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A new algorithm for optimum open pit design: Floating cone method III

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Abstract
Ultimate limits of an open pit, which define its size and shape at the end of the mine's life, is the pit with the highest profit value. A number of algorithms such as floating or moving cone method, floating cone method II and the corrected forms of this method, the Korobov algorithm and the corrected form of this method, dynamic programming and the Lerchs and Grossmann algorithm based on graph theory have been developed to find out the optimum final pit limits. Each of these methods has special advantages and disadvantages. Among these methods, the floating cone method is the simplest and fastest technique to determine optimum ultimate pit limits to which variable slope angle can be easily applied. In contrast, this method fails to find out optimum final pit limits for all the cases. Therefore, other techniques such as floating cone method II and the corrected forms of this method have been developed to overcome this shortcoming. Nevertheless, these methods are not always able to yield the true optimum pit. To overcome this problem, in this paper a new algorithm called *floating cone method III* has been introduced to determine optimum ultimate pit limits. The results show that this method is able to produce good outcome.

Keywords: *Open pit mining; Ultimate pit limit; Floating cone method; Floating cone method II; Floating cone method III*

1. Introduction

Open pit mining is an important general mining method that mineral deposit will be mined via pits. The shape of mining area at the end of mining operation or final limits of a mine must be designed before starting the operation. According to the designed final pit limits, mining operational parameters such as width, length and depth of mined pit, opening track ways, location of waste dump, stripping ration, mine life, minable ore tonnage, waste tonnage and production scheduling can also be determined [1].

Optimum pit limits are usually designed with the use of the block models. Geological block model, which presents the reserve as a combination of numerous small blocks, is determined by inverse distance or geostatistical methods. Then the economical block model is calculated by applying cost, price and other parameters to each block. In this model ore blocks have positive values, waste blocks have negative values and air blocks, and the blocks over the surface topography have zero values. Most of the optimum pit limits methods use the economical block models to determine the pit limits. The methodology is searching for a combination of blocks with the maximum economical value at current economical and technical condition [2].

Floating or moving cone method [3], floating cone II method [4], modified floating cone II methods [5], dynamic programming [6], [7] and [8], the Lerchs and Grossmann algorithm based on graph theory [9], Korobov algorithm [10], corrected form of the Korobov algorithm [11] and genetic algorithm [12] are some of the several algorithms

developed to determine optimum pit limits. Each of these methods has special advantages and disadvantages. For example, dynamic programming is just a 2D modeling method and although the Lerchs and Grossmann algorithm can always create an optimized and true limit of the pit, the method is too complicate to be applied easily.
Floating or moving cone algorithm is one of the easiest and fastest algorithms for determining the final pit limits. In addition, mining operational restrictions on various slopes can be applied to this method perfectly. Further, a few algorithms such as floating cone II method and modified floating cone II method are developed to overcome the shortcoming of this method. In despite of these corrections, these methods are not able to obtain the true optimum limit of the pit. Therefore it needs more corrections. This paper presents a new edition of the algorithm, called floating cone III method, in order to cover shortcomings of previous methods.

2- An overview on floating cone algorithms

2-1- Floating cone method

This method, which was first described by Carlson, Erickson, O'Brain and Pana (1966), works on an economical block model of the deposit [3]. For each positive (ore) block, this method involves constructing a cone with sides oriented parallel to the pit slope angles, and then determining the value of the cone by summing the values of blocks enclosed within it. If the value of the cone is positive, all blocks within the cone are mined. This process starts from the uppermost level and moves downward searching for positive blocks. The process continues until no positive cones remain in the block model. Although this algorithm is simple and easy to understand, it is not able to yield a true optimum pit limit. For example, for the 2-D economical block model in Figure 1, when final dip of pit is 1:1, floating cone algorithm cannot produce true optimum pit limit, as presented on Table 1. Nevertheless, by applying dynamic programming method [6] to this model an optimum pit limit with the value of 2 would be obtained (Figure 2).

	1	2	3	4	5	6	7	8	9
1	-3	-3	-3	-3	-3	-3	-3	-3	-3
2	-4	7	-4	6	-4	-4	-4	8	-4
3	-5	-5	-5	15	-5	-5	-5	-5	-5

Figure 1. 2-D economical block model

	1	2	3	4	5	6	7	8	9
1	-3	-3	-3	-3	-3	-3	-3	-3	-3
2	-4	7	-4	6	-4	-4	-4	8	-4
3	-5	-5	-5	15	-5	-5	-5	-5	-5

Figure 2. Optimum pit limit by dynamic programming method

Table 1. Cone value for example shown in Figure 1

Stage	Block No	Block value	Cone value	Minable?
1	(2,2)	+7	-2	No
2	(2,4)	+6	-3	No
3	(2,8)	+8	-1	No
4	(3,4)	+15	-2	No

2-2- Floating cone II algorithm

Floating cone II algorithm was introduced and presented by Wright in 1999 [4]. The methodology is similar to the floating cone approach except that first values of the cone of all ore blocks are calculated in each level and the cone with maximum value is removed from the block model. Next cumulative pit value is calculated and this process is carried out for remaining ore blocks. Then all the extraction cones of the block with highest cumulative pit value are included as a member of the optimum solution set. For the block model shown in Figure 1, when final dip of pit is 1:1, the result of this algorithm is illustrated in Table 2 and Figure 3, with the value of -3. A true optimum pit limit of this model is shown Figure 2 with the value of +2.

Table 2. Cumulative pit value for example shown in Figure 1

Level	Block No	Block value	Cone value	Cumulative value	Minable ?
	(2,8)	+8	-1	-1	Yes
2	(2,2)	+7	-2	-3	No
	(2,4)	+6	0	-3	No
3	(3,4)	+15	-2	-2	Yes

	1	2	3	4	5	6	7	8	9
1	-3	-3	-3	-3	-3	-3	-3	-3	-3
2	-4	7	-4	6	-4	-4	-4	8	-4
3	-5	-5	-5	15	-5	-5	-5	-5	-5

Figure 3. Optimum pit limit by floating cone II algorithm

2-3- Modified floating cone II, method 1

This algorithm is the same as the algorithm of floating cone II method except that when the highest cumulative pit value is positive, the blocks are added as part of the optimum pit solution [5]. Therefore using this method, for the block model shown in Figure 1, as illustrated in Table 2 3, no blocks would be mined. Nevertheless, the true optimum limit for this model is shown in Figure 2 with the value of +2.

2-4- Modified floating cone II, method 2

This method is a development of first method of modified floating cone II algorithm [5]. In the second method all levels are considered together. In other words, the value of the cones of all ore blocks are evaluated economically and the cone with maximum value is assumed to be as part of the pit limits and the cumulative pit value is calculated. This process is then continued until no positive block remains in the block model. Finally, the block with positive and maximum cumulative value and all other previous blocks are included as a member of the optimum solution set. As illustrated in Table 4 and Figure 4, this algorithm yields a pit limit with the value of +1 for the block model shown in Figure 1.

3- Floating cone III

Although the floating cone II method and its corrections overcomes some weaknesses of the floating cone method, in some circumstances these methods fail to yield a true optimum pit limit and therefore a new development is needed. In general, blocks in economical block models could be divided into two groups of dependent and independent blocks. The ore blocks which have no common overlying block with other ore blocks in their extraction cones are independent otherwise they are classified as dependent. Each of these groups with regard to the value of their extraction cones are also classified as effective or ineffective. Effective blocks have positive value and ineffective blocks have negative value. The optimum pit limit is determined according to the cones on effective blocks. A flowchart of the floating cone III method is shown in Figure 5. Stages of the algorithm are as follows:

3-1- The algorithm is the same as the floating cone algorithm except that after extraction of each mining cones, search for other limits would be continued from the first level for remaining blocks. The aim of this stage is finding independent effective blocks in economical model.

3-2- Finding ore blocks from the first level of economical block model to the other levels. If any ore block is found in any level, other ore blocks are considered from this level to the first level. The aim of this stage is to check the effect of levels on each other.

3-3- Constructing extraction cones for all ore blocks, with regard to the technical restrictions, then finding dependent and independent blocks.

3-4- Finding ineffective and independent blocks which have no positive values. If positive values are assigned against negative valued blocks, the effect of such blocks on optimum pit limit will be removed.

3-5- Finding ineffective and dependent blocks which have no positive values. If positive values are assigned against uncommon and negative overlying blocks, the effect of such blocks on optimum pit limit will be removed or decreased.

3-6- Finding effective and dependent blocks, these are all remaining blocks after carrying out the foregoing stages. Finding optimum pit limit is as follows:

3-6-1- Identifying common blocks for each of extraction cone and then calculating their weights, The weight of each block is equal to the number of cones enclosed within it.

Table 3. Cumulative pit value by floating cone II, method 1

Level	Block No	Block value	Cone value	Cumulative value	Minable ?
	(2,8)	+8	-1	-1	No
2	(2,2)	+7	-2	-3	No
	(2,4)	+6	0	-3	No
3	(3,4)	+15	-2	-2	No

Table 4: Cumulative pit value by floating cone II, method 2

Stage	Block No	Block value	Cone value	Cumulative value	Minable?
1	(2,8)	+8	-1	-1	Yes
2	(2,2)	+7	-2	-3	Yes
3	(3,4)	+15	+4	+1	Yes

	1	2	3	4	5	6	7	8	9
1	-3	-3	-3	-3	-3	-3	-3	-3	-3
2	-4	7	-4	6	-4	-4	-4	8	-4
3	-5	-5	-5	15	-5	-5	-5	-5	-5

Figure 4. Optimum pit limit by floating cone II, method 2

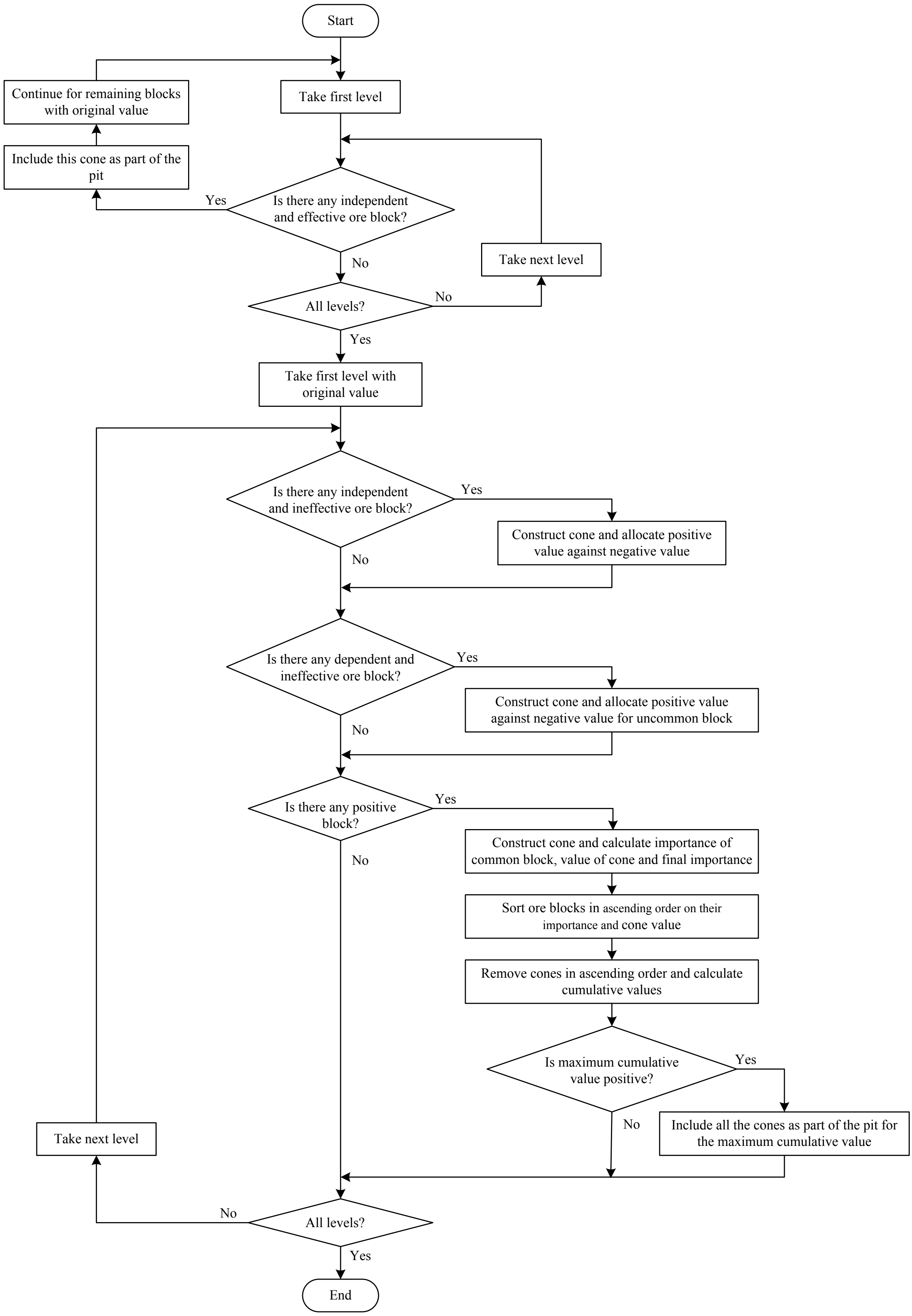

Figure 5. Flowchart of Floating Cone III algorithm

3-6-2- Calculating the weight of mining cones. It is the sum of weight of common blocks of the cone.
3-6-3- Calculating the cone values for each of mining cone.
3-6-4- Calculating the final importance for each of mining cone.Final importance is the ratio of the weight of cone to the absolute value of its cone value.
3-6-5- Sorting mineral blocks on final importance, then on value of mining cones in descending order and supposing extraction of first ore block.
3-6-6- Supposing extraction of other blocks would be continued according to fist block extraction. At this stage the value of other cones will be evaluated and the mining cone with maximum value will be extracted, a supposed extraction offcourse. The cumulative value of every supposed extraction must be calculated now.
3-6-7- Finding the maximum and positive cumulative value and determining optimum pit limit by including blocks from fist mining cone to this cone. Then we search for another limit from next levels by using original block model. If the maximum cumulative value is not positive, this means that there is no optimum limit to this level and the search will be continued from next level.

The floating cone III algorithm can best be explained by a simple example applied to an economical model shown in Figure 1. As shown below, this algorithm produces a final pit with the value of +2 (Figure 13) which is a true optimum pit limit.

a) The first level containing ore blocks is the second level and the algorithm applied for this level is as follows:

First stage: finding effective ore blocks. As can be seen in Table 1, since the extraction cone of all the ore blocks are negative, there is no independent effective block in the model.

Second stage: Removing independent and ineffective ore blocks from model, as shown in Figure 6.

	1	2	3	4	5	6	7	8	9
1	-3	-3	-3	-3	-3	-3	0	0	-1
2	-4	7	-4	6	-4	-4	-4	0	-4

Figure 6. Removing independent and ineffective ore blocks

Third stage: decreasing or removing the effect of dependent and ineffective blocks, as illustrated in Figure 7.

Forth stage: Determining the importance of common blocks. According to Figure 7, only one ore block remains. Therefore there is no common block here and the initial and final importance of this cone is 0, as is shown in Figure 8.

	1	2	3	4	5	6	7	8	9
1	0	0	-3	0	0	-3	0	0	-1
2	-4	1	-4	0	-4	-4	-4	0	-4

Figure 7Decreasing or removing the effect of dependent and ineffective blocks

	1	2	3	4	5	6	7	8	9
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0

Figure 8. Determining importance of common blocks

Fifth stage: Supposing the extraction of only remaining ore block which its value of extraction cone and its cumulative value of both are -2. Since the value is less than zero, this cone is not included as part of pit limit. In other words, there is no optimum limit to the second level. Hence the algorithm will be continued for the economical block with its original value as follows:

b) The second level with ore blocks is the third level and the stages of the algorithm for this level are as follows:

Sixth stage: finding effective ore blocks. As it can be seen from Table 1, there is no independent effective block in the model.

Seventh stage: Removing independent and ineffective ore blocks, as illustrated in Figure 9.

	1	2	3	4	5	6	7	8	9
1	-3	-3	-3	-3	-3	-3	0	0	-1
2	-4	7	-4	6	-4	-4	-4	0	-4
3	-5	-5	-5	15	-5	-5	-5	-5	-5

Figure 9. Removing independent and ineffective ore blocks

Eighth stage: decreasing or removing the effect of dependent and ineffective blocks, as shown in Figure 10.

	1	2	3	4	5	6	7	8	9
1	0	-3	-3	-3	-3	0	0	0	-1
2	-4	4	0	6	0	-4	-4	0	-4
3	-5	-5	-5	4	-5	-5	-5	-5	-5

Figure 10. Decreasing or removing the effect of dependent and ineffective blocks

Ninth stage: Determining the importance of common blocks, as shown in Figure 11.

	1	2	3	4	5	6	7	8	9
1	0	2	3	2	2	0	0	0	0
2	0	0	0	2	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0

Figure 11. Determining importance of common blocks

Tenth stage: Descending sort of ore blocks on their final importance of mining cones, and then on the value of cones, as shown in Table 5.

Table 5. Descending sort of the ore blocks

Ref	Block No	Block value	Final importance	Cone value
1	(3,4)	+4	5.5	-2
2	(2,4)	+6	3	-3
3	(2,2)	+4	2.5	-2

Eleventh stage: Supposed extraction of ore blocks as illustrated in Figure 12.

	1	2	3	4	5	6	7	8	9
1	-3						0	0	-1
2	-4	7				-4	-4	0	-4
3	-5	-5	-5		-5	-5	-5	-5	-5

Figure 12. First supposed limits

Twelfth stage: Finding maximum value of remaining blocks from sorted list and calculating their cumulative value (Table 6).

Table 6. Calculation of cumulative values

Ref	Block No	Block value	Cone value	Cumulative value	Minable ?
1	(3,4)	+4	-2	-2	Yes
2	(2,2)	+4	+4	+2	Yes

Thirteenth stage: With regard to Table 6, maximum cumulative value is positive. Therefore related mining cones is included as part of the pit and optimum limit is obtained with the value of +2 (Figure 13). The result of this algorithm is the same as the Dynamic programming method.

9	8	7	6	5	4	3	2	1	
-1	0	0	-3	-3	-3	-3	-3	-3	1
-4	0	-4	-4	-4	6	-4	7	-4	2
-5	-5	-5	-5	-5	15	-5	-5	-5	3

Figure 13.Final Optimum pit limit by floating Cone III

Example 2

In order to show the ability of the floating cone III algorithm, another simple example as shown in Figure 14, When final dip of pit is 1:1, is employed. Floating cone II algorithm and its modification produce a pit with value of +1 as illustrated in Figure 15, whereas floating cone III method as shown in Table 8 and Figure 16-c creates a true optimum pit with value of +2 as the same as the dynamic programming technique.

9	8	7	6	5	4	3	2	1	
-2	-2	-2	-2	-2	-2	-2	-2	-2	1
-4	-4	-4	-4	-4	-4	-4	-4	-4	2
-6	-6	17	-6	-6	16	14	-6	-6	3

Figure 14. An economical block model

Table 7. Stages of floating cone II and its modification algorithm

Level	Block No	Block value	Cone value	Cumulative value	Minable ?
	(3,7)	+17	-5	-5	Yes
3	(3,4)	+16	-2	-7	Yes
	(3,3)	+14	+8	+1	Yes

9	8	7	6	5	4	3	2	1	
-2	-2	-2	-2	-2	-2	-2	-2	-2	1
-4	-4	-4	-4	-4	-4	-4	-4	-4	2
-6	-6	17	-6	-6	16	14	-6	-6	3

Figure 15. Optimum pit limit by of floating cone II and its modification algorithm

9	8	7	6	5	4	3	2	1	
0	0	0	-2	-2	-2	-2	-2	0	1
-4	-1	0	0	0	-4	-4	0	-4	2
-6	-6	0	-6	-6	12	8	-6	-6	3

Figure 16-a. Dependent ineffective blocks removal- floating cone III algorithm

9	8	7	6	5	4	3	2	1	
0	0	0	2	2	2	2	2	0	1
0	0	0	0	0	2	2	0	0	2
0	0	0	0	0	0	0	0	0	3

Figure 16-b. Determining the importance of dependent and ineffective common blocks - floating cone III algorithm

9	8	7	6	5	4	3	2	1	
-2	-2	-2	-2	-2	-2	-2	-2	-2	1
-4	-4	-4	-4	-4	-4	-4	-4	-4	2
-6	-6	17	-6	-6	16	14	-6	-6	3

Figure 16-c. Optimum pit limit by floating cone III algorithm

3-7- Case study

In this section, floating cone algorithms are applied for a real data of a gold mine, located at 35 kilometers north-east of Sweden. First of all, an economical block model of this mine has been created using Pitwin32 software. This software with using grade block model and technical and economical parameters such as cut-off grade, dimension of blocks, ore and waste density, price,

cost and etc creates an economical block model of deposit.
The deposit is divided into 15m (east-west) x 10m (north-south) x 5m (vertical) blocks and the lock numbers bin the east west, north south and vertical directions are 101, 82 and 36 respectively. Each block is assigned the estimated (kriged) recoverable tonnage of ore above a cut-off grade and the estimated (kriged) average grade of this tonnage. Table 9 shows the overall results of this case study with the use of floating cone, floating cone II and its modifications methods. In addition, for implementation of the floating cone III method a C++ code was developed by using Visual C++ programming language. The result of this algorithm for this case is also shown in Table 9. It can be concluded from this table that compared with other floating cone methods, floating cone III produces a final pit limit with the highest value.

4- Summary

Although floating cone II algorithm and its modifications overcome some of the shortcomings of the floating cone method, for examples shown in this paper they produce a pit with less value and fail to determine true optimum pit limits. Since these methods do not take into account the effect of independent and dependent block to each other. On the other hand, the floating cone III method take into consideration this shortcoming and always create a pit with positive and high value. The algorithm is straightforward and using different pit slopes in different parts of the orebody is very simple.

Table 8. Descending sorting of dependent effective blocks - floating cone III algorithm

Ref	Block No	Block value	Final importance	Cone value	Cumulative value of cone	Minable?
1	(3,4)	+12	2.33	-6	-6	Yes
2	(3,3)	+8	1.5	+8	+2	Yes

Table 9. Overall results by different methods

Method	Number of blocks			Value (*10000)		
	Pit	Ore	Waste	Ore	Waste	Net
Floating cone	10053	4394	5660	76895.5	-11597.4	65298.1
Floating cone II	12351	4932	7419	82544.3	-15437.8	67106.5
Floating cone II- First modification	12351	4932	7419	82544.3	-15437.8	67106.5
Floating cone II- Second modification	13732	5341	8391	85182.1	-17374.7	67807.4
Floating cone III	15027	5584	9443	88138.1	-19553	68585.1
Lerchs& Grossmann	15030	5405	9625	89853.7	-20324.9	69528.8

References

[1] Khalokakaie, R., Dowd, P. A. and Fowell, R. J., 2000, "Lerchs-Grossmann algorithm with variable slope angles", Trans. Instn Min. Metall. (Sect. A: Min. industry), No. 109, p.p. A77-A85.

[2] Khalokakaie, R.; Dowd, P. A. and Fowell, R. J.; 2000; "A windows program for optimal open pit design with variable slope angles", International Journal of Surface Mining, Reclamation and Environment, 14, p.p. 261-275.

[3] Carlson, T. R.; Erickson, J. D.; O'Brain D. T. and Pana, M. T.; 1966; "Computer techniques in mine planning", Mining Engineering, Vol. 18, No. 5, p.p. 53-56.

[4] Wright, E. A.; 1999; "MOVING CONE II - A Simple Algorithm for Optimum Pit Limits Design", Proceedings of the 28th Symposium on the application of computers and operations research in the mineral industries (APCOM), (Colorado USA), p.p. 367-374.

[5] Khalokakaie, R. 2006; "Optimum open pit design with modified moving cone II methods", Journal of engineering in Tehran university, Vol. 4, No. 3 p.p 297-307 (in Persian).

[6]Koenigsberg E.; 1982; "The optimum contours of an open pit mine: an application of dynamic programming", Proceedings of the 17th Symposium on the application of computers and operations research in the mineral industries (APCOM), (New York: AIME), p.p. 247-287.

[7] Wilke, F. L. and Wright, E. A.; 1984; "Determining the optimal ultimate pit design for hard rock open pit mines using dynamic programming", Erzmetall, No. 37, p.p. 139-144.

[8] Yamatomi, J.; Mogi, G.; Akaike, A. and Yamaguchi, U.; 1995; "Selective extraction dynamic cone algorithm for three-dimensional open pit designs", Proceedings of the 25th Symposium on the application of computers and operations research in the mineral industries (APCOM), Brisbane, p.p. 267-274.

[9] Lerchs, H. and Grossmann, I. F.; 1965; "Optimumm design of open pit Mine"; CIM Bulletin, No. 58, pp. 47 – 54.

[10] David, M.; Dowd, P. A. and Korobov, S.; 1974; "Forecasting departure from planning in open pit design and grade control", Proceedings of the 12th Symposium on the application of computers and operations research in the mineral industries (APCOM), (Golden, Colo: Colorado School of Mines), Vol. 2, p.p. F131-F142.

[11] Dowd, P. A. and Onur, A. H.; 1993; "Open pit optimization - part 1: optimal open pit design", Trans. Instn Min. Metall. (Sect. A: Min. industry), No. 102, p.p. A95-A104.

[12] Denby, B. and Schofield, D.; 1994; "Open-pit design and scheduling by use of genetic algorithms", Trans. Instn Min. Metall. (Sect. A: Min. industry), No. 103, p. p. A21-A26.