 8. Cut-off grade vs. average grade curve for in-situ ore and the optimal ore body.

4.2. Sensitivity analysis of proposed stope boundary optimization model

Sensitivity analysis is one of the most interesting and preoccupying areas in optimization. It is used to identify how much variation in the input values for a given variable will impact the results of outputs. The metal price and mining cost vary with time, socio-economic changes, and the progress of the mine. Therefore, dynamic metal prices and varying mining costs will impact the optimization results in terms of contained ore tonnage, average grade, metal, profit, and life of the mine. The upside potential, as well as downside risks associated with mining the orebody can also be evaluated via sensitivity analysis. The upside potential helps to decide the entire mining plan to extract the deposit. The downside risk is also important for assessing the success factor of a project. When the upside potential is considered, the downside risk also needs to be considered before the finalization of any optimization model results. The analysis also helps to select a feasible mining method (for example, bulk mining or selective mining) for a deposit under various cost scenario analyses. From a business perspective, it is more important to reduce risk than to have large but risky upside potential. The impacts of changes in the metal price and mining cost on the stope boundary, ore tonnage, grade, metal, and profit from the ore body are analysed.

4.2.1. Impact of metal price uncertainty

Metal prices are highly uncertain and drastically change with the world economy. Historical data on copper price fluctuations show that the amplitude of price fluctuations has mostly varied from 10-40% over the last twenty years. To determine the impact of the metal price on the outputs of the stope boundary optimization results, metal price fluctuations are taken up to ±25% from the present price at an interval of 5% deviation. This sensitivity analysis aims to select the most robust scenario for addressing metal price fluctuations. To do so, a price sensitivity analysis for upside potential and downside risk has been conducted. The stope configuration, ore tonnage, ore grade, metal tonnage, and profit have also been examined.

In this ore body, the metal price variation has a large effect on the ore tonnage, average ore grade, metal tonnage, and profit. Ore tonnage variation is very high in this ore body, because of the increased metal price. When the metal (Cu) price increases from 5436 dollars to 6795 dollars per tonne, the ore tonnage increases from 21,750 tonnes to 1,58,250 tonnes. On the other hand, the downwards trend (price decrease) is almost flat. At a 25% reduction from the present price, the ore tonnage reduces to 3,000 tonnes from 21,750 tonnes. The above changes lead to a change in the average grade from 1.27% Cu to 1.70 % Cu. When the metal price increases to 25% of the present value, the ore tonnage expands from 21,750 tonnes to 158,250 tonnes, and the average grade varies from 1.27% Cu to 1.03% Cu. The metal price change also has a substantial effect on profit. For a 25% increase in metal price from the present value, the profit may shift from 0.6 million dollars to 4.8 million dollars. On the other hand, due to the reduction in the price of metal (Cu) to 25% of the present price, the profit decreases to 0.1 million dollar from 0.6 million dollars. Therefore, the upside potential is very attractive, whereas the downside risk is very low in this block. The graphical plots for the optimal ore, grade, metal, and profit are shown in Figure9, Figure10, Figure11, and Figure12, respectively, with the change in the metal price. As a reference, the ore body configurations with various metal prices are shown in Figure 13.

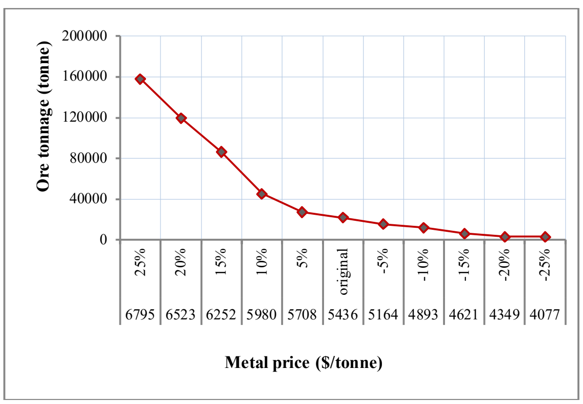

Figure 9. Metal price vs. optimal ore tonnage curve.

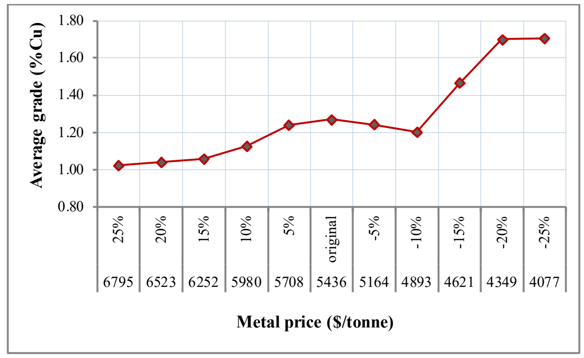

Figure 10. Metal price vs. average grade curve.

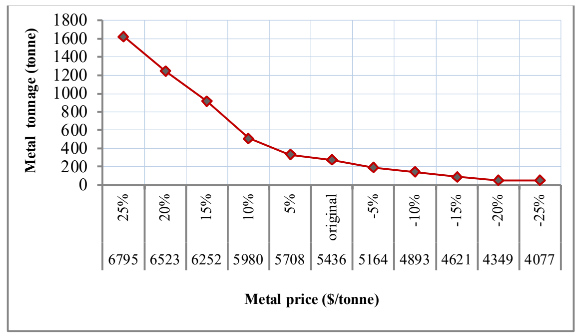

Figure 11. Metal price vs. metal tonnage curve.

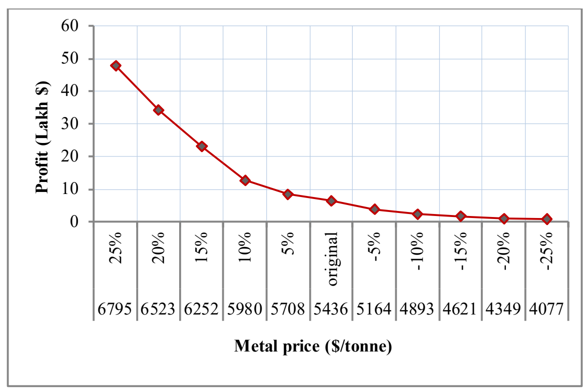

Figure 12. Metal price vs. optimal profit curve.

4.2.2. Impact of mining cost variation

The mining cost also differs in various parts of a mine because of cost variations in various mining activities, such as mine development, hauling, dumping, etc. Also, for a particular stope boundary selection scenario, the reduced cost may be considered for selecting a lower-valued block into the stope boundary, as the stope preparation activity cost is already incurred for this stope. This ore body is very thin and lenticular, and the grade is erratically distributed. Therefore, the sole aim of this study is to examine how mining cost variations make an impact on this kind of ore body in general, so that planning engineers might have an overall understanding of possible upside potential and downside risks for the remaining part of the deposit. In this orebody, with the change in mining cost, the stope size changes reasonably.

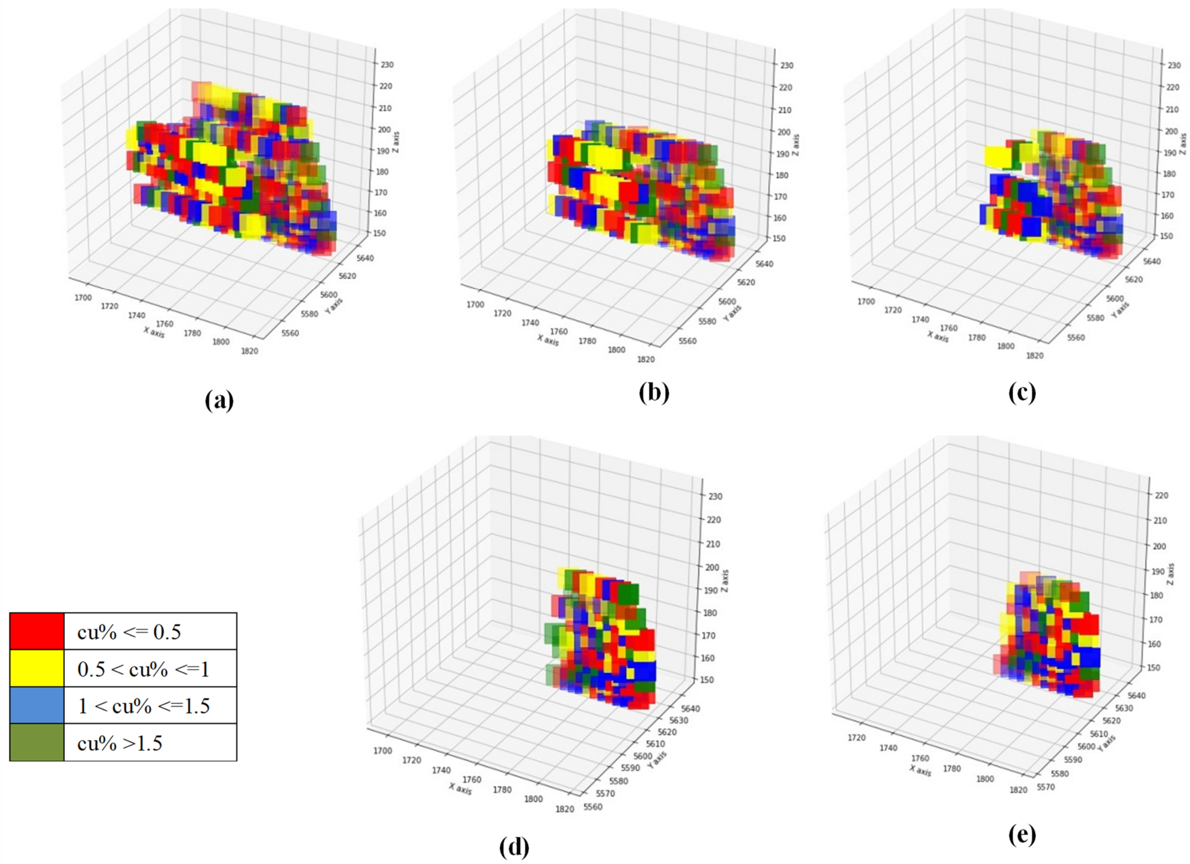

Figure 13. Orebody configurations for various metal prices: (a) +25% metal price, (b) +20% metal price, (c) +15% metal price, (d) +10% metal price, (e) +5% metal price.

At the present ore mining cost, 21,750 tonnes of ore can be mined from the stope, with an average grade of 1.27% Cu and a profit of 0.645 million dollars. At a 25% increase in mining cost from the present value, the ore tonnage decreases to 3,000 tonnes from 21,750 tonnes, and when the mining cost is shrunk by 25% from the present value, the ore tonnage in the stope increases to 237,750 tonnes. The average grade in the stope changes from1.27% to 1.70% and from 1.27 % to 0.98%, respectively. The profit varies significantly from 0.13 million dollars to 5.642 million dollars with the change in mining cost from – 25% to 25% of the present mining cost. It is also revealed that the mining cost has a substantial impact on profit, and with a 25% reduction in cost, there might be an upside potential for profit realization from 0.645 million dollars to 5.642 million dollars without much reduction in profit per tonne. On the other hand, with an increase of 25% in cost, the downside risk is only nominal from 0.645 million dollars to 0.13 million dollars. It is worth mentioning here that profit (dollars) per tonne is also an important consideration for any investment decision and financial evaluation of a mining project. Figure 14, Figure 15, Figure 16, and Figure17 show the change in ore tonnage, grade (%Cu), metal tonnage, and profit with respect to the change in mining cost.

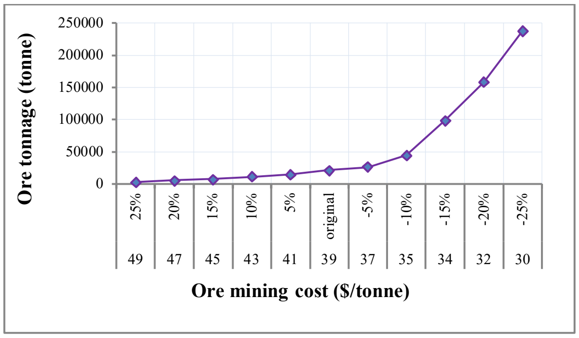

Figure 14. Optimal ore tonnage against changes in mining cost.

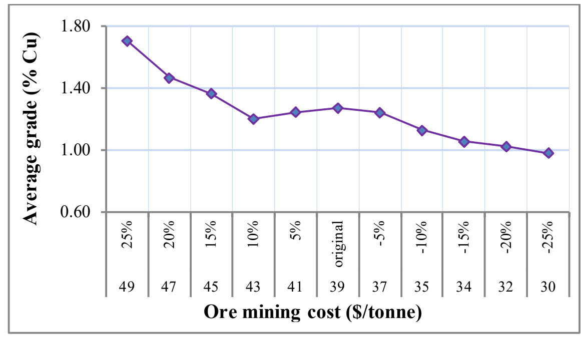

Figure 15. Grade variation against change in mining cost.

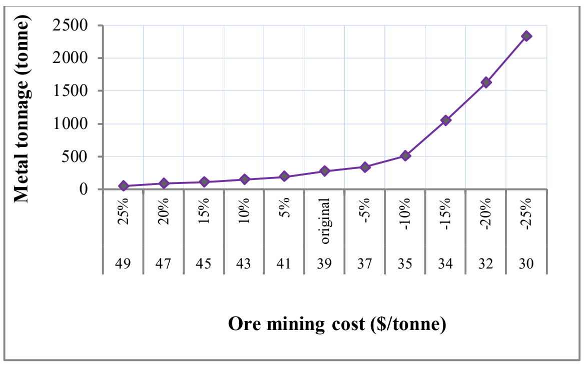

Figure 16. Optimal metal tonnage against changes in mining cost.

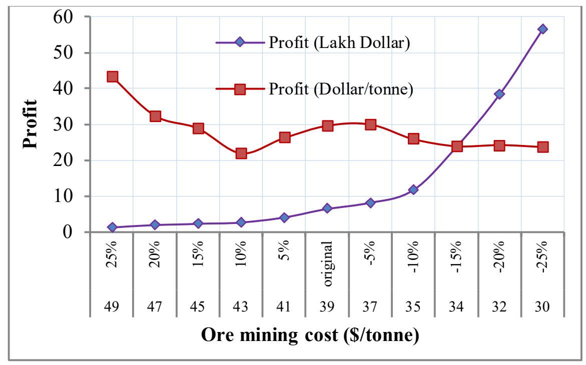

Figure 17. Optimal profit and profit per tonne against change in mining cost.

4.2.3. Comparison of manual design and real output with the optimal model recommendations

To gain confidence in the outputs of the developed model, an exercise has been carried out. The exercise consists of a performance test of the model against real mining outputs and manual design outputs by experienced field engineers of the mine. For this field application, a portion of the mined-out stope in this mine is considered. The mine management has been sheared all the relevant available data of the mined-out stope for this investigation. The outputs of three different entities, (i) manual design estimation of the stope, (ii) real mining output from the stope, and (iii) stope design results through the optimization model, are analysed. The manual design estimates ore 165,440 tonnes with a grade of 1.17% Cu. Considering the present mining cost, the estimated profit is 4.032 million dollars. The actual achievement from the stope is significantly different from the manual design. Due to soft rock formation, during mining, dilution occurred from H/W, which resulted in more tonnage but a lower average grade from this stope. The real output from this stope is 205,377 tonnes of ore with 0.98% Cu, and the profit realization is 2.816 million dollars. On the other hand, the results of the optimization model for this ore body are quite impressive. The ore tonnage is estimated to be 94,875 tonnes with 1.48 % Cu. The profit may reach 3.867 million dollars, which is 37.32% higher with an optimized stope design. The results shows that the optimization model efficiently rejects the lower-grade (loss-making block) ore block from the stope boundary. This exercise reveals that the output of the model is beneficial in all respects. A comparative result considering the manual design, real mining output, and optimal model output is given in Table 5.

Table 5. Summary of results for validation of model.

Sl. No.	Title	Ore tonnage (t)	Grade (%Cu)	Metal (t)	Profit (million $)
1	Manual design	165,440	1.17	1,942	4.032
2	Real mining output	205,377	0.98	2,008	2.816
3	Optimal model output	94,875	1.48	1,400	3.867

5. Conclusions

This article describes a new profit-based mixed integer programming model to solve the stope boundary optimization problem for underground metal mining. The model requires an ore body model as an input, and generates a stope boundary that maximizes the profit from the stope under practical mining constraints. This study addresses the physical mining constraint, more specifically, the stope dimension such as the maximum and minimum length, width, and height of the stope. An implementation study reflects that the new model is robust, and ensures the better economic value from mining a stope. The sensitivity analysis reveals that the model is efficient at addressing the dynamic metal price and changing mining cost for stope boundary optimization. The model assists in the comparative assessment of various cost-effective underground stoping methods such as selective mining or bulk mining. The upside potential and downside risk of a stope or whole ore body can also be analysed before any business decision for mining can be made. The case study shows that for an increase (25%) in the metal price from the present value, the profit may shift from 0.6 million dollars to 4.8 million dollars. On the other hand, due to the reduction, the profit decreases to 0.1 million dollars from 0.6 million dollars. Therefore, the upside potential is very attractive, whereas the downside risk is low in this block. The comparison study shows that the application of the proposed model may produce greater profit (37.32%) than the present practice in the mine. The approach provides a powerful tool for performing stope boundary optimization under various economic scenarios while designing a stope. Hence, the model offers a pragmatic solution for realistic underground mine planning operations.

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یک مدل برنامه‌ریزی عدد صحیح مختلط مبتنی بر سود برای بهینه‌سازی مرز کارگاه و پیاده‌سازی در ذخایر مس هند

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چکیده:

چیدمان بهینه کارگاه (مرز کارگاه) در یک معدن فلزی زیرزمینی، سود یک کانسار را با توجه به محدودیت‌های ژئوتکنیکی و عملیاتی استخراج مانند طول کارگاه، عرض کارگاه، ارتفاع کارگاه، به حداکثر می‌رساند. رویکردهای مختلفی برای پرداختن به مسئله بهینه‌سازی مرز کارگاه معرفی شده‌اند، اما به دلیل پیچیدگی محاسباتی و محدودیت‌های عملی متعدد، مدل‌های موجود راه‌حل‌های جزئی برای این مسئله ارائه می‌دهند. در کار حاضر، یک مدل برنامه‌ریزی عدد صحیح مختلط با گنجاندن محدودیت‌های استخراج در یک چارچوب سه‌بعدی توسعه داده شده است. این مدل بر اساس حداکثرسازی سود توسعه داده شده است. تحلیل حساسیت اعمال شده در یک معدن مطالعه موردی نشان می‌دهد که این مدل در ارزیابی پتانسیل مثبت و ریسک منفی سود تحت نوسانات قیمت فلز و هزینه‌های استخراج، کارآمد است. علاوه بر این، می‌توان آن را در مراحل مختلف طراحی معدن برای تسهیل ارزیابی منابع، انتخاب روش‌های کارگاه و برنامه‌ریزی جامع معدن به کار برد. در یک کاربرد عملی بر روی یک توده معدنی واقعی، نشان می‌دهد که مدل پیشنهادی می‌تواند تا ۳۷.۳۲٪ سود بیشتری در مقایسه با روش طراحی کارگاه فعلی در معادن ایجاد کند.

کلمات کلیدی: استخراج زیرزمینی، مدل بلوکی، برنامه‌ریزی عدد صحیح، محدودیت‌ها، تحلیل حساسیت.