

ON THE EDGE COVER POLYNOMIAL OF CERTAIN GRAPHS

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ABSTRACT. Let G be a simple graph of order n and size m . The edge covering of G is a set of edges such that every vertex of G is incident to at least one edge of the set. The edge cover polynomial of G is the polynomial $E(G, x) = \sum_{i=\rho(G)}^m e(G, i)x^i$, where $e(G, i)$ is the number of edge coverings of G of size i , and $\rho(G)$ is the edge covering number of G . In this paper we study the edge cover polynomials of cubic graphs of order 10. We show that all cubic graphs of order 10 (especially the Petersen graph) are determined uniquely by their edge cover polynomials.

1. INTRODUCTION

Let $G = (V, E)$ be a simple graph. The *order* and the *size* of G is the number of vertices and the number of edges of G , respectively. For every graph G with no isolated vertex, an edge covering of G is a set of edges of G such that every vertex is incident with at least one edge of the set. In other words, an edge covering of a graph is a set of edges which together meet all vertices of the graph. A minimum edge covering is an edge covering of the smallest possible size. The edge covering number of G is the size of a minimum edge covering of G and denoted by $\rho(G)$. In this paper we let $\rho(G) = 0$, if G has some isolated vertices. For a detailed treatment of these parameters, the reader is referred to [2, 3, 6]. Let $\mathcal{E}(G, i)$ be the family of all edge coverings of a graph G with cardinality i and let $e(G, i) = |\mathcal{E}(G, i)|$. The *edge cover*

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polynomial $E(G, x)$ of G is defined as

$$E(G, x) = \sum_{i=\rho(G)}^m e(G, i)x^i,$$

where $\rho(G)$ is the edge covering number of G . Also, for a graph G with some isolated vertices we define $E(G, x) = 0$. Let $E(G, x) = 1$, when both order and size of G are zero (see [2]).

For two graphs G and H , the corona $G \circ H$ is the graph arising from the disjoint union of G with $|V(G)|$ copies of H , by adding edges between the i th vertex of G and all vertices of i th copy of H [5]. The corona $G \circ K_1$, in particular, is the graph constructed from a copy of G , where for each vertex $v \in V(G)$, a new vertex u and a pendant edge $\{v, u\}$ are added. It is easy to see that the corona operation of two graphs does not have the commutative property.

Two graphs G and H are said to be *edge covering equivalent*, or simply \mathcal{E} -equivalent, written $G \sim H$, if $E(G, x) = E(H, x)$. It is evident that the relation \sim of being \mathcal{E} -equivalence is an equivalence relation on the family \mathcal{G} of graphs, and thus \mathcal{G} is partitioned into equivalence classes, called the \mathcal{E} -equivalence classes. Given $G \in \mathcal{G}$, let

$$[G] = \{H \in \mathcal{G} : H \sim G\}.$$

We call $[G]$ the equivalence class determined by G . A graph G is said to be *edge covering unique*, or simply \mathcal{E} -unique, if $[G] = \{G\}$.

For every vertex $v \in V(G)$, the degree of v is the number of edges incident with v and is denoted by $d_G(v)$. For simplicity we write $d(v)$ instead of $d_G(v)$. Let $a_i(G)$ be the number of vertices of G with degree i . The minimum degree of G is denoted by $\delta(G)$. A graph G is called *k-regular* if all vertices have the same degree k . One of the famous graphs is the Petersen graph which is a symmetric non-planar cubic graph. In the study of edge cover polynomials, it is interesting to investigate the edge coverings and edge cover polynomial of this graph. We denote the Petersen graph by P .

In [2] authors has characterized all graphs whose edge cover polynomials have exactly one or two distinct roots and moreover they proved that these roots are contained in the set $\{-3, -2, -1, 0\}$. In this paper, we construct some infinite families of graphs whose edge cover polynomials have only roots -1 and 0 . Also similar to [1], we study the edge coverings and edge cover polynomials of cubic graphs of order 10. As a consequence, we show that the all cubic graphs of order 10 (especially the Petersen graph) are determined uniquely by their edge cover polynomials.

In the next section, we use corona of a graph G with K_1 to construct some families of graphs whose edge cover polynomials have only roots -1 and 0 . In Section 3, we obtain the edge cover polynomial of the Petersen graph. In section 4, we list all ρ -sets of connected cubic graphs of order 10. This list will be used to study the \mathcal{E} -equivalence of these graphs in the last section. In the last section, using a procedure we study the \mathcal{E} -equivalence classes of all cubic graphs of order 10 and we show that all cubic graphs of order 10 are determined uniquely by their edge cover polynomial.

2. FAMILIES OF GRAPHS WITH EDGE COVER ROOT -1 AND 0

Theorem 18 of [2] introduces graphs whose edge cover polynomials have only roots -1 and 0 . Here we would like to look at these kind of graphs as corona of two graphs. We think that the results in this section can be a starting point for the study of the edge cover polynomial of corona of two graphs which has not yet been studied.

Let G be any graph with vertex set $\{v_1, \dots, v_n\}$. Add n new vertices $\{u_1, \dots, u_n\}$ and join u_i to v_i for every $i, 1 \leq i \leq n$. By definition we denote this new graph by $G \circ K_1$. We start this section with the following Lemma.

Lemma 2.1. *For any graph G of order n , $\rho(G \circ K_1) = n$.*

Proof. If E is an edge covering of $G \circ K_1$, then for every i , $\{v_i, u_i\} \in E$. This implies that $|E| \geq n$. Since $\{\{v_1, u_1\}, \dots, \{v_n, u_n\}\}$ is an edge covering of $G \circ K_1$, we have the result. \square

By Lemma 2.1, $e(G \circ K_1, i) = 0$, for every $i, i < n$, so we shall compute $e(G \circ K_1, i)$ for each $i, n \leq i \leq n + |E(G)|$.

Theorem 2.2. *For any graph G of order n and size m . For every $i, n \leq i \leq n + m$, we have $e(G \circ K_1, i) = \binom{m}{i-n}$. Hence $E(G \circ K_1, x) = x^n(1 + x)^m$.*

Proof. Suppose that E is an edge covering of $G \circ K_1$ of size i , and $|E \cap E(G)| = j$, for $0 \leq j \leq m$. Also E contains these edges $\{v_1, u_1\}, \dots, \{v_n, u_n\}$. For addition j edges of $E(G)$ to an edge covering E of size i , we have $\binom{m}{j}$ possibilities. \square

We have the following corollary:

Corollary 2.3. *Let $i \in \mathbb{N}$. For any graph G of order n and size m , $E(G \circ \overline{K}_i, x) = x^{in}(1 + x)^m$.*

The following corollary is a consequence of Corollary 2.3.

Corollary 2.4. *The edge cover polynomial of every graph H in the family $\{G \circ \overline{K_i}, (G \circ \overline{K_i}) \circ \overline{K_i}, ((G \circ \overline{K_i}) \circ \overline{K_i}) \circ \overline{K_i}, \dots\}$ have only two roots -1 and 0 .*

Proof. By Corollary 2.3, the graph $G \circ \overline{K_i}$ has only two roots -1 and 0 . Now suppose that $H_1 = G \circ \overline{K_i}$. Using Corollary 2.3, $H_1 \circ \overline{K_i}$ has only two roots -1 and 0 . We apply Corollary 2.3 consecutive, then we have the result. \square

Remark. Graphs in Corollary 2.3 and 2.4 are special cases of family of graphs in Theorem 18 of [2].

As we know the edge cover polynomials of binary graph operations, such as join, corona, Cartesian and lexicographic product have not been studied. Let us to state the following problem:

Problem. Compute and present recurrence formulae and properties of the edge cover polynomials of families of graphs obtained by various products.

3. EDGE COVER POLYNOMIAL OF THE PETERSEN GRAPH

In this section we shall investigate the edge cover polynomial of the Petersen graph. First, we state some properties of the edge cover polynomial of a graph.

Lemma 3.1. [2] *Let G be a graph of order n and size m with no isolated vertex. If $E(G, x) = \sum_{i=\rho(G)}^m e(G, i)x^i$, then the following hold:*

- (i) $E(G, x)$ is a monic polynomial of degree m .
- (ii) For $i = m - \delta + 1, \dots, m$, $e(G, i) = \binom{m}{i}$. Moreover, if $i_0 = \min\{i | e(G, i) = \binom{m}{i}\}$, then $\delta = m - i_0 + 1$.

Corollary 3.2. *Let G and H be two graphs (with no isolated vertex) of size m_1 and m_2 , respectively. If $E(G, x) = E(H, x)$, then $\rho(G) = \rho(H)$, $m_1 = m_2$ and $\delta(G) = \delta(H)$.*

The following theorem states that if G is a regular graph, then every element of $[G]$ is a regular graph too.

Theorem 3.3. [2] *Let G be a k -regular graph of order n and $k \geq 2$. Suppose that H is a graph and $E(H, x) = E(G, x)$. Then H is a k -regular graph of order n .*

We can determine some coefficients of $E(G, x)$ by having the degree sequence of G . Also some information about the degree sequence of G can be determined by the edge cover polynomial of G .

Theorem 3.4. [2] *Let G be a graph of order n and size m with no isolated vertex and $E(G, x) = \sum_{i=\rho(G)}^m e(G, i)x^i$, then the following hold:*

(i) *For every i ,*

$$e(G, i) \geq \binom{m}{i} - \sum_{v \in V(G)} \binom{m - d(v)}{i}.$$

(ii) *For every $i, i \geq m - 2\delta + 2$,*

$$e(G, i) = \binom{m}{i} - \sum_{v \in V(G)} \binom{m - d(v)}{i}.$$

There are exactly 21 cubic graphs of order 10 given in Figure 1 (see [7]). Clearly, edge covering number of a connected cubic graph of order 10 is 5. There are just two non-connected cubic graphs of order 10. Clearly, for these graphs, the edge covering number is also 5. Note that the graph G_{17} is the Petersen graph. For the labeled graph G_{17} in Figure 1, we obtain all edge coverings of size 5 in the following lemma.

Lemma 3.5. *For the Petersen graph P , $e(P, 5) = 6$.*

Proof. We list all edge coverings of P of cardinality 5, which are the ρ -sets of the labeled Petersen graph (graph G_{17}) given in Figure 1.

$$\begin{aligned} \mathcal{E}(P, 5) = & \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 10\}, \{6, 8\}, \{7, 9\} \}, \{ \{1, 2\}, \{3, 8\}, \{4, 5\}, \right. \\ & \{6, 9\}, \{7, 10\} \}, \{ \{1, 5\}, \{2, 3\}, \{4, 9\}, \{6, 8\}, \{7, 10\} \}, \{ \{1, 5\}, \{2, 7\}, \\ & \{3, 4\}, \{6, 9\}, \{8, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 9\}, \{8, 10\} \}, \{ \{1, 6\}, \\ & \left. \{2, 7\}, \{3, 8\}, \{4, 9\}, \{5, 10\} \} \right\}. \end{aligned}$$

Therefore, $e(P, 5) = |\mathcal{E}(P, \rho)| = 6$. \square

We need the following theorem for finding the edge cover polynomial of the Petersen graph.

Lemma 3.6. *Let G be a cubic graph of order 10. Then the following hold:*

- (i) $e(G, i) = \binom{m}{i}$, for $i = 13, 14, 15$.
- (ii) $e(G, i) = \binom{m}{i} - n \binom{m-3}{i}$, for $i = 11, 12$.
- (iii) $e(G, 10) = 2358$.

Proof.

- (i) It follows from Lemma 3.1 (ii).
- (ii) It follows from Theorem 3.4 (ii).
- (iii) It is easy to see that $e(G, 10) = \binom{m}{10} - n \binom{m-3}{10} + 15 = 2358$. \square

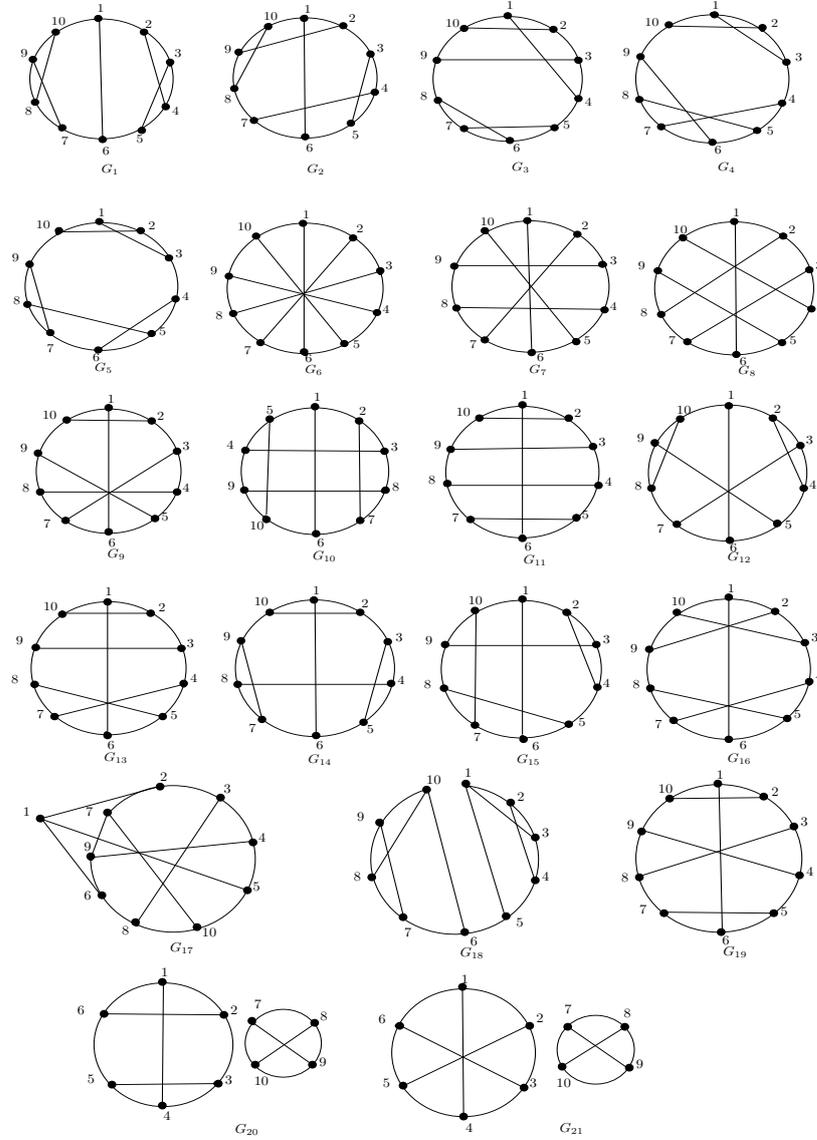


FIGURE 1. Cubic graphs of order 10.

Theorem 3.7. *The edge cover polynomial of the Petersen graph P is:*

$$E(P, x) = x^{15} + \binom{15}{14}x^{14} + \binom{15}{13}x^{13} + \left[\binom{15}{12} - 10 \right]x^{12} + \left[\binom{15}{11} - 120 \right]x^{11} + \left[\binom{15}{10} - 645 \right]x^{10} + 2985x^9 + 2400x^8 + 1101x^7 + 215x^6 + 6x^5.$$

Proof. The result follows from Lemmas 3.5 and 3.6. \square

4. ρ -SETS OF CUBIC GRAPHS OF ORDER 10.

In this section, we present all ρ -sets of connected cubic graphs G_1, \dots, G_{19} shown in Figure 1. The results here will be useful in studying the \mathcal{E} -equivalence of these graphs in the last section.

$$\mathcal{E}(G_1, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 9\}, \{8, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 9\}, \{8, 10\} \}, \{ \{1, 6\}, \{2, 4\}, \{3, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 4\}, \{3, 5\}, \{7, 9\}, \{8, 10\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 4\}, \{3, 5\}, \{6, 7\}, \{8, 9\} \} \right\}.$$

Therefore, $e(G_1, 5) = |\mathcal{E}(G_1, \rho)| = 8$.

$$\mathcal{E}(G_2, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 9\}, \{3, 5\}, \{4, 7\}, \{8, 10\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 7\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 9\}, \{3, 4\}, \{5, 6\}, \{7, 8\} \} \right\}.$$

Therefore $e(G_2, 5) = |\mathcal{E}(G_2, \rho)| = 6$.

$$\mathcal{E}(G_3, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 7\}, \{6, 8\}, \{9, 10\} \}, \{ \{1, 4\}, \{2, 3\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 4\}, \{2, 3\}, \{5, 7\}, \{6, 8\}, \{9, 10\} \}, \{ \{1, 4\}, \{2, 10\}, \{3, 9\}, \{5, 6\}, \{7, 8\} \}, \{ \{1, 4\}, \{2, 10\}, \{3, 9\}, \{5, 7\}, \{6, 8\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \} \right\}. \text{ Therefore } e(G_3, 5) = |\mathcal{E}(G_3, \rho)| = 7.$$

$$\mathcal{E}(G_4, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 8\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 3\}, \{2, 10\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 3\}, \{2, 10\}, \{4, 5\}, \{6, 9\}, \{7, 8\} \}, \{ \{1, 3\}, \{2, 10\}, \{4, 7\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 3\}, \{2, 10\}, \{4, 7\}, \{5, 8\}, \{6, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 9\}, \{7, 8\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 7\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 7\}, \{5, 8\}, \{6, 9\} \} \right\}.$$

Therefore $e(G_4, 5) = |\mathcal{E}(G_4, \rho)| = 10$.

$$\mathcal{E}(G_5, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 8\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 3\}, \{2, 10\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 3\}, \{2, 10\}, \{4, 6\}, \{5, 8\}, \{7, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 6\}, \{5, 8\}, \{7, 9\} \} \right\}.$$

Therefore $e(G_5, 5) = |\mathcal{E}(G_5, \rho)| = 6$.

$\mathcal{E}(G_6, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 10\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 2\}, \{3, 8\}, \{4, 5\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 8\}, \{4, 9\}, \{5, 10\}, \{6, 7\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 9\}, \{5, 10\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 4\}, \{5, 10\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 8\}, \{4, 5\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 8\}, \{4, 9\}, \{5, 10\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 9\}, \{5, 6\}, \{7, 8\} \}, \{ \{1, 10\}, \{2, 7\}, \{3, 4\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 7\}, \{3, 8\}, \{4, 9\}, \{5, 6\} \} \right\}$. Therefore $e(G_6, 5) = |\mathcal{E}(G_6, \rho)| = 13$.

$\mathcal{E}(G_7, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 10\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 2\}, \{3, 9\}, \{4, 8\}, \{5, 10\}, \{6, 7\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 4\}, \{5, 10\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 9\}, \{4, 8\}, \{5, 10\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 7\}, \{3, 4\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 7\}, \{3, 9\}, \{4, 8\}, \{5, 6\} \} \right\}$. Therefore $e(G_7, 5) = |\mathcal{E}(G_7, \rho)| = 9$

$\mathcal{E}(G_8, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 7\}, \{4, 10\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 10\}, \{5, 9\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 8\}, \{3, 7\}, \{4, 5\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 8\}, \{3, 7\}, \{4, 10\}, \{5, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 8\}, \{3, 4\}, \{5, 9\}, \{6, 7\} \} \right\}$.

Therefore $e(G_8, 5) = |\mathcal{E}(G_8, \rho)| = 8$.

$\mathcal{E}(G_9, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 7\}, \{4, 8\}, \{5, 6\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 4\}, \{5, 9\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 7\}, \{4, 5\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 7\}, \{4, 8\}, \{5, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 8\}, \{5, 9\}, \{6, 7\} \} \right\}$.

Therefore $e(G_9, 5) = |\mathcal{E}(G_9, \rho)| = 8$.

$\mathcal{E}(G_{10}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 10\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 2\}, \{3, 8\}, \{4, 5\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 8\}, \{4, 9\}, \{5, 10\}, \{6, 7\} \}, \{ \{1, 5\}, \{2, 3\}, \{4, 9\}, \{6, 10\}, \{7, 8\} \}, \{ \{1, 5\}, \{2, 7\}, \{3, 4\}, \{6, 10\}, \{8, 9\} \}, \{ \{1, 5\}, \{2, 7\}, \{3, 8\}, \{4, 9\}, \{6, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 9\}, \{5, 10\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 4\}, \{5, 10\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 8\}, \{4, 5\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 7\}, \{3, 8\}, \{4, 9\}, \{5, 10\} \} \right\}$. Therefore $e(G_{10}, 5) = |\mathcal{E}(G_{10}, \rho)| = 11$.

$$\mathcal{E}(G_{11}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 8\}, \{5, 7\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 4\}, \{5, 7\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 9\}, \{4, 5\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 9\}, \{4, 8\}, \{5, 7\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \} \right\}. \text{ Therefore } e(G_{11}, 5) = |\mathcal{E}(G_{11}, \rho)| = 7.$$

$$\mathcal{E}(G_{12}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 9\}, \{6, 7\}, \{8, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 4\}, \{3, 7\}, \{5, 9\}, \{8, 10\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 4\}, \{3, 7\}, \{5, 6\}, \{8, 9\} \} \right\}.$$

$$\text{Therefore } e(G_{12}, 5) = |\mathcal{E}(G_{12}, \rho)| = 6.$$

$$\mathcal{E}(G_{13}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 8\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 7\}, \{5, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 9\}, \{4, 5\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 9\}, \{4, 7\}, \{5, 8\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 7\}, \{5, 6\}, \{8, 9\} \} \right\}.$$

$$\text{Therefore } e(G_{13}, 5) = |\mathcal{E}(G_{13}, \rho)| = 8.$$

$$\mathcal{E}(G_{14}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 5\}, \{4, 8\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 10\}, \{3, 5\}, \{4, 8\}, \{7, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 8\}, \{5, 6\}, \{7, 9\} \} \right\}.$$

$$\text{Therefore } e(G_{14}, 5) = |\mathcal{E}(G_{14}, \rho)| = 6.$$

$$\mathcal{E}(G_{15}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 10\}, \{8, 9\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 8\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 10\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 4\}, \{3, 9\}, \{5, 8\}, \{7, 10\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 4\}, \{3, 9\}, \{5, 6\}, \{7, 8\} \}, \{ \{1, 10\}, \{2, 4\}, \{3, 9\}, \{5, 8\}, \{6, 7\} \} \right\}. \text{ Therefore } e(G_{15}, 5) = |\mathcal{E}(G_{15}, \rho)| = 9.$$

$$\mathcal{E}(G_{16}, \rho) = \left\{ \{ \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 4\}, \{5, 8\}, \{6, 7\}, \{9, 10\} \}, \{ \{1, 2\}, \{3, 10\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 2\}, \{3, 10\}, \{4, 7\}, \{5, 6\}, \{8, 9\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 3\}, \{4, 7\}, \{5, 8\}, \{9, 10\} \}, \{ \{1, 6\}, \{2, 9\}, \{3, 10\}, \{4, 5\}, \{7, 8\} \}, \{ \{1, 6\}, \{2, 9\}, \{3, 10\}, \{4, 7\}, \{5, 8\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\} \}, \{ \{1, 10\}, \{2, 3\}, \{4, 7\}, \{5, 8\} \} \right\}.$$

$\{5, 6\}, \{8, 9\}, \{\{1, 10\}, \{2, 9\}, \{3, 4\}, \{5, 6\}, \{7, 8\}\}, \{\{1, 10\}, \{2, 9\}, \{3, 4\}, \{5, 8\}, \{6, 7\}\}$.

Therefore $e(G_{16}, 5) = |\mathcal{E}(G_{16}, \rho)| = 12$.

$\mathcal{E}(G_{17}, \rho) = \left\{ \{\{1, 2\}, \{3, 4\}, \{5, 10\}, \{6, 8\}, \{7, 9\}\}, \{\{1, 2\}, \{3, 8\}, \{4, 5\}, \{6, 9\}, \{7, 10\}\}, \{\{1, 5\}, \{2, 3\}, \{4, 9\}, \{6, 8\}, \{7, 10\}\}, \{\{1, 5\}, \{2, 7\}, \{3, 4\}, \{6, 9\}, \{8, 10\}\}, \{\{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 9\}, \{8, 10\}\}, \{\{1, 6\}, \{2, 7\}, \{3, 8\}, \{4, 9\}, \{5, 10\}\} \right\}$.

Therefore, $e(G_{17}, 5) = |\mathcal{E}(G_{17}, \rho)| = 6$.

$\mathcal{E}(G_{18}, \rho) = \left\{ \{\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\}\}, \{\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 9\}, \{8, 10\}\}, \{\{1, 3\}, \{2, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\}\}, \{\{1, 3\}, \{2, 4\}, \{5, 6\}, \{7, 9\}, \{8, 10\}\} \right\}$.

Therefore, $e(G_{18}, 5) = |\mathcal{E}(G_{18}, \rho)| = 4$.

$\mathcal{E}(G_{19}, \rho) = \left\{ \{\{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}, \{9, 10\}\}, \{\{1, 2\}, \{3, 8\}, \{4, 5\}, \{6, 7\}, \{9, 10\}\}, \{\{1, 6\}, \{2, 3\}, \{4, 5\}, \{7, 8\}, \{9, 10\}\}, \{\{1, 6\}, \{2, 10\}, \{3, 4\}, \{5, 7\}, \{8, 9\}\}, \{\{1, 6\}, \{2, 10\}, \{3, 8\}, \{4, 9\}, \{5, 7\}\}, \{\{1, 10\}, \{2, 3\}, \{4, 5\}, \{6, 7\}, \{8, 9\}\}, \{\{1, 10\}, \{2, 3\}, \{4, 9\}, \{5, 6\}, \{7, 8\}\} \right\}$. Therefore $e(G_{19}, 5) = |\mathcal{E}(G_{19}, \rho)| = 7$.

5. \mathcal{E} -EQUIVALENCE CLASSES OF ALL CUBIC GRAPHS OF ORDER 10

In this section we study the \mathcal{E} -equivalence classes of all cubic graphs of order 10 and we show that all cubic graphs of order 10 are determined uniquely by their edge cover polynomials.

Theorem 5.1. *Let G be a cubic graphs of order 10 and H be a graph of order n . If $E(H, x) = E(G, x)$, then $H = G$.*

Proof. By Theorem 3.3, we find that H is a 3-regular graph of order 10. Since there are exactly 21 cubic graphs of order 10 (Figure 1), by comparing the edge cover polynomial of these graphs, we conclude that none of these polynomials are equal (see coefficients of these polynomials in Table 1). Therefore $H = G$. \square

The following procedure allows to compute the edge cover polynomial of a graph G with order n and size m . Let $W = \{S_1, \dots, S_{2^m}$

$\}_{-1}$ be the collection of nonempty subsets of $E(G)$. The algorithm starts with $E(G, x) = 0$ and continues with the following steps, for $1 \leq j \leq 2^m - 1$.

1. If $\bigcup_{i=1}^{|S_j|} S_{j_i} = \{1, \dots, n\}$, then go to step 2, else replace j by $j + 1$ and apply the algorithm again.
2. Add one term $x^{|S_j|}$ to $E(G, x)$.
3. Replace j by $j + 1$ and apply the algorithm again.

We show the number of edge covering of the cubic graphs of order 10 in Table 1.

j	5	6	7	8	9	10	11	12	13	14	15
$e(G_1, j)$	8	195	1055	2378	2981	2358	1245	445	105	15	1
$e(G_2, j)$	6	205	1073	2388	2983	2358	1245	445	105	15	1
$e(G_3, j)$	7	202	1065	2383	2982	2358	1245	445	105	15	1
$e(G_4, j)$	10	218	1080	2389	2983	2358	1245	445	105	15	1
$e(G_5, j)$	6	192	1052	2377	2981	2358	1245	445	105	15	1
$e(G_6, j)$	13	230	1100	2400	2985	2358	1245	445	105	15	1
$e(G_7, j)$	9	224	1098	2400	2985	2358	1245	445	105	15	1
$e(G_8, j)$	8	221	1097	2400	2985	2358	1245	445	105	15	1
$e(G_9, j)$	8	214	1085	2394	2984	2358	1245	445	105	15	1
$e(G_{10}, j)$	11	230	1100	2400	2985	2358	1245	445	105	15	1
$e(G_{11}, j)$	7	208	1074	2388	2983	2358	1245	445	105	15	1
$e(G_{12}, j)$	6	201	1071	2388	2983	2358	1245	445	105	15	1
$e(G_{13}, j)$	8	218	1087	2394	2984	2358	1245	445	105	15	1
$e(G_{14}, j)$	6	194	1059	2382	2982	2358	1245	445	105	15	1
$e(G_{15}, j)$	9	217	1086	2394	2984	2358	1245	445	105	15	1
$e(G_{16}, j)$	12	231	1101	2400	2985	2358	1245	445	105	15	1
$e(G_{17}, j)$	6	215	1095	2400	2985	2358	1245	445	105	15	1
$e(G_{18}, j)$	4	197	1057	2378	2981	2358	1245	445	105	15	1
$e(G_{19}, j)$	7	204	1072	2388	2983	2358	1245	445	105	15	1
$e(G_{20}, j)$	12	199	1050	2373	2980	2358	1245	445	105	15	1
$e(G_{21}, j)$	18	231	1080	2385	2982	2358	1245	445	105	15	1

Table 1. The number of edge coverings of G_n with cardinality j .

Regarding to this table we see that all cubic graphs of order 10 are determined uniquely by their edge cover polynomial. So we have the following corollary.

Corollary 5.2. *All cubic graphs of order 10 are determined uniquely by their edge cover polynomials.*

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REFERENCES

1. S. Alikhani and Y. H. Peng, The domination polynomials of cubic graphs of order 10, *Turk. J. Math.* **35** (2011), 355-366.
2. S. Akbari and M. R. Oboudi, On the edge cover polynomial of a graph, *Europ. J. Combin.* **34** (2013), 297-321.
3. J. A. Bondy and U. S. R. Murty, *Graph Theory*, Graduate Texts in Mathematics, 244. Springer, New York, 2008.
4. P. Erdős, A. Rényi and V. T. Sós, On a problem of graph theory, *Studia Sci. Math. Hungar.*, **1** (1966), 215-235.
5. R. Frucht and F. Harary, On the corona of two graphs, *Aequationes Mathematicae*, **4** (1970), 322-325.
6. M. Grohe and D. Marx, *Constraint solving via fractional edge covers*, in: Proceedings of the Seventeenth Annual ACM SIAM Symposium on Discrete Algorithm, SODA 06, New York, NY, USA, 2006, pp. 289-298.
7. G. B. Khosrovshahi, Ch. Maysoori and B. Tayfeh-Rezaie, A Note on 3-Factorizations of K_{10} , *J. Combin. Designs*, **9** (2001), 379-383.

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